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EXISTENCE OF SOLUTIONS FOR ELLIPTIC SYSTEM WITH NONLINEARITIES UNDER THE DIRICHLET BOUNDARY CONDITION

Hyewon Nam

Abstract. By linking theorem, we prove the existence of nontrivial solutions for the elliptic system with jumping nonlinearity and growth nonlinearity and Dirichlet boundary condition.

1. Introduction and main result

Presently there are many significant results with respect to the nonlinear elliptic equation and system with Dirichlet boundary condition [2, 6, 8, 9]. Many authors also investigated the nonlinear elliptic equation and system with jumping nonlinearity and subcritical growth nonlinearity and Dirichlet boundary condition [4, 5, 7].

In this paper, we consider the existence of nontrivial solutions to the elliptic system

$$(1)\quad \begin{cases} -\triangle u = au + bv + \delta_1(u^+)^{p_1} - \eta_1(u^-) + f_1(x,u,v) & \text{in } \Omega, \\ -\triangle v = bu + cv + \delta_2(v^+)^{p_2} - \eta_2(v^-) + f_2(x,u,v) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega. \end{cases}$$

where $u^+ = \max\{0, u(x)\}$, $u^- = -\min\{0, u(x)\}$ and $\Omega \subset R^N$ be a smooth bounded domain with $N \geq 2$.

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The nonlinearities will be assumed both superlinear and subcritical, that is, $1 < p_1, p_2 < 2^* - 1$, where $2^* = \frac{2N}{N-2}$ if $N \geq 3$ and $2^* = \infty$ if $N = 2$.

And there exists a function $F : \bar{\Omega} \times R^2 \to R$ such that $\frac{\partial F}{\partial u} = f_1$ and $\frac{\partial F}{\partial v} = f_2$ without loss of generality, we set

$$F(x, u, v) = \int_{(0,0)}^{(u,v)} f_1(x, u, v)du + f_2(x, u, v)dv.$$

Then $F \in C^1(\bar{\Omega} \times R^2, R)$.

We consider the following assumptions.

(F1) There exist $M > 0$ and $\alpha > 2$ such that

$$0 < \alpha F(x, u, v) \leq uF_u(x, u, v) + vF_v(x, u, v)$$

for all $(x, u, v) \in \bar{\Omega} \times R^2$ with $u^2 + v^2 > M^2$.

(F2) There exist constants $a_1 > 0$ and $a_2 > 0$ such that

$$|F_u(x, u, v)| + |F_v(x, u, v)| \leq a_1 + a_2(|u|^r + |v|^r)$$

where $1 \leq r < \frac{(N+2)}{(N-2)}$ if $N > 2$ and $1 \leq r < \infty$ if $N = 2$.

(F3) For $(0, v) \to (0, 0)$,

$$\frac{F(x, 0, v)}{v^2} \to 0.$$

Remark 1.1. The condition (F1) shows that there exist constants $b_1 > 0$ and b_2 such that(cf. [1])

$$F(x, u, v) \geq b_1(|u|^\alpha + |v|^\alpha) - b_2.$$

Our main result is the following.

Theorem 1.1. *Assume F satisfies (F1), (F2) and (F3) with $\alpha = r+1$. If a, b, c, δ_1, δ_2, η_1, and η_2 are positive with $a + b + \eta_1 < \lambda_1$ and $b + c + \eta_2 < \lambda_1$ then system (1) has at least two nontrivial solutions.*

In this paper we prove the existence of two nontrivial solutions for the elliptic system with jumping nonlinearity and growth nonlinearity and Dirichlet boundary condition. In Section 2, we use a variational approach to look for critical points of the functional I on a Hilbert space H. In Section 3, we prove the Palais Smale star condition for the linking theorem. And we prove the Lemmas in order to applyting the linking theorem, so we prove Theorem 1.1.

2. Preliminaries

Let H be a Hilbert space and V a C^2 complete connected Finsler manifold. Suppose $H = H_1 \oplus H_2$ and let $H_n = H_{1n} \oplus H_{2n}$ be a sequence of closed subspaces of H such that

$$H_{in} \subset H_i, \quad 1 \leq \dim H_{in} < +\infty \quad \text{for each} \quad i = 1, 2 \quad \text{and} \quad n \in N$$

Moreover suppose that there exist $e_1 \in \cap_{n=1}^{\infty} H_{1n}$, and $e_2 \in \cap_{n=1}^{\infty} H_{2n}$, with $\|e_1\| = \|e_2\| = 1$.

For any Y subspace of H, consider $B_\rho(Y) := \{u \in Y | \|u\| \leq \rho\}$ and denote by $\partial B_\rho(Y)$ the boundary of $B_\rho(Y)$ relative to Y. Furthermore define, for any $e \in H$,

$$Q_R(Y, e) := \{u + ae \in Y \oplus [e] | u \in Y, a \geq 0, \|u + ad\| \leq R\}$$

and denote by $\partial Q_R(Y, e)$ its boundary relative to $Y \oplus [e]$, and denote by $X = H \times V$.

We recall the two critical points theorem in [3].

THEOREM 2.1. *Suppose that f satisfies the $(PS)^*$ condition with respect to H_n. In addition assume that there exist ρ, R, such that $0 < \rho < R$ and*

$$\sup_{\partial Q_R(H_2, e_1) \times V} f \quad < \quad \inf_{\partial B_\rho(H_1) \times V} f,$$
$$\sup_{Q_R(H_2, e_1) \times V} f < +\infty, \qquad \inf_{B_\rho(H_1) \times V} f < -\infty,$$

Then there exist at least 2 critical levels of f. Moreover the critical levels satisfy the following inequalities

$$\inf_{B_\rho(H_1) \times V} f \leq c_1 \leq \sup_{\partial Q_R(H_2, e_1) \times V} f < \inf_{\partial B_\rho(H_1) \times V} f \leq c_2 \leq \sup_{Q_R(H_2, e_1) \times V} f,$$

and there exist at least $2 + 2$ cuplength(V) critical points of f.

