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Modelling and Development of Control Algorithm of Endoscopy

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요 약 본 논문에서는 효율적인 제어를 위하여 캡슐 앤도스코피에 대한 모델링을 실시하였다. 방법론적으로 루그레 모델에 대한 시스템 특성파악을 위한 수학적 모델링을 이용하였다. 비선형 마찰 모델인 루그레 모델에 바탕을 둔 stick-slip 모션 시스템이 인체내에서의 캡슐 앤도스코피의 활동을 묘사하는 시뮬레이션 모델로 이용하였다. 다양한 상황을 고려하여 루그레 마찰모델에 대한 시뮬레이션을 Matlab Simulink 를 작성하여 수행하였다. 전체적인 모션과 파라미터의 영향이 엔도스코피의 속도에 미치는 영향에 대한 것에 주안을 두어 실시하였다.

Abstract In this paper, basic backgrounds about capsule endoscopy are introduced, and the aims and objectives are also illustrated. Methodology and mathematical model for LuGre model were investigated to analyse system characteristics. A nonlinear friction model, the stick-slip motion system based on LuGre friction model was used to simulate the motion of capsule endoscopy inside human body. Under the different situation, LuGre friction model was simulated by Matlab Simulink software. The entire cycle of motion and the influence of parameters towards to velocity are fully simulated.

Keywords : Friction Force, LuGre Model, Endoscopy, Nonlinear Modelling

1. Introduction

Control system plays an important role not only in the engineering field but also social system, economics and management. In the automatic control system, both linear and nonlinear control structure has been applied widely [1, 2]. Acontroller in the control system may not always be linear time-invariant, and there is no common method to analyse nonlinear system. Thus, such analysis methods for linear cases cannot be applied to nonlinear control system directly. Moreover, the method of how to linearize a nonlinear system is significant for modern control system. Friction plays a major role in control system. The precision of a system can be influenced by friction. Sometimes, friction causes instabilities to system. In a particular, capsule endoscopy has been used medically to carry out inspection about digestive tract, especially, applied in stomach and intestine. The advantages of capsule endoscopy is a convenient, harmless, and painless for patient compared with other traditional inspection method, for instance, gastroscopies, Colonoscopy, and barium meal examination etc. Furthermore, detailed colourful image and results can be provided to doctor. Several nonlinear dynamic friction models can be used to simulate the movement of capsule endoscopy inside human body. Among multiple nonlinear models, LuGre model describes the characteristics of friction, which is suitable to analyze.

In order to make capsule stay longer in somewhere

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valued inside the human intestine to take more pictures to help diagnosis, a model need to be simulated in order to find the factors that influenced velocity of capsule. In this case, a dynamics nonlinear model of stick-slip driving system based on LuGre friction model was proposed. In order to testify and simulate the effect, several block diagrams for the nonlinear LuGre model should be built with Matlab Simulink. On the modelling and simulation of friction, the bristle model was brought up to catch the characteristics of tiny contact point [3]. The stain of bristle produces the friction force. Mathematically, the magnitude of friction can be described as $F=\sum_i^N \sigma_0(x_i-b_i)$, where N is the number of bristle, σ_0 is stiffness of bristle, x_i is relative position of bristle, b_i is the position of crossover took place [3].

Efficient Simulation of a Dynamic System with LuGre Frictionwas illustrated in reference [3]. This paper shows that LuGre model exhibits very slow dynamics during periods of sticking and very fast dynamics during periods of slip. Different simulation strategies are investigated. It is found that the Runge-Kutta method had better accuracy, but the Radau-IIA method required less integration steps [4]. In the future, controller or observer design is followed withthe result of this study on nonlinear system modelling and analysis.

LuGre-model based friction compensation was written by L. Freidovich *et. al.*, the main content of the result wasto give some new methods the numerical real-time implementation of a compensator for disturbances describable by one of various LuGre models with the knowledge of Lyapunov function [5].The Dynamics study of the stick-slip driving system based on LuGre dynamic friction modelwas proposed by B. Zhong *et. al.*, they gavea simulation to dynamics model of stick-slip system based on LuGre friction model with PiezoelectricTransducer (PZT) in Matlab Simulink software [6].

