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# Neighborhood Properties for Certain Subclasses of Analytic Functions of Complex Order with Negative Coefficients

Serap Bulut
*Kocaeli University, Civil Aviation College, Arslanbey Campus, 41285 İzmit-Kocaeli, Turkey*
*e-mail* : serap.bulut@kocaeli.edu.tr

Abstract. In the present investigation, by making use of the familiar concept of neighborhoods of analytic and multivalent functions, we prove several inclusion relations associated with the $(n, \delta)$-neighborhoods of certain subclasses of analytic functions of complex order, which are introduced here by means of the Al-Oboudi derivative. Several special cases of the main results are mentioned.

## 1. Introduction and Definitions

Let $\mathcal{T}_n(p)$ denote the class of functions of the form

$$f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k \qquad (a_k \geq 0;\ n, p \in \mathbb{N} = \{1, 2, \ldots\}), \tag{1.1}$$

which are analytic and multivalent in the open unit disc

$$\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}.$$

A function $f \in \mathcal{T}_n(p)$ is $p$-valently starlike of complex order $\gamma$ $(\gamma \in \mathbb{C} \setminus \{0\})$, that is, $f \in \mathcal{S}_{n,p}(\gamma)$, if it satisfies the following inequality:

$$\Re\left\{p + \frac{1}{\gamma}\left(\frac{zf'(z)}{f(z)} - p\right)\right\} > 0 \quad (z \in \mathbb{U}). \tag{1.2}$$

Furthermore, a function $f \in \mathcal{T}_n(p)$ is $p$-valently convex of complex order $\gamma$ $(\gamma \in \mathbb{C} \setminus \{0\})$, that is, $f \in \mathcal{C}_{n,p}(\gamma)$, if it satisfies the following inequality:

$$\Re\left\{p + \frac{1}{\gamma}\left(1 + \frac{zf''(z)}{f'(z)} - p\right)\right\} > 0 \quad (z \in \mathbb{U}). \tag{1.3}$$

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We note that $f \in \mathcal{C}_{n,p}(\gamma)$ if and only if $\frac{1}{p} z f' \in \mathcal{S}_{n,p}(\gamma)$.

**R**emark 1.1. (i) For $n = p = 1$, we have the classes

$$\mathcal{S}_{1,1}(\gamma) \equiv \mathcal{S}(\gamma) \quad \text{and} \quad \mathcal{C}_{1,1}(\gamma) \equiv \mathcal{C}(\gamma)$$

which are the classes of starlike and convex functions of complex order $\gamma$ in $\mathbb{U}$, respectively, considered by Nasr and Aouf [13] and Wiatrowski [20].

(ii) For $n = p = 1$ and $\gamma = 1 - \alpha$, we have the classes

$$\mathcal{S}_{1,1}(\gamma) \equiv \mathcal{S}(1-\alpha) \equiv \mathcal{S}^*(\alpha) \quad \text{and} \quad \mathcal{C}_{1,1}(1-\alpha) \equiv \mathcal{C}(1-\alpha) \equiv \mathcal{K}(\alpha)$$

which are the classes of starlike and convex functions of order $\alpha$ $(0 \le \alpha < 1)$ in $\mathbb{U}$, respectively.

Following the works of Goodman [10] and Ruscheweyh [16] (see also [1, 4, 6, 7, 11, 12, 15, 18, 19]), Altıntaş [3] defined the $(n,\delta)$-neighborhood of a function $f \in \mathcal{T}_n(p)$ by

(1.4)
$$\mathcal{N}_{n,p}^{\delta}(f) = \left\{ g \in \mathcal{T}_n(p) : g(z) = z^p - \sum_{k=n+p}^{\infty} b_k z^k \quad \text{and} \quad \sum_{k=n+p}^{\infty} k\,|a_k - b_k| \le \delta \right\}.$$

It follows from (1.4) that, if

$$h(z) = z^p \qquad (p \in \mathbb{N}), \tag{1.5}$$

then

$$\mathcal{N}_{n,p}^{\delta}(h) = \left\{ g \in \mathcal{T}_n(p) : g(z) = z^p - \sum_{k=n+p}^{\infty} b_k z^k \quad \text{and} \quad \sum_{k=n+p}^{\infty} k\,|b_k| \le \delta \right\}. \tag{1.6}$$

