A study on sliding surface design

Yifan.Zhang[1], Sanghyuk Lee[1*]
[1]Electronic Engineering Xi'an Jiaotong-Liverpool University, Suzhou, China
E-mail : Sanghyuk.Lee@xjtlu.edu.cn

요 약 비선형시스템에 대한 슬라이딩 모드제어 기법에 대한 연구를 수행하였다. 비선형 시스템의 파라미터가 제어성능과 간인성에 대한 관계를 구명하였다. 제어성능을 파악하기 위하여 역진자 시스템에적용하여 보았고, 다른 초기값, 슬라이딩 표면 그리고 입력값의 변화를 통하여 비교결과를 얻었다. 제어값은 제한적이었으며 슬라이딩 표면 역시 예외없이 제한폭을 나타냈다. 채터링 현상은 피할수 없이 존재하였으며, 이를 극복하기 위하여 수정된 불연속 제어기를 사용하여 현상을 감소시켰다.

Abstract Sliding mode design and analysis for nonlinear system was carried out. A designer will determine the parameters to know about the performance and robustness of the system dynamics. To investigate the characteristics of sliding mode control, an inverted pendulum model is applied by the sliding mode control and the state concerned is output. Comparison is made by evaluating different initial conditions, sliding numerical components for sliding surface, and input gain, the dynamic of output will be investigated to conclude the generality. Control approaches have their limitations and sliding mode control is no exception. The chattering problem is its main negative effect to overcome. This effect is displayed and in this project chattering problem is suppressed by a modified discontinuous controller.

Keywords : Sliding mode; Inverted pendulum Sliding surface; Equilibrium

1. Introduction

For a dynamical system, overcoming external factors such as uncertainties and variations is always an interesting topic. Robust control methods including control has been proposed, however it has challenge to solve nonlinear system. It still left much thing to consider overcome nonlinear system stabilization. Hence, the control of uncertainties for nonlinear system is another important requirement. Variable Structure Control was recognized as a robust method to deal with uncertain systems due to its invariance to disturbances. Sliding mode control has such characteristics of VSC in that state repeatedly crosses the sliding surface (also called hyper-plane, switching surface) and asymptotically stable [1].

Sliding mode control (SMC) is an efficient control method by designing robust controller. It was proposed that SMC provide less sensitive under the variation of parameter and external disturbances [2]. Design process is also rather simple, with consideration of constructing hyper plane as sliding surface, and simple controller structure make system stable even it is nonlinear structure. However, it is not easy for high order system, because it is not easy to show hyper plane as graphical representation. In this literature, we focus on sliding mode controller design for higher order system with uncertainty. Two modes after dividing

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*교신저자 : Sanghyuk Lee(Sanghyuk.Lee@xjtlu.edu.cn)

with hyper plane are appeared, the reaching mode and the sliding mode in sequence. In the reaching mode, the state trajectory tends to be tangent to the sliding surface. After in the sliding mode, the state slides on the surface and is eventually motionless at the origin.

Generally, the design of sliding mode controller is composed of two steps. First, the selection of sliding surface is considered. The selection of the sliding surface is conducted mainly by design the sliding coefficients and therefore a sliding surface is created by the designer. The sliding coefficients contribute to the equivalent control. Next, the controller design refers to the combination of equivalent control and discontinuous control. The discontinuous control is composed of a positive control when the state is in the upper region of sliding surface and a negative control when the state is in the lower region of the surface. Main issue of sliding mode control is how and which variable is proper to stabilize the whole system with additional control input.

Simple example was provided to the sliding mode control, inverted pendulum. The pendulum with a bob is connected to a cart underneath and the cart is powered by the built-in electrical motor which is controlled by a controller. The motor plays a role as sliding mode control that provide the input control and try to drive the pendulum to the equilibrium position. The objective of the project is to balance the pendulum bob to the reference midline with the designed sliding mode controller. The characteristics of the sliding mode controller are investigated by modifying and adjusting the parameters.

