Optimal Adaptive Multiband Spectrum Sensing in Cognitive Radio Networks

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Abstract

In this paper, optimal sensing time allocation for adaptive multiband spectrum sensing-transmission procedure is investigated. The sensing procedure consists of an exploration phase and a detection phase. We first formulate an optimization problem to maximize the throughput by designing not only the overall sensing time, but also the sensing time for every stage in the exploration and detection phases, while keeping the miss detection probability for each channel under a pre-defined threshold. Then, we transform the initial non-convex optimization problem into a convex bilevel optimization problem to make it mathematically tractable. Simulation results show that the optimized sensing time setting in this paper can provide a significant performance gain over the previous studies.

Keywords: Cognitive radio, adaptive spectrum sensing, multiband sensing, sensing-throughput tradeoff, bilevel optimization

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1. Introduction

The explosive increase in the wireless service demand has made radio spectrum scarcity a serious problem. Cognitive radio has been proposed to improve the spectrum utilization by allowing secondary users (SUs) to opportunistically access the vacant frequency bands [1]. Since a cognitive radio network is designed to be aware of its surroundings, it is necessary for SUs to sense whether primary users (PUs) are active or not. Spectrum sensing is a critical task for cognitive radio system mainly due to noise, channel fading and shadowing [2][3][4].

Recently, sequential spectrum sensing strategy has received growing attention [5][6], in which the SUs sequentially sense the channels according to a pre-defined order and stop to sense when specific criterions are met. Sequential sensing strategy only senses one channel at a time. However, when the bandwidth is wide and the channel occupancy rate is high, the sequential sensing strategy needs a large amount of sensing resources causing the sensing efficiency to be greatly reduced. An adaptive multiband sensing approach has been proposed for rapidly identifying multiple spectrum holes in wideband cognitive radio network. The sensing method consists of two phases, a exploration phase and a detection phase [7]. In the exploration phase, the size of candidate idle channel set is reduced by excluding channels that are likely to be occupied by the PUs. In the detection phase, the final detection is performed to determine the idle channels. The sensing samples are distributed to focus the limited sensing resources on the more promising channels. This adaptive sensing strategy provides a significant performance gain under the scenario that holes are sparsely scattered across the wideband spectrum. However, the exploration phase in the adaptive sensing approach lengthens the sensing time, which accordingly decreases the transmission time. Consequently, there exists a sensing-throughput tradeoff in sensing time setting and transmission time setting. In [8], the optimal tradeoff is investigated so as to optimally utilize the transmission opportunities in a single channel for a single SU. In [9]-[12], the optimal tradeoff of cooperative sensing with multiple SUs are studied in a single channel.

In this paper, we optimize both the overall sensing time and the sensing time for every stage in the exploration and detection phases for multiband spectrum sensing. An optimization problem is formulated to maximize the average achievable throughput by jointly designing the overall sensing time, and the sensing time for every stage in the exploration and detection phases, while keeping the miss detection probability for each channel under a pre-defined threshold. The initial non-convex optimization problem is further transformed into a convex bilevel optimization problem to make it mathematically tractable. Simulations show that compared with the existing studies, the proposed sensing time setting can provide a significant throughput performance gain.

The rest of this paper is organized as follows. In Section II, the system model is given. In Section III, we formulate the optimization problem and prove that the problem is a convex bilevel problem. In Section V, we provide simulation results to demonstrate the performance. Finally, conclusions are drawn in Section VI.

2. System Model

A cognitive radio system with N channels is considered. Each channel has a bandwidth of W. Time is divided into slots, each with a fixed length T. In each slot, the primary user in the channel is assumed to be either active or idle for the whole slot. The channels among primary and secondary users are assumed to remain unchanged within each slot.

For channel n, we have the following hypotheses:

$$\begin{aligned} \mathrm{H}_n^0 &: y_n(m) = w_n(m), \\ \mathrm{H}_n^1 &: y_n(m) = s_n(m) + w_n(m), \end{aligned} \qquad n = 1, 2, \ldots, N. \tag{1}$$

where H_n^0 and H_n^1 mean that the primary user in channel n is idle and busy respectively, m is the sample index, $y(.)$ is the received signal of channel n at the secondary user. $w_n(.)$ is the additive white Gaussian noise with variance σ_w^2 corresponding to the channel n. $s_n(.)$ is PU's signal sample at the channel n and is assumed to be independent and identically distributed with variance $(\sigma_s^n)^2$. Denote $\gamma_n = \frac{(\sigma_s^n)^2}{\sigma_w^2}$ as the received signal-to-noise (SNR) in channel n.