Next, for $f \in \mathcal{T}_n(p)$, we consider the following operator:

$$D_{\lambda,p}^{0} f(z) = f(z), \tag{1.7}$$

$$D_{\lambda,p}^{1} f(z) = (1-\lambda) f(z) + \frac{\lambda}{p} z f'(z) = D_{\lambda,p} f(z), \quad \lambda \ge 0 \tag{1.8}$$

$$D_{\lambda,p}^{m} f(z) = D_{\lambda,p}(D_{\lambda,p}^{m-1} f(z)), \quad (m \in \mathbb{N}). \tag{1.9}$$

If $f$ is given by (1.1), then from (1.8) and (1.9) we see that

$$D_{\lambda,p}^{m} f(z) = z^p - \sum_{k=n+p}^{\infty} \left(1 + \left(\frac{k}{p} - 1\right)\lambda\right)^m a_k z^k, \quad (m \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}), \tag{1.10}$$

with $D_{\lambda,p}^{m}f(0)=0$.

For $f\in\mathcal{T}_1(p)=\mathcal{T}_p$, this operator was introduced by Bulut [8]. For more details, see Al-Oboudi [2] and Sălăgean [17].

Finally, in terms of the derivative operator $D_{\lambda,p}^{m}$, defined by (1.10) above, we define new subclasses of analytic and multivalent functions.

**Definition 1.2.** We denote by $\mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$ the subclass of $\mathcal{T}_n(p)$ consisting of functions $f$ which satisfy

(1.11)
$$\left|\frac{1}{\gamma}\left(\frac{\mu\alpha z^3\left(D_{\lambda,p}^{m}f(z)\right)'''+(2\mu\alpha+\mu-\alpha)z^2\left(D_{\lambda,p}^{m}f(z)\right)''+z\left(D_{\lambda,p}^{m}f(z)\right)'}{\mu\alpha z^2\left(D_{\lambda,p}^{m}f(z)\right)''+(\mu-\alpha)z\left(D_{\lambda,p}^{m}f(z)\right)'+(1-\mu+\alpha)D_{\lambda,p}^{m}f(z)}-p\right)\right|<\beta$$

$$(z\in\mathbb{U};\ p,n\in\mathbb{N};\ m\in\mathbb{N}_0;\ \gamma\in\mathbb{C}-\{0\};\ \lambda\geq 0;\ 0\leq\alpha\leq\mu\leq 1;\ 0<\beta\leq 1).$$

**Definition 1.3.** We denote by $\mathcal{R}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$ the subclass of $\mathcal{T}_n(p)$, which consists of functions $f\in\mathcal{T}_n(p)$ which satisfy

(1.12)
$$\left|\frac{1}{\gamma}\left(\mu\alpha z^2\left(D_{\lambda,p}^{m}f(z)\right)'''+(2\mu\alpha+\mu-\alpha)z\left(D_{\lambda,p}^{m}f(z)\right)''+\left(D_{\lambda,p}^{m}f(z)\right)'-p\right)\right|<\beta$$

$$(z\in\mathbb{U};\ p,n\in\mathbb{N};\ m\in\mathbb{N}_0;\ \gamma\in\mathbb{C}-\{0\};\ \lambda\geq 0;\ 0\leq\alpha\leq\mu\leq 1;\ 0<\beta\leq 1).$$

**Remark 1.4**. We have the following relationships.

(i) For $\mu=\alpha=0$, we have the class

$$\mathcal{S}_{n,p}(m;\gamma,\lambda,0,0,\beta)\equiv\mathcal{H}_n(m,p,\lambda,\gamma,\beta)$$

introduced by El-Ashwah and Aouf [9].

(ii) For $p=1$, $m=0$ and $\alpha=0$, we have the classes

$$\mathcal{S}_{n,p}(0;\gamma,\lambda,\mu,0,\beta)\equiv\mathcal{S}_n(\gamma,\mu,\beta)$$

and

$$\mathcal{R}_{n,p}(0;\gamma,\lambda,\mu,0,\beta)\equiv\mathcal{R}_n(\gamma,\mu,\beta)$$

studied by Altıntaş *et al.* [5].

(iii) For $n=p=1$, $m=0$ and $\beta=1$, we have the classes

$$\mathcal{S}_{n,1}(0;\gamma,\lambda,\mu,\alpha,0)\equiv\mathcal{P}_\alpha(\mu,\gamma)$$

and

$$\mathcal{R}_{n,1}(0;\gamma,\lambda,\alpha,\beta)\equiv\mathcal{R}_\alpha(\mu,\gamma)$$

studied by Orhan and Kamalı [14].