In this paper, sliding mode control methodology was illustrated in next chapter. In Chapter 3, Inverted pendulum was provided, as a linearized structure. System controllability was also verified near the operating point. In Chapter 4, sliding mode controller was designed ad applied to the system. Control input variable was selected to minimize settling time and maximum overshoot. Finally, conclusion was followed in Chapter 5.

2. Sliding mode control

The sliding mode control is operated into two steps, the selection of sliding surface in the state space where the state trajectory is constrained, and control law design which is composed of equivalent control and discontinuous control. Due to the existence of the discontinuous control, the output of the system suffers from chattering problem.

A sliding surface (also called sliding manifold and hyper-plane in high dimension sliding mode) can be defined as the following equation:

$$s = Sx = 0 \tag{1}$$

In this case, S is a 1 x n matrix with n constant elements and usually S is written as:

$$\mathrm{S} = [S_1 \;\; S_2 \;\; S_3 \;\; \cdots S_n] \tag{2}$$

This matrix is the major design of the sliding mode control and is determined by the designer. The state is a converged motion on the sliding surface s(x) and the sliding surface will always include the origin x=0[3-5]. In the sliding mode, the state point will approach the surface asymptotically and this process is called reaching mode [5].

Lyapunov function is a useful criterion of the stability of the system and in sliding mode control. The Lyapunov function is introduced as

$$V = \frac{1}{2} s^T s \tag{3}$$

where is the sliding surface of the system. A controller will be designed in a way such that

$$\dot{V} < 0 \tag{4}$$

$\dot{V}(x)$ is negative in order that is negative. We assume s(x) is negative when the system trajectory departs from sliding surface, at the same time the derivative of s(x) is positive which drive the system trajectory to converge to the sliding surface and vice versa. This condition built is to ensure the state trajectory will approach the sliding surface in a period of time.

In order to stabilize the system, the purpose of the sliding mode control is to force the state variables to slide on the sliding surface and eventually the state variables approach the origin along the surface. The trajectory of the sliding mode control goes in two phase: the reaching mode and sliding mode. In this design, Lyapunov stability criterion is applied to obtain control law $u(t)$ which is the sum of two controllers u_{eq}(equivalent control) and u_c (equivalent control) [3].

A trajectory of the state start to move from its initial position, it is expected to approach the sliding surface asymptotically, when it requires an equivalent control.

Consider a linear state space equation:

$$\dot{x} = Ax + Bu \tag{5}$$

The sliding mode motion is expected to tangent to switching surface:

$$s^T\dot{s} = \frac{\partial s}{\partial x}(Ax + Bu) = S(Ax + Bu) = 0 \tag{6}$$

The equivalent control is derived as:

$$u_{eq} = -[SB]^{-1}SAx \tag{7}$$

The sliding mode dynamics (with only equivalent control):

$$\dot{x} = [I - B(SB)^{-1}S]Ax \tag{8}$$

The corrective control (u_c) is discontinuously defined as follows:

$$u_c = \begin{cases} u^+ & s > 0 \\ u^- & s < 0 \end{cases} \tag{9}$$

The corrective control mainly plays a role in reaching mode. In this mode, the trajectory of system from the initial position will be forced to approach the sliding surface in finite time.

2. Inverted Pendulum Application

In the diagram, the inverted pendulum is mounted on a cart on the frictionless surface. The differential equations are created by summing the forces in the horizontal direction and about the pivot point in terms of the angular rotation $\theta(t)$ and the position of the cart y(t). Assume that M >> m and the rotation angle θ is small enough in order that the equation is linear. [6]

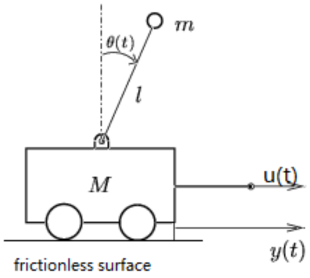

Figure 1. Inverted Pendulum

The force summation in the horizontal direction:

$M\ddot{y} + ml\ddot{\theta} - u(t) = 0$

where u(t) represents the force on the cart, and is the length of the rod with respect to the pivot point.

