

Analysis of a Dipole Antenna Using Maxwell-SCHRÖDINGER Equation

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맥스웰-슈뢰딩거 방정식을 이용한 다이폴 안테나 해석

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Abstract We present a quantitative analysis of a dipole antenna and its characteristics from the viewpoint of quantum mechanics. The method makes use of a Maxwell equation used in an existing antenna propagation formula. This includes radiation resistance, input reactance, and antenna efficiency as functions of frequency and antenna length. Particular attention is paid to the Schrödinger equation. We accomplish E-field and H-field analyses of a dipole antenna by combining the Maxwell and Schrödinger wave equations. When comparing the existing Maxwell wave equation with the Schrödinger wave equation, quantum-electric movement is more accurate than using the Maxwell wave equation alone.

요약 본 논문은 양자역학 관점으로 본 다이폴 안테나의 정량적 분석과 그것의 특성에 관한 논문이다. 분석 방법으로 현존하는 안테나 전파 방정식에 이용되는 맥스웰 방정식을 사용한다. 이는 안테나의 길이와 주파수에 관한 함수로, 방사저항, 입력 리액턴스, 안테나 효율을 포함한다. 본 논문의 주요 관심사는 슈뢰딩거 방정식이다. 또한 본 논문은 맥스웰과 슈뢰딩거 방정식을 결합하여 다이폴 안테나의 전계와 자계를 해석한다. 현존하는 맥스웰 방정식과 슈뢰딩거 방정식을 비교함으로써, 단일 맥스웰 방정식을 썼을 때 보다 양자-전기 이동의 정확성이 향상됨을 보인다.

Key Words : Dipole antenna, Maxwell equation, Schrodinger wave equation, Quantum mechanics

1. Introduction

Recently, various studies have conducted numerical analysis and experiments for a more accurate analysis of communication devices at microwave (gigahertz) frequencies. There has been a renewed interest in analyzing antenna propagation characteristics using quantum mechanics in order to take complex antenna environments into account. For instance,

unconventional materials with non-linear or homogeneous conductivity and permittivity properties are often combined with novel antennas [1-3]. Also, for an antenna placed in a complex electromagnetic environment, its radiation performance is significantly affected by the surrounding materials or an undesirable radiation source [4,5]. The classical wave equations (e.g., Maxwell equation, acoustic equation) are not sufficient to accurately analyze antenna fields in the

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mentioned environments. In order to overcome this limitation, numerical tools based on the method of moment (MoM), finite difference time domain (FDTD), and finite element method (FEM) are frequently used[6-9]. However, there is still demand for closed-form expressions for quick and physically intuitive analysis. For this reason, quantum mechanics has been attempted for antenna analysis [10]. The Schrödinger equation is the key equation in quantum mechanics. It can be effectively relevant to construct a wave function that can satisfactorily describe the probability of finding a freely traveling particle within a given space at a given time. We were introduced by Schrödinger equation for wave equation, unlikely the existing analysis methods.

2. Dipole antenna analysis by maxwell equation

2.1 Electric Field and Magnetic Field Intensity of Dipole Antenna

Based on the known current distribution, it is straightforward to calculate the radiated electric and magnetic fields. We follow Balanis [11]. For the current distribution of the along the z-axis, the electric field in the far field region must be in the θ direction, and is given by

$$E_{\theta} = i\eta \frac{ke^{-ikr}}{4\pi r} \sin \theta \left[\int_{-l/2}^{+l/2} I(z) e^{ikz \cos \theta} dz \right] \quad (1)$$

Here, η is the characteristic impedance of free space. For a traditional wire antenna, the current distribution is periodic with a wave vector. Therefore, the distribution of the total field is a product of element factor and space factor. We can express the electric field as

$$E_{\theta} = i\eta \frac{ke^{-ikr}}{4\pi r} \sin \theta \times \left\{ \int_{-l/2}^0 I_0 \sin \left[k \left(\frac{l}{2} + z \right) \right] e^{jkz \cos \theta} dz + \int_0^{l/2} I_0 \sin \left[k \left(\frac{l}{2} - z \right) \right] e^{jkz \cos \theta} dz \right\} \quad (2)$$

After evaluating Equation (2), the result is

$$E_{\theta} = i\eta \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos \left(\frac{kl}{2} \cos \theta \right) - \cos \left(\frac{kl}{2} \right)}{\sin \theta} \right] \quad (3)$$

The magnetic field component has been obtained in a similar way, as follows.