(iv) For $\mu = 0$ and $\mu = 1$ with $m = 0$, $\alpha = 0$, $\beta = 1$, we have the following relationships for the class $\mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$, respectively:

$$\mathcal{S}_{n,p}(0;\gamma,\lambda,0,0,1) \subset \mathcal{S}_{n,p}(\gamma)$$

and

$$\mathcal{S}_{n,p}(0;\gamma,\lambda,1,0,1) \subset \mathcal{C}_{n,p}(\gamma).$$

## 2. A Set of Coefficient Inequalities

In this section, we prove the following results which yield the coefficient inequalities for functions in the subclasses

$$\mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta) \quad \text{and} \quad \mathcal{R}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta).$$

**Theorem 2.1.** *Let $f \in \mathcal{T}_n(p)$ be defined by* (1.1). *Then $f$ is in the class $\mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$ if and only if*

$$\sum_{k=n+p}^{\infty}\left(1+\left(\frac{k}{p}-1\right)\lambda\right)^{m}(k-p+\beta|\gamma|)\Psi(k)a_k \le \beta|\gamma|\Psi(p), \tag{2.1}$$

*where*

$$\Psi(s) := (s-1)(\mu\alpha s+\mu-\alpha)+1. \tag{2.2}$$

*Proof.* We first suppose that $f \in \mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$. Then, by appealing to the condition (1.11), we readily obtain

(2.3)

$$\Re\left\{\frac{\mu\alpha z^3\left(D^m_{\lambda,p}f(z)\right)'''+(2\mu\alpha+\mu-\alpha)z^2\left(D^m_{\lambda,p}f(z)\right)''+z\left(D^m_{\lambda,p}f(z)\right)'}{\mu\alpha z^2\left(D^m_{\lambda,p}f(z)\right)''+(\mu-\alpha)z\left(D^m_{\lambda,p}f(z)\right)'+(1-\mu+\alpha)D^m_{\lambda,p}f(z)}-p\right\}$$

$$> -\beta|\gamma|$$

for $z \in \mathbb{U}$, or, equivalently,

$$\Re\left\{\frac{-\sum_{k=n+p}^{\infty}\left(1+\left(\frac{k}{p}-1\right)\lambda\right)^{m}(k-p)\Psi(k)a_k z^k}{\Psi(p)z^p-\sum_{k=n+p}^{\infty}\left(1+\left(\frac{k}{p}-1\right)\lambda\right)^{m}\Psi(k)a_k z^k}\right\} > -\beta|\gamma| \tag{2.4}$$

for $z \in \mathbb{U}$, where we have made use of (1.10) and the definition (1.1). We now choose values of $z$ on the real axis and let $z \to 1^-$ through real values. Then the inequality (2.4) immediately yields the desired condition (2.1).

Conversely, suppose that the inequality (2.1) holds true and let

$$z \in \partial\mathbb{U} = \{z \in \mathbb{C} : |z| = 1\}.$$

Then we find hat

$$\begin{aligned}
&\left|\frac{\mu\alpha z^3\left(D^m_{\lambda,p}f(z)\right)''' + (2\mu\alpha+\mu-\alpha)z^2\left(D^m_{\lambda,p}f(z)\right)'' + z\left(D^m_{\lambda,p}f(z)\right)'}{\mu\alpha z^2\left(D^m_{\lambda,p}f(z)\right)'' + (\mu-\alpha)z\left(D^m_{\lambda,p}f(z)\right)' + (1-\mu+\alpha)D^m_{\lambda,p}f(z)} - p\right| \\
= &\left|\frac{\sum_{k=n+p}^{\infty}\left(1+\left(\frac{k}{p}-1\right)\lambda\right)^m (k-p)\Psi(k)a_k z^{k-p}}{\Psi(p) - \sum_{k=n+p}^{\infty}\left(1+\left(\frac{k}{p}-1\right)\lambda\right)^m \Psi(k)a_k z^{k-p}}\right| \\
\le &\ \frac{\beta|\gamma|\left[\Psi(p) - \sum_{k=n+p}^{\infty}\left(1+\left(\frac{k}{p}-1\right)\lambda\right)^m \Psi(k)a_k\right]}{\Psi(p) - \sum_{k=n+p}^{\infty}\left(1+\left(\frac{k}{p}-1\right)\lambda\right)^m \Psi(k)a_k} \\
= &\ \beta|\gamma|.
\end{aligned}$$

Hence, by the Maximum Modulus Theorem, we have

$$f \in \mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$$

which evidently completes the proof of Theorem 2.1. □

Similarly, we can prove the following result.