In the adaptive multiband spectrum sensing method, each slot includes two phases: a multi-stage exploration phase and a detection phase [7]. The exploration phase is essentially a coarse sensing process, and it consists of K iteration stages. In each stage, the size of candidate idle channel set is reduced by excluding channels that are likely to be occupied by the PU. We denote I_k as the set of surviving channels at the end of k-th stage, then after the k-th stage, the number of remaining channels is $N_k = |I_k| = N\alpha^k$, where $0 < \alpha < 1$ is called the distillation ratio, which is the percentage of channels that survive at each stage. **Fig. 1** shows the adaptive multiband spectrum sensing structure.

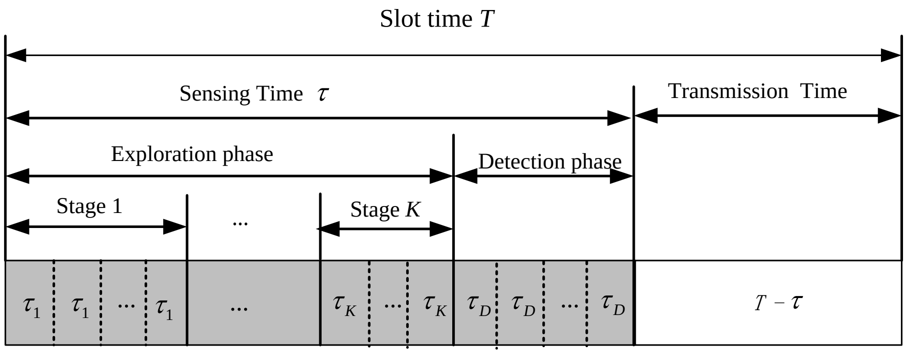

Fig. 1. Adaptive multiband spectrum sensing structure

The statistic of the received energy in channel n at the k-th stage is

$$\Gamma_n^k = \Gamma_n^{k-1} + \sum_{m=1}^{\mu\tau_k} |y_n(m)|^2, \quad n \in I_{k-1} \tag{2}$$

where μ is the sampling rate of the received signal, τ_k is the sampling time for per channel at the k-th stage. (2) is equivalent to $\Gamma_n^k = \sum_{m=1}^{\mu T_k} |y_n(m)|^2$, $n \in I_{k-1}$, where $T_k = \sum_{i=1}^{k} \tau_i$. Sort the $N_{k-1} = |I_{k-1}|$ energy values $\{\Gamma_n^k, n \in I_{k-1}\}$ in ascending order, to obtain N_k channels with the smallest energy values as the new set of surviving channels I_k. Although the new set is obtained by comparing the energy values between the N_{k-1} channels, the decision process at the k-th stage can be seen as comparing the statistic in (2) with a virtual threshold λ_n^k. The probability distribution function (pdf) of Γ_n^k in channel n at the k-th stage is

$$\begin{aligned}&\mathrm{H}_n^0 : \Gamma_n^k \sim \mathcal{N}\left(\sigma_w^2, \sigma_w^4/(\mu T_k)\right),\\&\mathrm{H}_n^1 : \Gamma_n^k \sim \mathcal{N}\left(\sigma_w^2 + (\sigma_s^n)^2, \left(\sigma_w^2 + (\sigma_s^n)^2\right)^2 \big/ (\mu T_k)\right),\end{aligned} \tag{3}$$

where $\mathcal{N}$ denotes Gaussian distribution. So, the false alarm probability (i.e., the probability that, under hypothesis H_n^0, the SU falsely declares that the primary signal is active) of channel *n* at the *k*-th stage is given as

$$P_{n,k}^{fa} = \Pr(\Gamma_n^k > \lambda_n^k \mid \mathrm{H}_n^0) = Q\left((\frac{\lambda_n^k}{\sigma_w^2} - 1)\sqrt{\mu T_k} \right) \tag{4}$$

and miss detection probability (i.e., the probability that, under hypothesis H_n^1, the SU falsely declares that the primary signal is inactive) of channel *n* at the *k*-th stage is given as

$$P_{n,k}^{md} = \Pr(\Gamma_n^k \le \lambda_n^k \mid \mathrm{H}_n^1) = 1 - Q\left((\frac{\lambda_n^k}{\sigma_w^2} - \gamma_n - 1)\sqrt{\mu T_k / (\gamma_n + 1)^2} \right) \tag{5}$$

where $Q(.)$ is the Q function, defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp(-\frac{z^2}{2}) dz .$$