$$H_{\phi} = i \frac{I_0 e^{-jkr}}{2\pi r} \left[\frac{\cos \left(\frac{kl}{2} \cos \theta \right) - \cos \left(\frac{kl}{2} \right)}{\sin \theta} \right] = \frac{E_{\theta}}{\eta} \quad (4)$$

2.2 Vector Potential Energy of Dipole Antenna

To find the fields radiated by the current element, it is important to extract vector potential energy [11]. The vector potential (A) is useful in solving for the EM field generated by given harmonic electric current (J). By Maxwell's law, the magnetic flux (B) doesn't always radiate. Therefore, it can be represented as the curl of another vector. (i.e. vector potential (A))

$$\nabla \cdot \nabla \times A = 0 \quad (5)$$

Where, A is arbitrary vector form. Thus,

$$B_a = \mu H_a = \nabla \times A \quad (6)$$

Therefore, H_a can be represented.

$$H_a = \frac{1}{\mu} \nabla \times A \quad (7)$$

Here, subscript a indicates the field due to the A potential. Substituting (7) into Maxwell's curl equation

$$\nabla \times E_a = -j\omega \mu H_a = -j\omega \nabla \times A \quad (8)$$

which can be written as

$$\nabla \times [E_a + j\omega A] = 0 \quad (9)$$

It can be represented that

$$E_a + j\omega A = -\nabla \phi_e \quad (10)$$

The scalar function ϕ_e represents an arbitrary electric scalar potential which is a function of position.

$$\nabla \times (\mu H_a) = \nabla (\nabla \cdot A) - \nabla^2 A \quad (11)$$

Equating Maxwell's equation leads to

$$\mu J + j\omega\mu\epsilon E_a = \nabla(\nabla \cdot A) - \nabla^2 A \quad (12)$$

Eventually, we obtain wave equation.

$$\nabla^2 A + k^2 A = -\mu J + \nabla(\nabla \cdot A) + \nabla(j\omega\mu\epsilon\phi_e) \quad (13)$$

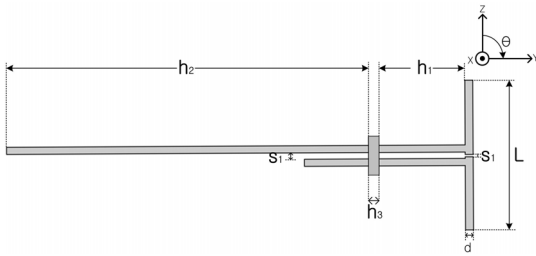
Where, $k^2 = \omega^2\mu\epsilon$.

Following Lorentz equation, we represent magnitude of potential energy.

$$A = \frac{\mu}{4\pi} \iiint_v J \frac{e^{-jkR}}{R} dv' \quad (14)$$

Therefore, we obtain potential energy of linear dipole antenna.

$$V(l) = \frac{\mu I_0}{4\pi} \int_{-l/2}^{+l/2} \frac{\sin[k(l/2) - |z'|]}{r} e^{-ikr} dz' \quad (15)$$



[Fig. 1] Proposed conventional dipole antenna

[Table 1] Dimension of the proposed Dipole Antenna

Symbol	Unit(mm)
L	70
D	3.6
h1	40.6
h2	180
h3	4
s1	0.5
s2	3

3. Introduction of The Schrödinger Equation

3.1 The one dimensional time-dependent Schrödinger equation

The Schrödinger equation is the main equation in

the analysis using the quantum mechanical model. The one-dimensional Schrödinger equation is used when the particle interest is confined to one spatial dimension. The particles traveling along the x-axis are assumed. To derive the one-dimensional Schrödinger equation, we start with the total energy equation (i.e., the sum of kinetic and potential energy [12-14].

$$\frac{p^2}{2m} + U(x) = E_{total} \quad (16)$$

Here, p is the momentum of the particle, m is the mass of the particle, $U(x)$ is the potential energy in the combined particle, and E_{total} is the total energy of the system achieved by combining particles.

The substitution of the dynamical variables with their quantum mechanical operator, which acts on the wave function $\Psi(x,t)$, yields the one-dimensional time-dependent Schrödinger equation:

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x,t) + U(x)\Psi(x,t) = -\frac{\hbar}{i} \frac{\partial}{\partial t} \Psi(x,t) \quad (17)$$

The left side of this equation can be rewritten using the Hamiltonian operator (or total energy operator).

$$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + U(x) \quad (18)$$

Using the Hamiltonian operator (H), formula (15) can be expressed as follows.

$$H\Psi(x,t) = -\frac{\hbar}{i} \frac{\partial}{\partial t} \Psi(x,t) \quad (19)$$

Since the Schrödinger equation is a partial differential equation, the product method can be used to separate the equation into spatial and temporal parts.