**Theorem 2.2.** *Let* $f \in \mathcal{T}_n(p)$ *be defined by* (1.1). *Then* $f$ *is in the class* $\mathcal{R}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$ *if and only if*

$$\sum_{k=n+p}^{\infty}\left(1+\left(\frac{k}{p}-1\right)\lambda\right)^m k\Psi(k)a_k \le \beta|\gamma| + p(\Psi(p)-1), \tag{2.5}$$

*where the function* $\Psi$ *is defined by* (2.2).

**Remark 2.3**. Special cases of Theorem 2.1 and Theorem 2.2 when

$$p = 1, \quad m = 0 \quad \text{and} \quad \alpha = 0 \tag{2.6}$$

was given by Altıntaş *et al.* [5].

## 3. Inclusion Relations Involving the $(n,\delta)$-neighborhood $\mathcal{N}^{\delta}_{n,p}(h)$

In this section, we establish several inclusion relations for the normalized $p$-valently analytic function classes

$$\mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta) \quad \text{and} \quad \mathcal{R}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$$

involving the $(n,\delta)$-neighborhood defined by (1.6).

**Theorem 3.1.** *If*

$$\delta := \frac{\beta\left|\gamma\right|\left(n+p\right)\Psi\left(p\right)}{\left(1+\frac{n}{p}\lambda\right)^{m}\left(n+\beta\left|\gamma\right|\right)\Psi\left(n+p\right)}, \tag{3.1}$$

*then*

$$\mathcal{S}_{n,p}\left(m;\gamma,\lambda,\mu,\alpha,\beta\right)\subset\mathcal{N}_{n,p}^{\delta}\left(h\right), \tag{3.2}$$

*where the function* $\Psi$ *is defined by* (2.2).

*Proof.* For a function $f\in\mathcal{S}_{n,p}\left(m;\gamma,\lambda,\mu,\alpha,\beta\right)$ of the form (1.1), Theorem 2.1 immediately yields

$$\left(1+\frac{n}{p}\lambda\right)^{m}\left(n+\beta\left|\gamma\right|\right)\Psi\left(n+p\right)\sum_{k=n+p}^{\infty}a_k\leq\beta\left|\gamma\right|\Psi\left(p\right),$$

so that

$$\sum_{k=n+p}^{\infty}a_k\leq\frac{\beta\left|\gamma\right|\Psi\left(p\right)}{\left(1+\frac{n}{p}\lambda\right)^{m}\left(n+\beta\left|\gamma\right|\right)\Psi\left(n+p\right)}. \tag{3.3}$$

On the other hand, we also find from (2.1) that

$$\left(1+\frac{n}{p}\lambda\right)^{m}\frac{\left(n+\beta\left|\gamma\right|\right)\Psi\left(n+p\right)}{n+p}\sum_{k=n+p}^{\infty}ka_k\leq\beta\left|\gamma\right|\Psi\left(p\right),$$

that is, that

$$\sum_{k=n+p}^{\infty}ka_k\leq\frac{\beta\left|\gamma\right|\left(n+p\right)\Psi\left(p\right)}{\left(1+\frac{n}{p}\lambda\right)^{m}\left(n+\beta\left|\gamma\right|\right)\Psi\left(n+p\right)}:=\delta, \tag{3.4}$$

which, in view of the definition (1.6), proves Theorem 3.1.

Similarly, by applying Theorem 2.2 instead of Theorem 2.1, we can prove Theorem 3.2 below.

**Theorem 3.2.** *If*

$$\delta := \frac{\beta\left|\gamma\right|+p\left(\Psi\left(p\right)-1\right)}{\left(1+\frac{n}{p}\lambda\right)^{m}\Psi\left(n+p\right)}, \tag{3.5}$$

*then*

$$\mathcal{R}_{n,p}\left(m;\gamma,\lambda,\mu,\alpha,\beta\right)\subset\mathcal{N}_{n,p}^{\delta}\left(h\right), \tag{3.6}$$

*where the function* $\Psi$ *is defined by* (2.2).

**Remark 3.3.** Applying the parametric substitutions listed in (2.6), Theorem 3.1 and Theorem 3.2 would yield a set of known results due to Altıntaş *et al.* [5].