The summation about the pivot point is:

$ml\ddot{y} + ml^2\ddot{\theta} - mlg\theta = 0$

The state variables are chosen as $(x_1, x_2, x_3, x_4) = (y, \dot{y}, \theta, \dot{\theta})$.

To sum up, write them as four first order equations as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{mg}{M}x_3 + \frac{1}{M}u(t) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{g}{l}x_3 - \frac{1}{Ml}u(t) \end{cases}$$

Write the differential equations as state space form $\dot{x} = Ax + Bu$

$$\text{Where A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g/l & 0 \end{bmatrix}, \text{B} = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ -/(Ml) \end{bmatrix}$$

Let the system parameters be

$l = 0.098\, m,\ g = 9.8\, m/s^2,\ m = 0.825\, kg,\ M = 8.085\, kg$

In this case, the system state and input matrices are

$$\text{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 100 & 0 \end{bmatrix}, \text{B} = \begin{bmatrix} 0 \\ 0.1237 \\ 0 \\ -1.2621 \end{bmatrix}$$

Identify the eigenvalues of matrix A:

$\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 10, \lambda_4 = -10$

Therefore it is an unstable system.

Controllability matrix is generated as:

$P_c = [B\ AB\ A^2B\ A^3B]$

Substitute the matrix A and B into Pc and Pc matrix is obtained as:

$$P_c = \begin{bmatrix} 0 & 0.1237 & 0 & 1.2621 \\ 0.1237 & 0 & 1.2621 & 0 \\ 0 & -1.2621 & 0 & -126.21 \\ -1.2621 & 0 & -126.21 & 0 \end{bmatrix}$$

The system is controllable.

4. Sliding mode controller design

The system is originally a nonlinear system, but it can be linearized by assuming $\sin\theta = \theta$ and $\cos\theta = 1$. As a result, the inclination angle θ has the restriction range to implement and it should be valued within an interval [0, 0.244] (13.98 degrees for 0.244). When inclination angle θ is π, where the pendulum start from the downward position, the sliding control seems still effective for the linearized system, but the original non-linear system cannot be equivalent to the linearized model once the inclination angle exceed its range [7].

The sliding numerical components and the initial condition are specified as $S = [S_1\ S_2\ S_3\ S_4]$ = [1 1 1 1], $x_0 = \begin{bmatrix} 0 \\ 0 \\ 0.031 \\ 0 \end{bmatrix}$. The input gain $|u_c|$ = 0.01

The sliding surface for the sliding mode control in this pendulum is a four dimension which cannot be displayed in 3-D space. Therefore, three of four variables x_1, x_2, x_3 are chosen to display the trajectory and the results are shown in Figure 2.

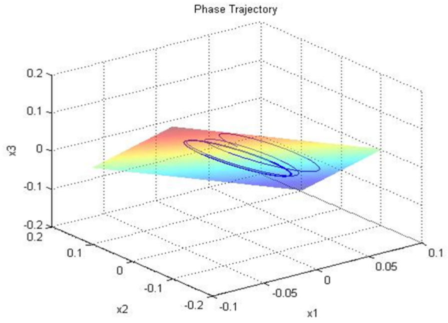

Figure 2. Trajectory of marginally stable system

As is shown in Figure 2, the sliding surface in color is drawn as a sliding surface. The state trajectory is expected to follow the sliding surface. The initial point of the state is [0 0 0.031 0]', when sliding mode control is implemented, the initial state starts to approach the sliding surface and crosses it spirally and finally all three states approach zero.