Nonlinear modelling and evaluation of rolling friction was also derived by Y. Maeda *et. al.*, and they presented a nonlinear friction modeling and its simulation for rolling friction behaviors in ball screw-driven table systems. The paper evaluate rolling friction characteristic with Bode plot [7]. Very detailed characteristics of rolling friction were evaluated in the paper. As can be seen clearly from the above literature review, there has been a great deal of researches about friction especially LuGre friction. However, there still have many limitations. For instance, the research of efficient simulation of a dynamic system with LuGre friction and many other papers give a very detailed study in simulation the characteristics of different friction model. But the controller or observer design has not introduced or studied in detail. Therefore, the study of controller or observer design should be carried out in this study to make specialized research in the future.

2. Modelling and Analysis

There has been a great deal of studies about friction, which indicates friction is a complex phenomenon with many different types and characteristics. The magnitude of friction depends not only on the geometrical shape of two contact surfaces, but also the mass, relative velocity, and displacement. Due to complexity of friction, different friction models have been presented.

C. Coulomb studied and summarized the experiments and researches of L. Vinci and G. Amonton. Then he carried out a series of his experiments. Finally, the Coulomb friction law was brought. The Coulomb friction model, the friction force depends on the velocity direction, which is a linear viscous friction. Another friction model was developed in 1950s, known as Dahl model. The stick-slip motion cannot be simulated by Dahl model. That is because that even Dahl model gives a consideration of many properties of friction but does not consider the Stribeck effect.

LuGre model [8 – 10], is collaboration between control groups in Lund and Grenoble, an extension of Dahl friction model with many of the observed features of frictional behavior, which is derived from Dahl model and captures the Stribeck effect. Many kinks of dynamic and staticstate characteristics are described by LuGre model accurately, for example, stick slip, hunting, pre-sliding displacement, friction memory, rising static friction, and static Stribeck curve [9]. Relative velocity leads to bristle deformation. The deformation of bristle can cause friction, as shown in Figure 1.

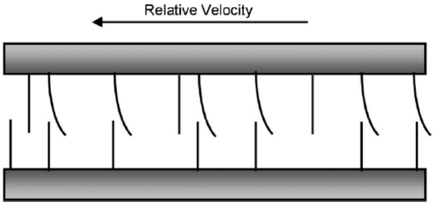


Figure 1. Bristle contact with each other

The LuGre model can be described by following equations,

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{g(v)} z = v - h(v) z \qquad (1)$$

$$F = \sigma_0 z + \sigma_1 \dot{z} + f(v) \qquad (2)$$

The parameter v represents the velocity between the two surface in contact, parameter z is the internal friction state variable, $\mathbf{F}$ is the predicted friction force, the parameter σ_0 is the stiffness, σ_1 is the micro damping, and $f(v)$ represents viscous friction and usually equals to $\sigma_2 v$. Compared with Dahl friction model, use a constant instead of $g(v)$, shown in equation (3).

$$\frac{dz}{dt} = v - \sigma_0 \frac{|v|}{F_c} z \qquad (3)$$

For constant velocity, the steady state friction force is described by equation (4).

$$F_{ss}(v) = g(v) sgn(v) + f(v) \qquad (4)$$

A reasonable approximation for $g(v)$ of the Stribeck is,

$$g(v) = F_c + (F_s - F_c) e^{-\left|\frac{v}{v_s}\right|^{\alpha}} \qquad (5)$$

F_s is the stiction friction force, and F_c is the Coulomb friction force. The general curve of $g(v)$ is shown in Figure 1. As can been seen clearly from Figure 1 that $g(v)$ is in the range of $F_c \leq g(v) \leq F_s$. Moreover, v_s determine how quickly $g(v)$ approach to F_c.

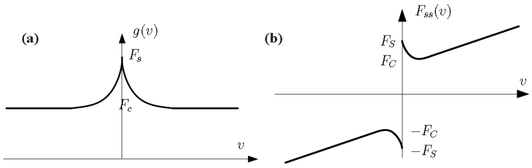


Figure 2. (a) Coulomb friction and Stribeck effect
(b) $F_{ss}(v) = g(v) sgn(v) + f(v)$.

A classic example, spring-mass experiment for stick-slip motion system is brought up to simulate the LuGre friction model.