## 4. Neighborhood Properties for the Classes $\mathcal{S}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta)$ and $\mathcal{R}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta)$

In this section, we determine the neighborhood for each of the function classes

$$\mathcal{S}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta) \quad \text{and} \quad \mathcal{R}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta),$$

which we define here as follows.

**Definition 4.1.** A function $f \in \mathcal{T}_n(p)$ is said to be in the class $\mathcal{S}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta)$ if there exists a function $g \in \mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$ such that the following inequality holds true:

$$\left|\frac{f(z)}{g(z)} - 1\right| < p - b \quad (z \in \mathbb{U};\ 0 \leq b < p). \tag{4.1}$$

**Definition 4.2.** A function $f \in \mathcal{T}_n(p)$ is said to be in the class $\mathcal{R}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta)$ if there exists a function $g \in \mathcal{R}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$ such that the inequality (4.1) holds true.

**Theorem 4.3.** *If* $g \in \mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$ *and*

$$b = 1 - \frac{\left(1+\frac{n}{p}\lambda\right)^m (n+\beta|\gamma|)\,\Psi(n+p)\,\delta}{(n+p)\left[\left(1+\frac{n}{p}\lambda\right)^m (n+\beta|\gamma|)\,\Psi(n+p) - \beta|\gamma|\,\Psi(p)\right]}, \tag{4.2}$$

*then*

$$\mathcal{N}_{n,p}^{\delta}(g) \subset \mathcal{S}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta), \tag{4.3}$$

*where the function* $\Psi$ *is defined by* (2.2).

*Proof.* Assuming that $f \in \mathcal{N}_{n,p}^{\delta}(g)$, we find from the definition (1.4) that

$$\sum_{k=n+p}^{\infty} k\,|a_k - b_k| \leq \delta, \tag{4.4}$$

which readily implies the following coefficient inequality:

$$\sum_{k=n+p}^{\infty} |a_k - b_k| \leq \frac{\delta}{n+p}. \tag{4.5}$$

Since $g \in \mathcal{S}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$, we have [*cf.* Eq. (3.3)]

$$\sum_{k=n+p}^{\infty} b_k = \frac{\beta|\gamma|\Psi(p)}{\left(1+\frac{n}{p}\lambda\right)^m (n+\beta|\gamma|)\Psi(n+p)}, \tag{4.6}$$

so that

$$\begin{aligned} \left|\frac{f(z)}{g(z)}-1\right| &< \frac{\sum_{k=n+p}^{\infty}|a_k-b_k|}{1-\sum_{k=n+p}^{\infty}b_k} \\ &\leq \frac{\delta}{n+p}\cdot\frac{\left(1+\frac{n}{p}\lambda\right)^m (n+\beta|\gamma|)\Psi(n+p)}{\left(1+\frac{n}{p}\lambda\right)^m (n+\beta|\gamma|)\Psi(n+p)-\beta|\gamma|\Psi(p)} \\ &=: 1-b, \end{aligned} \tag{4.7}$$

provided that $b$ is given precisely by (4.2). Thus, by Definition 4.1, we conclude that

$$f \in \mathcal{S}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta)$$

for $b$ given by (4.2). This evidently completes the proof of Theorem 4.3.

The proof of Theorem 4.4 below is much akin to that of Theorem 4.3, and so the details involved are being omitted here.

**Theorem 4.4.** *If* $g \in \mathcal{R}_{n,p}(m;\gamma,\lambda,\mu,\alpha,\beta)$ *and*

$$b = 1-\frac{\left(1+\frac{n}{p}\lambda\right)^m (n+p)\Psi(n+p)\delta}{(n+p)\left\{\left(1+\frac{n}{p}\lambda\right)^m (n+p)\Psi(n+p)-[\beta|\gamma|+p(\Psi(p)-1)]\right\}}, \tag{4.8}$$

*then*

$$\mathcal{N}_{n,p}^{\delta}(g) \subset \mathcal{R}_{n,p}^{(b)}(m;\gamma,\lambda,\mu,\alpha,\beta), \tag{4.9}$$

*where the function* $\Psi$ *is defined by* (2.2).

**Remark 4.5.** Applying the parametric substitutions listed in (2.6), Theorem 4.3 and Theorem 4.4 would yield a set of known results due to Altıntaş *et al.* [5].

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