At the end of the exploration phase, a small subset of the original *N* channel is obtained, for which the final detection to determine the identified idle channels is performed. In the detection phase, we update the energy of each channel in $\mathcal{I}_k$ as

$$\Gamma_n^D = \Gamma_n^K + \sum_{m=1}^{\mu\tau_D} | y_n(m) |^2 \quad n \in \mathcal{I}_k \tag{6}$$

where τ_D is the sampling time for each channel in $\mathcal{I}_k$ during the detection phase. (6) is equivalent to $\Gamma_n^D = \sum_{m=1}^{\mu T_D} | y_n(m) |^2$, $n \in \mathcal{I}_K$, where $T_D = \tau_D + \sum_{i=1}^K \tau_i$. The candidate set of idle channels is $\mathcal{I}_D = \{n \in \mathcal{I}_K : \Gamma_n^D \le \lambda_n^D\}$, where λ_n^D is the decision threshold. Similar to the *k*-th stage, we can get the false alarm probability $P_{n,D}^{fa}$ and miss detection probability $P_{n,D}^{md}$ of the detection phase as

$$P_{n,D}^{fa} = \Pr(\Gamma_n^D > \lambda_n^D \mid \mathrm{H}_n^0) = Q\left((\frac{\lambda_n^D}{\sigma_w^2} - 1)\sqrt{\mu T_D} \right) \tag{7}$$

and

$$P_{n,D}^{md} = \Pr(\Gamma_n^D \le \lambda_n^D \mid \mathrm{H}_n^1) = 1 - Q\left((\frac{\lambda_n^D}{\sigma_w^2} - \gamma_n - 1)\sqrt{\mu T_D / (\gamma_n + 1)^2} \right) \tag{8}$$

The overall false alarm probability of channel *n* is seen as the probability that the SU falsely declares that the primary signal is active in every stage of the exploration phase and the detection phase under hypothesis H_n^0. The overall miss detection probability of channel *n* is seen as the probability that the SU falsely declares that the primary signal is inactive in every stage of the exploration phase and the detection phase under hypothesis H_n^1. The overall false alarm probability and the overall miss detection probability are calculated as

$$P_n^{fa} = 1 - (1 - P_{n,D}^{fa}) \prod_{k=1}^K (1 - P_{n,k}^{fa}) \tag{9}$$

and

$$P_n^{md} = P_{n,D}^{md} \prod_{k=1}^{K} P_{n,k}^{md}, \tag{10}$$

which are similar to the OR-Rule in the cooperative sensing scheme.

When channel n is indeed free and is detected to be free, the achievable transmission rate of channel n is $C_n^0 = \log_2(1+P_s/N_0)$, where P_s is the received power of the secondary user and N_0 is the noise power [8]. When channel n is busy and is detected to be free, the achievable transmission rate of channel n is $C_n^1 = \log_2(1+P_s/(P_p+N_0))$, where P_p is the interference power of primary user measured at the secondary receiver [8]. Then, the average throughput of secondary user is given as

$$C = (1-\frac{\tau}{T})\sum_{n=1}^{N}[\Pr(\mathrm{H}_n^0)(1-P_n^{fa})C_n^0 + \Pr(\mathrm{H}_n^1)P_n^{md}C_n^1], , \tag{11}$$

where τ is the overall sensing time and $\tau = \tau_D N\alpha^K + \sum_{i=1}^{K}(\tau_i N\alpha^{i-1})$. Here we consider the scenario that all the candidate channels for the secondary user are licensed to a single primary user (e.g. a base station of cell networks), which means that the SNR of every candidate channel is approximately the same. For simplicity, we assume that the active probability is equal in all the channels, which means that $\gamma_n = \gamma$, and $\Pr(\mathrm{H}_n^0) = \Pr(\mathrm{H}_0)$, $\Pr(\mathrm{H}_n^1) = \Pr(\mathrm{H}_1)$, $P_{n,k}^{fa} = P_k^{fa}$, $P_{n,k}^{md} = P_k^{md}$, $P_n^{fa} = P^{fa}$, $P_n^{md} = P^{md}$, $\lambda_n^k = \lambda_k$, $\lambda_n^D = \lambda_D$, $C_n^0 = C_0$, $C_n^1 = C_1$. Therefore, the throughput in (11) is equivalent to

$$C = (1-\frac{\tau}{T})N\left[\Pr(\mathrm{H}_0)(1-P_D^{fa})\prod_{k=1}^{K}(1-P_k^{fa})C_0 + \Pr(\mathrm{H}_1)P_D^{md}\prod_{k=1}^{K}P_k^{md}C_1\right], \tag{12}$$