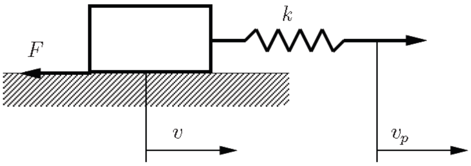


Figure 3. The spring-mass experiment for stick-slip motion.

Figure 3 illustrates a typical stick-slip experiment, where mass is pulled by a spring. Experiment of stick-slip motion is analysed using the LuGre friction model.

$$l = v_p - v \tag{6}$$

$$m\dot{v} = kl - F \tag{7}$$

$$\dot{z} = v - \sigma_0 \frac{|v|}{g(v)} z = v - h(v)z \tag{8}$$

The $h(v) = \sigma_0 \frac{|v|}{g(v)}$ can be derived from equation (8). The friction F is the friction between two surfaces can be described as,

$$F = \sigma_0 z + \sigma_1 \dot{z} + f(v) = \sigma_1 v + f(v) + [\sigma_0 - \sigma_1 h(v)] \tag{9}$$

l is the elongation of pulling spring.

Jacobian matrix gives an optimal linear approximation of a differentiable equation. Therefore, Jacobian matrix can be used to linearize the LuGre model. In this article, Jacobian matrix J of equation (6), (7), (8), is,

$$J = \begin{bmatrix} 0 & -1 & 0 \\ \frac{k}{m} & -\frac{\sigma_1\left(1 - zh'(v)\right) - f'(v)}{m} & -\frac{\sigma_0 - \sigma_1 h(v)}{m} \\ 0 & 1 - zh' & -h \end{bmatrix} \tag{10}$$

As mentioned before, $f(v)$ represents viscous friction and usually equals to $\sigma_2 v$. The linear approximation is given by $z = v = 0$. Then the Jacobian matrix can be approximated to,

$$J = \begin{bmatrix} 0 & -1 & 0 \\ \frac{k}{m} & -\frac{\sigma_1 + \sigma_2}{m} & -\frac{\sigma_0}{m} \\ 0 & 1 & 0 \end{bmatrix} \tag{11}$$

The characteristic equation of equation (11) is,

$$a(s) = s^3 + s\left(\frac{\sigma_1 + \sigma_2}{m} + \frac{\sigma_0 + k}{m}\right) \tag{12}$$

LuGre friction model is described by function (6), (7), (8), and (9). The parameter F_c, F_s, σ_2, v_s can be determined by the friction measured in steady state. From equation (9), when system is in steady state is obtained as,

$$F_{ss}(v) = \sigma_0 g(v) sgn(v) + \sigma_2 v = [F_c + (F_s - F_c)e^{-\left|\frac{v}{v_s}\right|^{\alpha}}] sgn(v) + \sigma_2 v \tag{10}$$

Equation (10) also indicates that LuGre model is classical model in steady state. However, LuGre model considers an important characteristic, which friction relate to position.