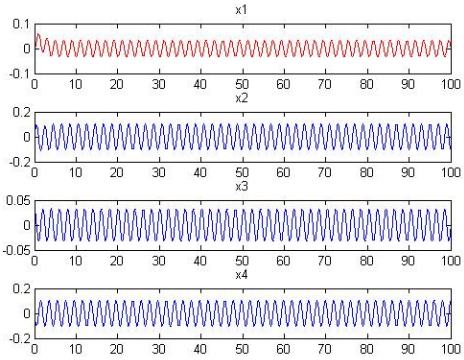

Figure 3. Output state variables

In Figure 3, the output state variables are displayed and all the four variables show vibration near the zero with almost constant amplitude. In this condition, the system is considered as marginally stable. In the inverted pendulum model, the pendulum swings left and right repeatedly in the neighborhood of equilibrium position, and the amplitude of the swings are invariant. As for the cart, the displacement is represented by state variable x_1 and the cart moves back and forth to stabilize the pendulum in the middle.

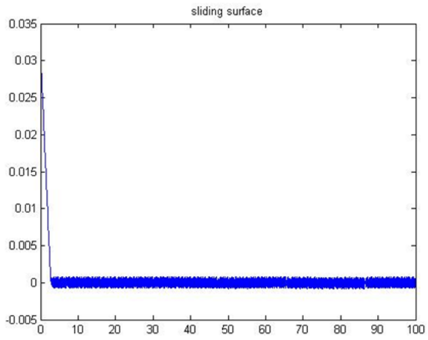

Figure 4. Value of sliding surface

Seen from the equation (1) defined for sliding surface, the equation can be rewritten as:

$$s = S_1x_1 + S_2x_2 + S_3x_3 + S_4x_4 = 0$$

Substitute $S = [S_1\ S_2\ S_3\ S_4] = [1\ 1\ 1\ 1]$ and the sliding surface derived as:

$$s = x_1 + x_2 + x_3 + x_4 = 0$$

Given each state at the time, we can substitute into the sliding surface to obtain value, when s > 0, the state is located in the upper region of the sliding surface and it may try to approach the sliding surface and probably goes in lower region of the sliding surface when s < 0. In Figure 4, the initial point makes s >0, and then the equivalent controller forces the value of to zero while may surpass the equilibrium position to s < 0, the discontinuous controller try to pull back to the zero position. Therefore, the state vibrates with amplitude.

The sliding numerical components and the initial condition are specified as $S = [S_1\ S_2\ S_3\ S_4]$ = [0.01 0.1 1 1], $x_0 = \begin{bmatrix} 0 \\ 0 \\ 0.031 \\ 0 \end{bmatrix}$. The input gain $|u_c|$ = 0.01

Repeat the previous procedures and obtain the information of phase trajectory, state variables and sliding surface.

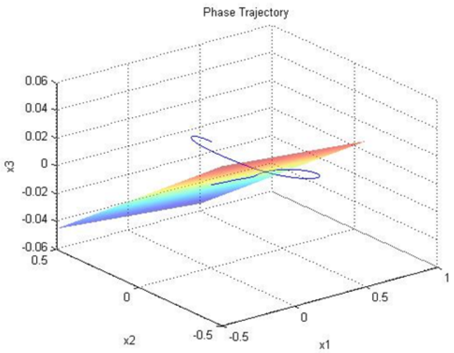

Figure 5. phase trajectory of stable system

As can be seen from Figure 5, the state starts from the initial position and reach the sliding surface over time and then it slides on the surface and goes towards the origin.