3. Optimal Sensing Time Setting for Multi-stage Exploration Phase and Detection Phase

In this section, an optimal sensing time allocation problem for multi-stage exploration phase, the detection phase and overall sensing procedure is formulated and addressed resulting in maximizing the average throughput. The miss detection probability in each channel should be smaller than a threshold that is denoted P_{th} so as to protect the activities of primary users. Then, the problem can be formulated as

$$\max_{\tau,\{\tau_k\},\tau_D} \quad C_P = (1-\frac{\tau}{T})$$

$$\cdot[\Pr(\mathrm{H}_0)(1-P_D^{fa}(T_D,\lambda_D))\prod_{k=1}^{K}(1-P_k^{fa}(T_k,\lambda_k))C_0 \tag{13a}$$

$$+\Pr(\mathrm{H}_1)P_D^{md}(T_D,\lambda_D)\prod_{k=1}^{K}P_k^{md}(T_k,\lambda_k)C_1]$$

$$\text{s.t.} \quad P^{md} = P_D^{md}(T_D,\lambda_D)\prod_{k=1}^{K}P_k^{md}(T_k,\lambda_k) \le P_{th}, \tag{13b}$$

$$T_D = \tau_D + \sum\nolimits_{i=1}^{K}\tau_i,\; T_k = \sum\nolimits_{i=1}^{k}\tau_i,\; k=1,2,...,K \tag{13c}$$

$$\tau = \tau_D N\alpha^K + \sum_{i=1}^{K}(\tau_i N\alpha^{i-1}) \tag{13d}$$

$$\tau_D \geq 0, \tau_i \geq 0. \ i = 1, 2, ..., K \tag{13e}$$

The problem above is not a convex problem. To solve it, we use the bilevel optimization [13][14], in which the lower level problem is to optimize $\{\tau_1, \tau_2, ..., \tau_K, \tau_D\}$ with a fix τ, whereas the upper level problem is to optimize the overall sensing time τ. Specifically, the lower level problem is

$$\begin{aligned} PL1: \max_{\{\tau_k\},\tau_D} \quad U_P = & [\Pr(\mathrm{H}_0)(1 - P_D^{fa}(T_D, \lambda_D)) \prod_{k=1}^{K} (1 - P_k^{fa}(T_k, \lambda_k)) C_0 \\ & + \Pr(\mathrm{H}_1) P_D^{md}(T_D, \lambda_D) \prod_{k=1}^{K} P_k^{md}(T_k, \lambda_k) C_1] \end{aligned}, \tag{14}$$

which is subject to the constraints (13b)-(13e).

Lemma 1: The objective function U_p in problem PL1 achieves the maximal value when $P^{md} = P_{th}$.

Proof: In the multi-stage exploration phase, at the k-th stage, we select $N_k = \alpha N_{k-1}$ channels as the surviving channels from N_{k-1} candidate channels. Because the idle channels are sparsely distributed among a large number of channels, the miss detection probability at the k-th stage is approximately calculated as $P_k^{md}(T_k, \lambda_k) \approx \frac{N_{k-1}}{N_k} = \alpha$. Using (4) and (5), the false alarm probability at the k-th stage can be written as a function of the detection probability

$$P_k^{fa}(T_k, \lambda_k) = Q\left((\gamma + 1)Q^{-1}(1 - \alpha) + \gamma\sqrt{\mu T_k}\right). \tag{15}$$

It can be seen that the false alarm probability and the miss detection probability at the k-th stage are irrelevant to the threshold λ_k. There are K stages in the exploration phase. From (9) and (10), we can obtain the overall miss detection probability and the overall false alarm probability as

$$P^{md} = \alpha^K P_D^{md}(T_D, \lambda_D) \tag{16}$$

and

$$P^{fa} = (1 - P_D^{fa}(T_D, \lambda_D)) \prod_{k=1}^{K} \left(1 - Q\left((\gamma + 1)Q^{-1}(1 - \alpha) + \gamma\sqrt{\mu T_k}\right)\right) \tag{17}$$

From Equation (7) and (8), it can be seen that $(1 - P_D^{fa}(T_D, \lambda_D))$ and $P_D^{md}(T_D, \lambda_D)$ grow with the increase of λ_D. When the overall miss detection probability P^{md} reaches the limit P_{th}, the overall false alarm probability P^{fa} also reaches the maximum value. Therefore, The objective function U_p achieves the maximum value when $P^{md} = P_{th}$.

This completes the proof.

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Based on Lemma 1, the lower level problem PL1 is equivalent to the problem

$$PL2: \max_{\{\tau_k\},\tau_D} \quad S_P = (1 - P_D^{fa}(T_D, \lambda_D)) \prod_{k=1}^{K} (1 - P_k^{fa}(T_k)), \tag{18}$$

which is subject to the constraints (13c)-(13e). The objective function in problem PL1 can be rewritten as $U_P = \Pr(\mathrm{H}_0) C_0 S_P + \Pr(\mathrm{H}_1) C_1 P_{th}$.