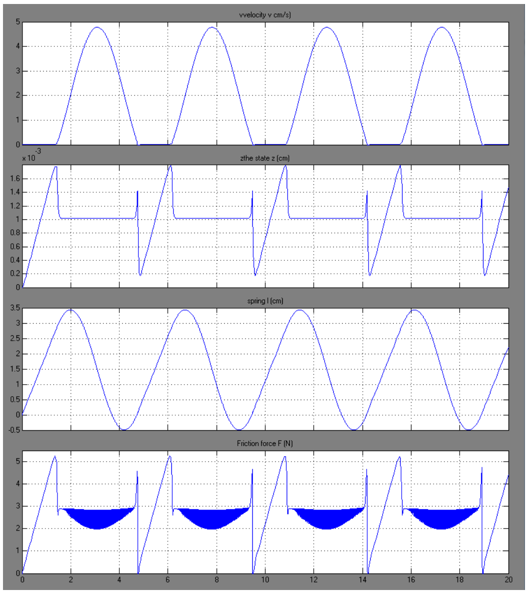


Figure 4. Simulink simulation results for LuGre friction model

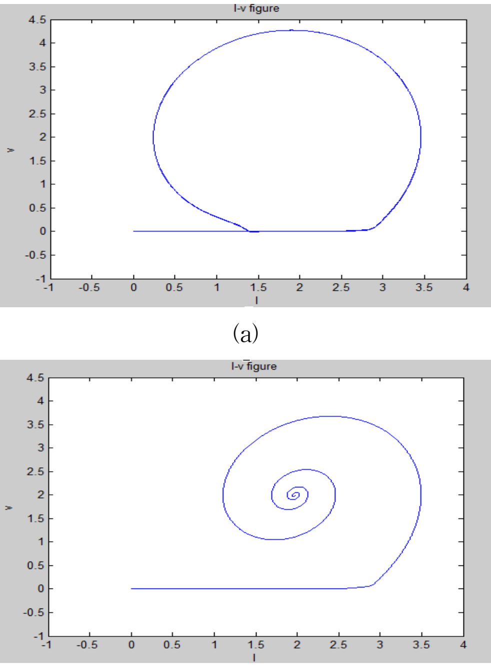
The first curve is the velocity v (cm/s). The second curve is state variable z. The third curve is elongation of spring l (cm). The fourth curve is friction force F (N). The parameter used in the model is $m = 1$, $k = 2$, $v_p = 2$, $f = 0$. The parameter for function (9) is $\sigma_0 = 2900$, $\sigma_1 = 107$, $f(v) = 0$. The parameter for function (5) is $F_c = 2.94, F_s = 5.88, v_s = 0.1, \alpha = 1$.

Figure 4 fully illustrates the characteristics of stick-slip motion. All of four curves are periodic, and period is 4.75s.Furthermore, there are two different modes, in the stick slip motion. In the stick mode, as shown in the first curve in the figure, the velocity is small. In the meantime, Friction force increase when it reaches maximum static friction. The system settles at steady state with a small displacement. Then the friction is large enough to stop the motion. The reason friction state and friction force decrease in 4.75s before the increase is to compensate spring force.The mass moves when spring force is large enough, and the system enter to the slip mode.In the slip mode, the mass begin to move. And the velocity keeping rising until after the spring begin to compress. The elongation of spring repeats periodically from the third curve. Finally, the velocity and friction return to zero. The next period begin immediately.

The same block diagram for LuGre friction is used to determine which parameters influence the whole system. The velocity of mass depends on the parameters. Because we want capsule endoscopy to stay longer inside human body to carry out inspection in detail. It is necessary to investigate the parameters that influence velocity mass. The following block diagrams are design to find out the parameters that influence the model.

- Viscous damping $f(v) = \sigma_2 v$

The first parameter that we investigate is viscous damping $f(v) = \sigma_2 v$. The $l - v$ figures describe the relationship between spring elongation and velocity. And the full simulations of system illustrate the detail movement under different viscous damping.

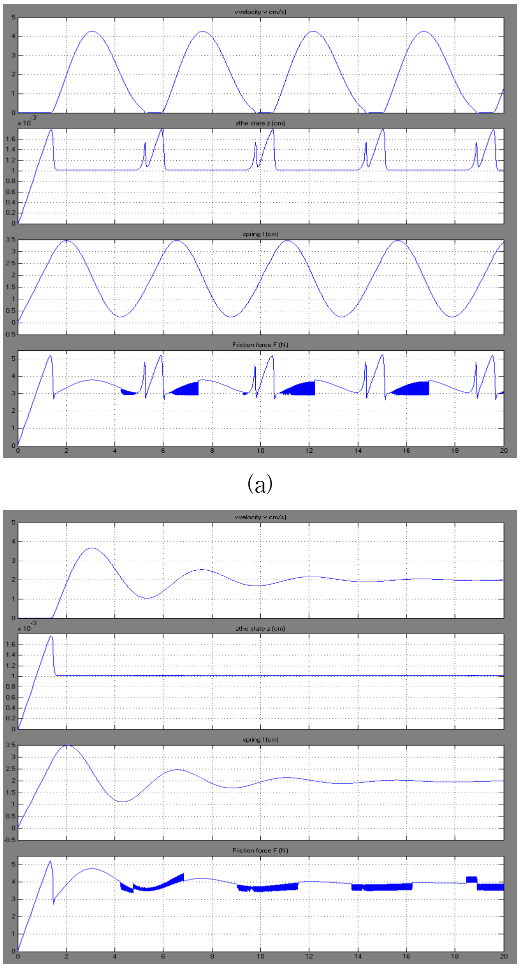


(a)