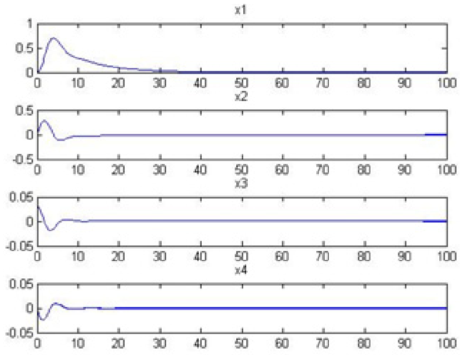

Figure 6. State variables of stable system

In Figure 6, all four state variables tend to be stable in overtime. All the four states eventually go to zero and the performance of the system can be investigated through settling time, overshoot and peak time. Although all the output state variables appears stable and smooth, once the sliding mode control is implemented, the output will suffer from the chattering problem which has an negative effect on the performance of system dynamics.

Take the output state x_2 as a reference and zoom in to identify the chattering phenomenon.

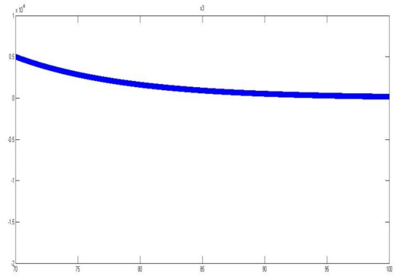

Figure 7. chattering problem of state x2

In Figure 7, the state x2 is zoomed in and displayed in an interval of -2×10^{-6} and 1×10^{-6}. As is displayed in the figure, the state x2 asymptotically approaches to zero but still has minor oscillations to the magnitude of 10^{-6}.

As the Initial condition and input gain make a difference to the performance of system dynamics with the sliding mode control, more data of initial angle and input gain will be listed to see the trend.

Table 1 Initial value x_0=[0 0 0.031 0]

u_c	0.01	0.05	0.1
Settling time(s)	9.1454	8.7817	8.7333
Max	-0.063	-0.0046	-0.0044
Peak time/Max time	2.7164	2.5790	2.5726

Table 2 Initial value x_0 = [0 0 0.244 0]

u_c	0.01	0.05	0.1
settling time	26.9499	10.6477	9.5673
Min	-0.0812	-0.1392	-0.1410
Peak time	3.9511	3.8735	2.9899

As is listed in tables, for the same initial condition when the input gain u_c is increased, the settling time and peak time are decreased. The system spends less time to reach the steady state, since when the input gain is increase, more power is provided by the controller to force the pendulum to the equilibrium position and therefore saves the time. For different initial positions, the dynamic characteristics of them also are distinct. As is shown in the figure, for the same gain applied, when the inclination angle θ is larger, the settling time and peak time increase. The pendulum with bob will take more time to approach the midline for the same gain input. In terms of the state position, it means that the state is further away from the sliding surface, and it takes more time to reach the sliding surface. However, for small input gain u_c, the distinction is quite obvious while when larger input power is poured, the difference is quite minor. What can be inferred is that, when the input power is large enough, the initial position is not the determinant to reduce the settling time and peak time.

5. Conclusions

In this paper, system control for higher order was carried with sliding mode control methodology. By increasing the input gain can reduce settling time. The initial angle can make a difference for settling time with small input gain, but when continuing adding input gain, the settling time indicates less distinction. Additional control was considered by linear slope. Then the slope of the linear section for the boundary layer controller is increased, the settling time may have a minimum value when the slope satisfied 0.2, and then settling time approaches to 9.84s. The slope of the linearity is invariant and the input gain is increased from 0.01 to 0.09, the settling time, overshoot and peak time all decrease when input gain is small but remain unchanged when the slope d is 4. A modified controller has practically reduce the chattering problem in magnitude, but cannot complete eliminate it.

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저 자 소 개

Yifan.Zhang

- Student at Dept. of Electrical Engineering, Xi'anJiaotong -Liverpool University, China

Sanghyuk Lee

- 1998. Feb: Seoul National University, Korea, Electrical Engineering, (Ph.D.)
- 2011. Aug.~ Present: Associate Professor at Dept. of Electrical Engineering, Xi'anJiaotong-Liverpool University, China

<관심분야> : Information theory, control theory, Data mining