Lemma 2: Problem PL2 is a convex problem under condition C1 which includes that $\tau_1 \geq \left(\frac{(\gamma+1)Q^{-1}(1-\alpha)}{\gamma\sqrt{\mu}}\right)^2$ when $P_{th} \geq \alpha^{K+1}$ at $\alpha \leq 0.5$ and $P_{th} > \alpha^K(1-\alpha)$ at $\alpha > 0.5$, $\tau_1 \geq \left(\frac{(\gamma+1)Q^{-1}(1-\alpha)}{\gamma\sqrt{\mu}}\right)^2$ and $\tau_D \geq \frac{(\gamma+1)^2}{\gamma^2\mu}\left(Q^{-1}(1-\alpha) - Q^{-1}(1-\frac{P_{th}}{\alpha^K})\right)^2$ when $P_{th} < \alpha^{K+1}$ at $\alpha \leq 0.5$ and $P_{th} \geq \alpha^K(1-\alpha)$ at $\alpha > 0.5$.

Proof: In the proof of Lemma 1, we have the miss detection probability $P_k^{md}(T_k) \approx \alpha$ and $P_D^{md}(T_D, P_{th}) \approx P_{th} / \alpha^K$. Thus, the false alarm probability in the multi-stage exploration phase and detection phase are given as

$$P_k^{fa}(T_k) = Q\left((\gamma+1)Q^{-1}(1-\alpha) + \gamma\sqrt{\mu T_k}\right) \qquad k = 1, 2, \ldots K \tag{19}$$

and

$$P_k^{fa}(T_D, P_{th}) = Q\left((\gamma+1)Q^{-1}(1-\frac{P_{th}}{\alpha^K}) + \gamma\sqrt{\mu T_D} \right). \tag{20}$$

Taking logarithm to the objective function S_p, we have

$$S_p^l = \log S_p = \log(1 - P_D^{fa}(T_D, P_{th})) + \sum_{k=1}^{K} \log(1 - P_k^{fa}(T_k)). \tag{21}$$

Based on (19), we have

$$\begin{aligned}\frac{\partial^2 P_k^{fa}(T_k)}{\partial T_k^{\,2}} = \frac{\gamma}{4\sqrt{2\pi}} \exp&\left(-\frac{\left((\gamma+1)Q^{-1}(1-\alpha) + \gamma\sqrt{\mu T_k}\right)^2}{2} \right) \\ &\cdot \sqrt{\frac{\mu}{T_k^3}}\left(1 + \sqrt{\mu T_k}\gamma\left((\gamma+1)Q^{-1}(1-\alpha) + \gamma\sqrt{\mu T_k}\right)\right)\end{aligned} . \tag{22}$$

Further, we have

$$\begin{aligned}\frac{\partial^2 \log(1 - P_k^{fa}(T_k))}{\partial T_k^2} &= -\frac{1}{1 - P_k^{fa}(T_k)} \frac{\partial^2 P_k^{fa}(T_k)}{\partial T_k^2} \\ &\quad - \frac{1}{\left(1 - P_k^{fa}(T_k)\right)^2}\left(\frac{\partial P_k^{fa}(T_k)}{\partial T_k}\right)^2.\end{aligned} \tag{23}$$

Since spectrum opportunity is sparsely distributed in the sensing band, we expect that the false alarm probability is no greater than 0.5 in every stage of the exploration phase. It is equivalent to the following two inequalities

$$\begin{aligned}&(\gamma+1)Q^{-1}(1-\alpha) + \gamma\sqrt{\mu T_k} \geq 0, \;\; k = 1, 2, \ldots, K \\ &(\gamma+1)Q^{-1}(1-\alpha) + \gamma\sqrt{\mu T_D} \geq 0,\end{aligned} \tag{24}$$

which mean that

$$T_k \geq \left(\frac{(\gamma+1)Q^{-1}(1-\alpha)}{\gamma\sqrt{\mu}}\right)^2 \quad k = 1, 2, \ldots, K \tag{25}$$

and

$$T_D \geq \left(\frac{(\gamma+1)Q^{-1}(1-P_{th}/\alpha^K)}{\gamma\sqrt{\mu}}\right)^2. \quad (26)$$

Since $\tau_k \geq 0$ for any k, (25) is equivalent to $\tau_1 \geq \left(\frac{(\gamma+1)Q^{-1}(1-\alpha)}{\gamma\sqrt{\mu}}\right)^2$. In the detection phase, we expect the miss detection probability is no larger than 0.5, and we can achieve this by properly setting parameter a and K to obtain $\frac{P_{th}}{\alpha^K} < 0.5$. And when $P_{th} < \alpha^{K+1}$ at $\alpha \leq 0.5$ and $P_{th} \geq \alpha^K(1-\alpha)$ at $\alpha > 0.5$, we have $\left(Q^{-1}(1-\frac{P_{th}}{\alpha^K})\right)^2 > \left(Q^{-1}(1-\alpha)\right)^2$ which means $T_D > T_k$. So it should be satisfied that $\tau_D \geq \frac{(\gamma+1)^2}{\gamma^2\mu}\left(Q^{-1}(1-\alpha) - Q^{-1}(1-\frac{P_{th}}{\alpha^K})\right)^2$.