(b)

Figure 5. Simulink simulation $l - v$ figure for of LuGre model, (a) $\sigma_2 = 0.2$, (b) $\sigma_2 = 0.5$

As can be seen clearly from Figure 5, the velocity fluctuate become smaller with the increase of viscous friction σ_2. The small viscous friction showed in the in Figure 9(a) makes the system to become a cycle movement. The system becomes a non-periodic system, when viscous friction σ_2 reaches a larger value. And the system is under damped with overshoot. Finally, the critical damping is reached, when $\sigma_2 = 2\sqrt{2}$. The velocity gets into a constant value without any overshoot.



(a)

(b)

Figure 6. Simulation results corresponding to different viscous damping $f(v) = \sigma_2 v$, (a) for $\sigma_2 = 0.2$, (b) for $\sigma_2 = 0.5$.

The velocity gets into a constant value without any overshoot. The system under critical damping gives a good method to make capsule endoscopy move with a constant velocity inside human body.

- **Pulling velocity v_p**

The second parameter that we investigate is pulling velocity v_p. The $l - v$ figures describe the relationship between spring elongation and velocity. And the full simulations of system illustrate the detail movement under different pulling velocity. In order to achieve high velocity need high pulling velocity v_p.

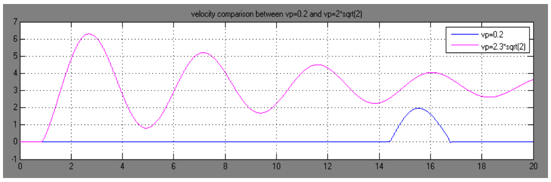


Figure 7. Velocity comparison between $v_p = 0.2$ and $v_p = 2.3\sqrt{2}$

All the simulation results indicate that small velocity v_p is expected to make the velocity of mass keep zero for the most time of a period, which means small velocity v_p is necessary to keep a capsule endoscopy stay inside body for long time to take more detailed diagnosis. However, if a capsule endoscopy is expected to move fast inside body, large velocity v_p is preferred.

- **Spring stiffness k**

The third parameter that we investigate is spring stiffness k. The $l - v$ figures describe the relationship between spring elongation and velocity. And the full simulations of system illustrate the detail movement under different spring stiffness k. The change of velocity spring stiffness k doesn't give much change to velocity but to the spring elongation l.

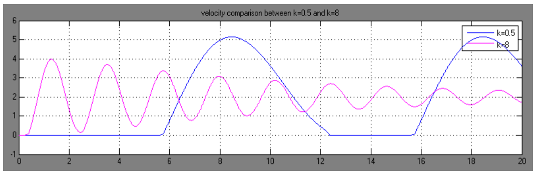


Figure 8. Velocity comparison between $k = 0.5$ and $k = 8$

Compare with the huge influence the spring stiffness k caused to spring elongation l, the change of spring stiffness k from 0.5 to 8, only change 1 to velocity from Figure 8. The simulation results show that the change of spring stiffness k mainly influences the spring elongation l and cycle of movement consequently.

3. Conclusions

The stick-slip motion system based on LuGre friction model is fully investigated in this paper to find out how and what parameters influence the system. The Matlab Simulink software is used to simulate LuGre friction model and explore the parameters influence the system. Firstly, viscous damping $f(v) = \sigma_2 v$ can be set as a critical damping value in order to make velocity keeps constant when mass move. Secondly, pulling velocity v_p influence the velocity of mass, small pulling velocity v_p would make the velocity of mass remain zero for 14.3s then only move 1s, which is a good method to keep mass at the same place for most of the movement cycle. Thirdly, the change of spring stiffness k mainly influences the spring elongation l and cycle of movement consequently. However, the velocity doesn't change much with spring stiffness k. Every parameter is investigated to find out how it influencesthe mass velocity, which will be important to future research about control method about LuGre model.

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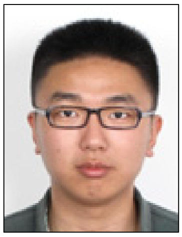

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