The wave function $\Psi(x,t)$ that describes the quantum state in time for the wave equation for the change is to make the particles are described as follows.

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r},t) = \left\{ -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}) \right\} \Psi(\vec{r},t) \quad (20)$$

3.2 Schrödinger equation in the spherical coordinate system

If the potential of the physical system to be examined is spherically symmetric, then the Schrödinger equation in spherical polar coordinates can be used to advantage. For a three dimensional problem, Laplacian in spherical polar coordinates is used to express the Schrödinger equation in the condensed form

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \Psi}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \Psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Psi}{\partial \phi^2} - \frac{2m}{\hbar^2} (V(r) - E) \Psi = 0 \quad (21)$$

4. Hamiltonian based in The Classical Electromagnetic Wave

In this section, we accomplish to substitute the quantum mechanical momentum for the classical. Here, we can think Lorentz gauge and Coulomb gauge about the electromagnetic wave.

In classical physics, gauge transformation is the invariance of the fields [15-16]. When the charged particle (electron ; e) moves in an electric field, the particle is taken to the Lorentz force.

$$\vec{F} = e \left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right) \quad (22)$$

In case of the absence of electromagnetic fields, the Hamiltonian that describes the movement of the charged particles of mass m is

$$H = \frac{\hat{p}^2}{2m} \quad (23)$$

The \hat{p} and H are expressed within electromagnetic field,

$$\begin{aligned} \hat{p} &\rightarrow \hat{p} - \frac{e}{c} \vec{A} \\ H &\rightarrow H - e\phi \end{aligned} \quad (24)$$

Therefore, Hamiltonian is

$$H = \frac{1}{2m} \left(\hat{p} - \frac{e}{c} \vec{A} \right)^2 + e\phi \quad (25)$$

From the Hamiltonian, Schrödinger Equation is

$$i\hbar \frac{\partial \Psi(\vec{r}, t)}{\partial t} = \frac{1}{2m} \left[\left(\frac{\hbar}{i} \nabla - \frac{e}{c} \vec{A}(\vec{r}) \right)^2 + e\phi(\vec{r}) \right] \Psi(\vec{r}, t) \quad (26)$$

If the Schrödinger equation were invariant by gauge transformation, wave equation $\Psi(\vec{r}, t)$ is phase transformation. Therefore, we were supposed to be able to phase transformation $\Psi(\vec{r}, t)$.

$$\psi(\vec{r}, t) = e^{-\frac{ie}{\hbar c} \lambda(\vec{r}, t)} \Psi(\vec{r}, t) \quad (27)$$

If charged particles were moved in the electromagnetic field, we have to accomplish gauge transformation in order to have the same physical meaning Schrödinger equation. Therefore, Schrödinger equation is

$$i\hbar \frac{\partial \psi(\vec{r}, t)}{\partial t} = \left[\frac{1}{2m} \left(\frac{\hbar}{i} \nabla - \frac{e}{c} \vec{A} \right)^2 + e\phi \right] \psi(\vec{r}, t) \quad (28)$$

5. Hamiltonian based on The Poynting Vector and Analysis of Dipole Antenna by The Maxwell-Schrödinger Equation

Antenna radiation characteristics are converted into spherical coordinates. So, it is important that antenna propagation analysis represents using the spherical system of the Schrödinger equation [11-14]. Quantized Hamiltonian of electromagnetic waves is electromagnetic waves with an average pointing vector [12-14].

$$H = \frac{1}{2} \int_V \left(\epsilon_0 |E|^2 + \mu_0 |H|^2 \right) dV \quad (29)$$

Therefore, when the propagation produces total system energy, the dipole antenna with radiation can be expressed as follows.

$$E_{total} = \frac{I_0^2 \eta}{8\pi^2 r^2} \times \left[\sin \theta \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{1 - \cos^2 \theta} \right]^2 \quad (30)$$

Finally the dipole antenna using the Schrödinger equation can be expressed as follows.

$$\begin{aligned} & \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \Psi \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \Psi \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \Psi}{\partial \phi^2} \\ & = \frac{2m}{\hbar^2} \left(\frac{\mu I_0}{4\pi} \int_{-l/2}^{l/2} \frac{\sin[k(l/2) - |z|]}{r} e^{-ikr} dz - \frac{I_0^2 \eta}{8\pi^2 r^2} \times \left[\sin \theta \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{1 - \cos^2 \theta} \right]^2 \right) \Psi \end{aligned} \quad (31)$$