Under the above conditions, we have $\frac{\partial^2 P_k^{fa}(T_k)}{\partial T_k^2} > 0$. Then it can be obtained that $\frac{\partial^2 S_p^l}{\partial T_k^2} < 0$ and $\frac{\partial^2 S_p^l}{\partial T_D^2} < 0$. Therefore, S_p^l is a concave function for $\{T_1, T_2, ..., T_K, T_D\}$. Because the objective function S_p is the exponent function of S_p^l and exponent function is concave and nondecreasing, S_p is also a concave function for $\{T_1, T_2, ..., T_K, T_D\}$ [14, page 84].

Denote $\mathbf{T} = [T_1, T_2, ..., T_K, T_D]^T$, $\boldsymbol{\tau} = [\tau_1, \tau_2, ...\tau_K, \tau_D]^T$. From the constraint (13c), we have $\mathbf{T} = \mathbf{A}\boldsymbol{\tau}$, where $\mathbf{A}$ is a $(K+1)$? $(K$ $1)$ lower triangular matrix with the nonzero elements equal to 1. Since S_p is a concave function for $\{T_1, T_2, ..., T_K, T_D\}$, after the operation of composition with an affine mapping [14, page 79], S_p is also a concave function for $\{\tau_1, \tau_2, ...\tau_K, \tau_D\}$.

This completes the proof.

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Since the problem PL2 is convex, the lower level problem PL1 is also convex, and it can be solved by convex optimization methods. So, the optimal solution of $\{\tau_1, \tau_2, ...\tau_K, \tau_D\}$ for a given τ can be obtained. By denoting $U_p^*(\tau)$ as the optimal objective value of the lower level problem PL1 with a specific τ, the upper level problem is given as:

$$PU1: \quad \max_{\tau} \quad C_p(\tau) = (1-\frac{\tau}{T}) \cdot U_p^*(\tau). \qquad \text{s.t.} \quad 0 \leq \tau \leq T \quad (27)$$

Lemma 3: Problem PU1 is a convex problem under condition C1.

Proof: Define two variables $\tau^{(1)}$ and $\tau^{(2)}$, assume that $\tau^{(1)} < \tau^{(2)}$. For $\tau = \tau^{(1)}$, the optimal solution to problem PL1 is $\{\tau_1^{(1)}, \tau_2^{(1)}, ...\tau_K^{(1)}, \tau_D^{(1)}\}$, i.e. $\frac{\tau^{(1)}}{N} = \tau_D^{(1)}\alpha^K + \sum_{i=1}^{K}(\tau_i^{(1)}\alpha^{i-1})$, and the optimal objective value is $U_p^*(\tau^{(1)})$. For $\tau = \tau^{(2)}$, the optimal solution to problem PL1 is

$\{\tau_1^{(2)}, \tau_2^{(2)}, ...\tau_K^{(2)}, \tau_D^{(2)}\}$, i.e. $\frac{\tau^{(2)}}{N} = \tau_D^{(2)}\alpha^K + \sum_{i=1}^{K}(\tau_i^{(2)}\alpha^{i-1})$, and the optimal objective value is $U_p^*(\tau^{(2)})$.

Denote a new set $\{\tau_1^{(1)}, \tau_2^{(1)}, ...\tau_K^{(1)}, \tau_D^{'}\}$ which satisfies

$$\frac{\tau^{(2)}}{N} = \tau_D^{'}\alpha^K + \sum_{i=1}^{K}(\tau_i^{(1)}\alpha^{i-1}). \tag{28}$$

It can be seen that the new set is a feasible solution to problem PL1 with $\tau = \tau^{(2)}$. Because $U_p^*(\tau^{(2)})$ is the optimal objective value to problem PL1 with $\tau = \tau^{(2)}$, we have

$$U_p^*(\tau^{(2)}) \geq \Pr(\mathrm{H}_0)C_0(1 - P_D^{fa}(T_D^{'}, P_{th}))\prod_{k=1}^{K}(1 - P_k^{fa}(T_k^{(1)})) + \Pr(\mathrm{H}_1)C_1 P_{th}, \tag{29}$$

where $T_k^{(1)} = \sum_{i=1}^{k}\tau_i^{(1)}$, $T_D^{'} = \tau_D^{'} + \sum_{i=1}^{K}\tau_i^{(1)}$. Since $\tau^{(1)} < \tau^{(2)}$, we have $\tau_D^{'} > \tau_D^{(1)}$, and further we have $(1 - P_D^{fa}(T_D^{'}, P_{th})) > (1 - P_D^{fa}(T_D^{(1)}, P_{th}))$. Therefore, it can be obtained that $U_p^*(\tau^{(2)}) > U_p^*(\tau^{(1)})$. So $U_p^*(\tau)$ is an increasing function with respect to τ. Then, we have $\frac{\mathrm{d}U_p^*(\tau)}{\mathrm{d}\tau} > 0$.

We rewrite the optimal objective value of the lower level problem PL1 as $U_p^*(\tau) = \sup_{\tau \in A} U_p(\tau, \boldsymbol{\tau})$, where A is the feasible domain defined in problem PL1. It has been proved that $U_p(\tau, \boldsymbol{\tau})$ is the concave function with respect to $\boldsymbol{\tau}$. Since τ is the linear combination of the elements in $\boldsymbol{\tau}$, with the operation of composition with an affine mapping, $U_p(\tau, \boldsymbol{\tau})$ is also the concave function for τ. Then, according to the pointwise supremum property [14, page 79], it can be obtained that $U_p^*(\tau)$ is the concave function with respect to τ. So, we have $\frac{\mathrm{d}^2U_p^*(\tau)}{\mathrm{d}\tau^2} < 0$.

The second order derivative of $C_p(\tau)$ is given as

$$\frac{\mathrm{d}^2C_p(\tau)}{\mathrm{d}\tau^2} = (1 - \frac{\tau}{T})\frac{\mathrm{d}^2U_p^*(\tau)}{\mathrm{d}\tau^2} - \frac{2}{T}\frac{\mathrm{d}U_p^*(\tau)}{\mathrm{d}\tau}. \tag{30}$$

Since $\frac{\mathrm{d}^2U_p^*(\tau)}{\mathrm{d}\tau^2} < 0$ and $\frac{\mathrm{d}U_p^*(\tau)}{\mathrm{d}\tau} > 0$, from Equation (30) we have $\frac{\mathrm{d}^2C_p(\tau)}{\mathrm{d}\tau^2} < 0$. Therefore, problem PU1 is a convex problem under condition C1.

This completes the proof.

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By Lemma 2 and Lemma 3, we prove that the problem is a convex bilevel problem, which can be solved by existing methods.

4. Simulation Results

In this section, simulations are provided to illustrate the performance of the proposed algorithm. Similar to [7], we consider a system consisting of N=100 channels. The sampling frequency is set to be $\mu = 600\,\text{kHz}$. The overall miss detection probability constraint is set as P_{th} = 0.0625. The distillation ratio of the adaptive method is set to be $\alpha = 0.5$ and the number of stages in the exploration phase is set as $K = 3$.

Fig.2 shows the maximum achievable normalized throughput of the proposed algorithm and compares the proposed algorithm with the exhaustive search algorithm, the no optimization scheme [7] with the equal sampling budget allocation and fixed overall sensing time, the adaptive partial optimization scheme [8] with the total sensing time as the optimization variable. The SNR is -10dB. It is obvious that the concave-shaped curve of proposed algorithm is consistent with the conclusion in Lemma 3. Compared with the exhaustive search curve, the proposed algorithm is close to the optimal. Compared with the no optimization scheme [7] and partial optimization scheme [8], the proposed algorithm provides a significant performance gain.

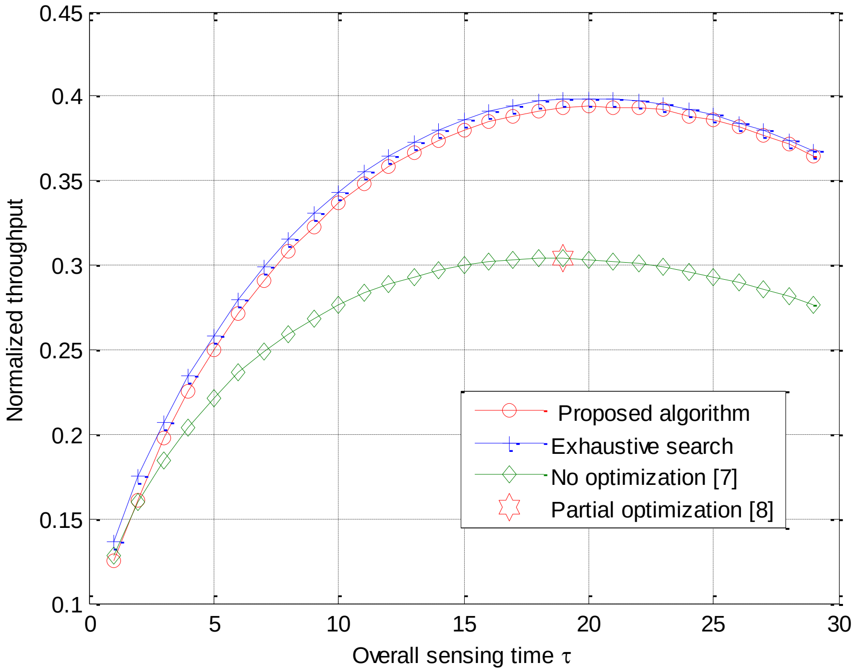

Fig. 2. Normalized throughput versus t.