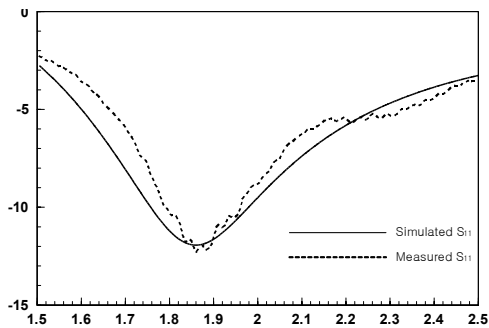
Where, Ψ is need to revise Ψ_E , Ψ_H to accomplish E-field and H-field.

$$\begin{aligned} \Psi_E &= \sum \left(i\eta \frac{I_0 e^{-ikr}}{2\pi r} \sin \theta \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{1 - \cos \theta} \right) \\ \Psi_H &= \sum \left(i \frac{I_0 e^{-ikr}}{2\pi r} \sin \theta \frac{\cos\left(\frac{kl}{2} \cos \theta\right) - \cos\left(\frac{kl}{2}\right)}{1 - \cos \theta} \right) \end{aligned} \quad (32)$$

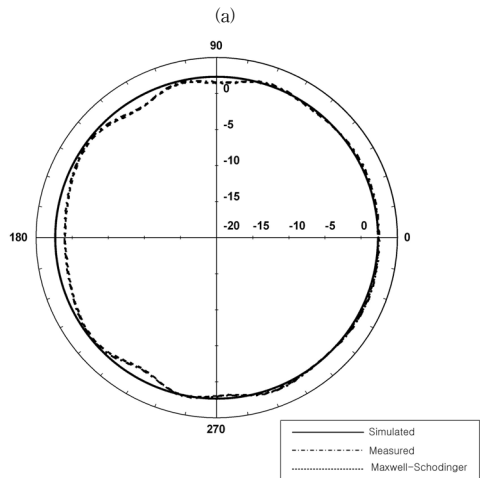
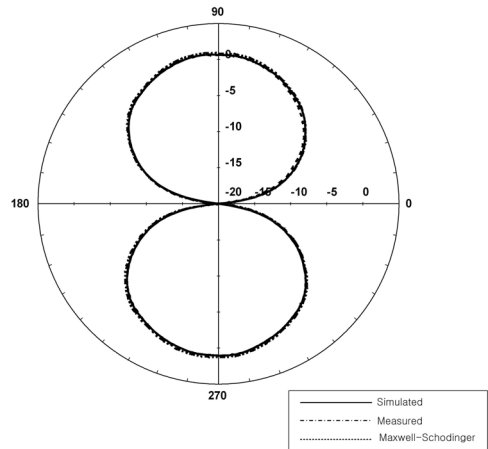
and m is mass of electron.



[Fig. 2] Fabricated Conventional Dipole Antenna



[Fig. 3] Measured and Simulated S-parameter of the Dipole Antenna.



(a) E-Field Pattern (@1.85GHz_Horizental)
(b) H-Field Pattern (@1.85GHz_Vertical)

[Fig. 4] E-H field pattern pattern of the Dipole Antenna (@1.85GHz) parameter of the Dipole Antenna.

6. Demonstration of Maxwell-Schrödinger Equation

The equation (32) is existing and other expression on the propagation of the dipole antenna. This is combination type of Maxwell's equation and Schrödinger equation. So as to prove equation (32), we proposed dipole antenna, as shown in Fig.1 and each of the length information is summarized in table.1. Also,

Fig. 2 is fabricated conventional dipole antenna. The simulated and measured return losses (S11) of conventional dipole antenna are shown in Fig.3. In order to derive equation (32), we accomplish E-plane and H-plane plot in the center frequency (@ 1.85). If you plot the equation (32) at the resonance frequency (@1.85GHz), you will be expressed as Fig.4. As shown, the E-plane and H-plane patterns which are very similar to simulation, measurement, and induced combining the Maxwell equation and the Schrödinger equation that is Maxwell-Schrödinger equation.

7. Conclusion

The Maxwell equation is fundamental in interpreting the movement of electromagnetic waves. The Schrödinger equation is based on treating wave movements that generate particles during exercise. In this paper, to analyze the dipole antenna, its propagation is derived using a formula that combines the Maxwell and Schrödinger equations. To verify the equation, the E/H-fields were plotted in the far field. As a result, the derived E/H-field patterns of the Maxwell-Schrödinger equation formula and the E/H-field patterns of measurement are similar. In the future, it will be possible to fabricate antennas using non-linear material.

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