Fig.3 compares the throughput performance between the proposed algorithm, the exhaustive search algorithm, the no optimization scheme [7] and the adaptive partial optimization scheme [8] in different SNR. For every SNR, there exists an optimal overall sensing time for a fix frame duration T [8]. When SNR is low, it needs more sensing time to maintain the given target probability of detection, thus, the optimal overall sensing time is relatively long. For example, when SNR=-10dB the optimal sensing time is 19ms, which is shown in figure 2. So the performance of no optimization τ =10ms and 15ms are better than that of no optimization τ =5ms. When SNR is 0dB, the case is opposite. The curve of partial optimization in figure 3 shows the performance with the optimal overall sensing time. The optimal overall sensing time at SNR=0dB is close to 5ms. So the curves of no optimization τ =10ms and τ =15ms are worse than that of no optimization τ =5ms when SNR is 0dB. The curve of proposed algorithm is almost coincide to the exhaustive search algorithm. Comparing with the scheme without optimization [7] under different fixed overall sensing time, the throughput of the proposed algorithm shows significant performance improvement. The performance of the proposed algorithm is also better than that of the partial optimization algorithm [8]. The gain is obtained from the optimization of the sensing time for each stage of the exploration phase.

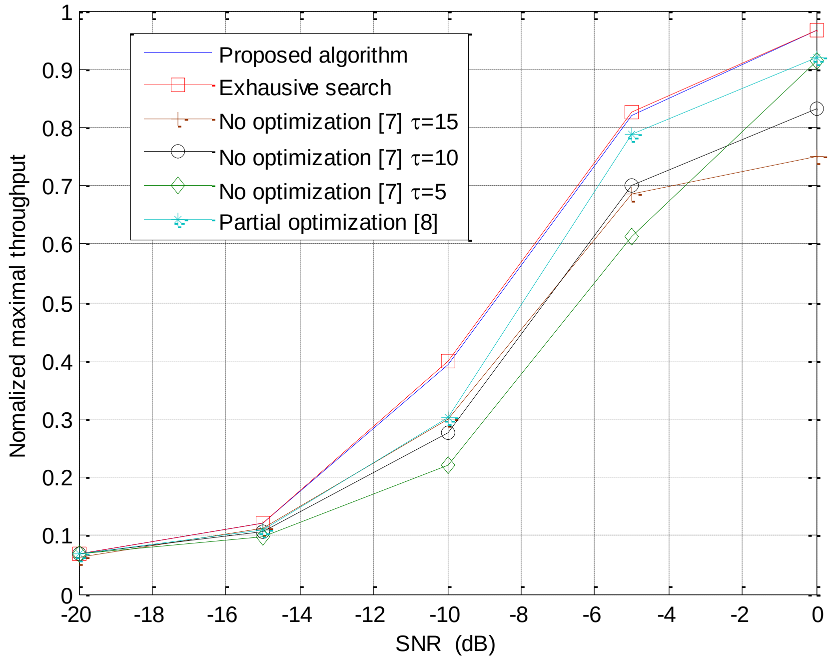

Fig. 3. Normalized maximal throughput versus SNR.

The computational complexity of the proposed algorithm is summarized as follows. When optimizing the overall sensing time, we divide the overall sensing time into M parts for searching. For each sensing time in the exploration and detection phases, it needs K operations. There is K+1 sensing time in the proposed algorithm, so the computational complexity is $O(MK^2)$. The comparison of computational complexity in our computer between the proposed algorithm, the no optimization scheme [7] and the adaptive partial optimization scheme [8] is given in **Table 1**. It is observed that the proposed algorithm obtains throughput improvement at the price of higher complexity. Considering the fact that in practice, the number of stages K in the exploration stage is very small (e.g., K<10), the complexity of the proposed algorithm is allowable for general systems.

Table 1. Computational complexity

Scheme	**Computational complexity**
No optimization scheme [7]	$O(K)$
Adaptive partial optimization scheme [8]	$O(MK)$
Proposed algorithm	$O(MK^2)$

5. Conclusions

In this paper, we have studied the problem of throughput tradeoff in adaptive multiband spectrum sensing procedures. This has been achieved by optimizing not only the overall sensing time but also the sensing time for the exploration and detection phases. We have transformed the initial non-convex optimization problem to a convex bilevel optimization problem. Comparing the adaptive method with equal sampling budget allocation and fixed overall sensing time, the proposed scheme provides a significant improvement on the throughput of the secondary user.

In this research, the problem is formulated when the distillation ratio of the adaptive method a and the number of stages in exploration phase K are fixed. How to implement the joint optimization of the distillation ratio a, the number of stages in the exploration phase K, overall sensing time t and the sensing time for the exploration and detection phases are interesting research topics for further investigation.

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