3. Main result

We will prove the existence of nontrivial solutions by using linking theorem.

3.1. The variational structure.

Throughout the paper, we will denote by λ_k the eigenvalues and by e_k the corresponding eigenfunctions, suitably normalized with respect to $L^2(\Omega)$ inner product, of the eigenvalue problem $-\Delta u = \lambda u$ in Ω, with Dirichlet boundary condition, where each eigenvalue λ_k is respected as often as its multiplicity. We recall that $0 < \lambda_1 < \lambda_2 \le \lambda_3 \le \cdots, \lambda_i \to +\infty$ and that $e_1 > 0$ for all $x \in \Omega$. Then $H = \text{span}\{e_i | i \in N\}$, where $H = W_0^{1,p}(\Omega)$, the usual Sobolev space with the norm $\|u\|^2 = \int_\Omega |\nabla u|^2 dx$.

Let $e_i^1 = (e_i, 0)$ and $e_i^2 = (0, e_i)$. We define $H_j = \text{span}\{e_i^j | i \in N\}$, for $j = 1, 2$ and $E = H_1 \oplus H_2$ with the norm $\|(u,v)\|_E^2 = \|u\|^2 + \|v\|^2$.

We define the energy functional associated to (1) as

$$
\begin{aligned}
I(u,v) &= \frac{1}{2}\int_\Omega \left(|\nabla u|^2 + |\nabla v|^2\right) dx - \frac{1}{2}\int_\Omega \left(au^2 + 2buv + cv^2\right) dx \\
&\quad - \int_\Omega \left(\frac{\delta_1}{p_1+1}(u^+)^{p_1+1} + \frac{\delta_2}{p_2+1}(v^+)^{p_2+1}\right) dx \qquad (2)\\
&\quad + \int_\Omega \left(\frac{\eta_1}{2}(u^-)^2 + \frac{\eta_2}{2}(v^-)^2\right) dx - \int_\Omega F(x,u,v) dx
\end{aligned}
$$

It is easy to see that $I \in C^1(E, R)$ and thus it makes sense to lock for solutions to (1) in weak sense as critical points for I *i.e.* $(u,v) \in E$ such that $I'(u,v) = 0$, where

$$
\begin{aligned}
I'(u,v)\cdot(\phi,\psi) &= \int_\Omega (\nabla u \nabla \phi + \nabla v \nabla \psi)\, dx \\
&\quad - \int_\Omega (au\phi + bv\phi + bu\psi + cv\psi)\, dx \\
&\quad - \int_\Omega \left(\delta_1 (u^+)^{p_1}\phi + \delta_2 (v^+)^{p_2}\psi\right) dx \\
&\quad + \int_\Omega \left(\eta_1 (u^-)\phi + \eta_2 (v^-)\psi\right) dx \\
&\quad - \int_\Omega (f_1(x,u,v)\phi + f_2(x,u,v)\psi)\, dx.
\end{aligned}
$$

3.2. The Palais Smale star condition.

In this section we will prove the $(PS)_c^*$ condition which was required for the application of Theorem 2.1. In the following, we consider the following sequence of subspaces of E :

$$E_n = \text{span}\{e_i^j | i = 1, \cdots, n \quad \text{and} \quad j = 1, 2\}, \qquad \text{for } n \geq 1.$$

LEMMA 3.1. *Assume F satisfies (F1) and (F2) with $\alpha = r + 1$. If $a+b+\eta_1 < \lambda_1$ and $b+c+\eta_2 < \lambda_1$, then any $(PS)_c^*$ sequence is bounded.*

Proof. Let $\{(u_n, v_n)\} \subset E$ be a sequence such that

$$(u_n, v_n) \in E_n, \quad I(u_n, v_n) \to c, \quad I'(u_n, v_n) \to 0 \quad \text{as} \quad n \to \infty.$$

To show the contradiction, we assume that $\{(u_n, v_n)\}$ is not bounded *i.e.* $\|(u_n, v_n)\|_E \to \infty$.

In the following we denote different constants by C_1, C_2 etc.

$$\begin{aligned}
C_1 &+ \frac{1}{2}o(1)\left(\|u_n\| + \|v_n\|\right) \\
&\geq I(u_n, v_n) - \frac{1}{2}I'(u_n, v_n) \cdot (u_n, v_n) \\
&= \int_\Omega \left(\frac{\delta_1(p_1 - 1)}{2(p_1 + 1)}({u_n}^+)^{p_1+1} + \frac{\delta_2(p_2 - 1)}{2(p_2 + 1)}({v_n}^+)^{p_2+1} \right) dx \\
&\quad + \frac{1}{2}\int_\Omega (u_n f_1 + v_n f_2) dx - \int_\Omega F(x, u_n, v_n) dx.
\end{aligned} \tag{3}$$

(F1) and Remark imply that

$$\begin{aligned}
\frac{1}{2}\int_\Omega (u_n f_1 + v_n f_2) dx \quad &- \int_\Omega F(x, u_n, v_n) dx \\
&\geq \left(\frac{\alpha}{2} - 1\right) \int_\Omega F(x, u_n, v_n) dx \\
&\geq \left(\frac{\alpha}{2} - 1\right) b_1 \int_\Omega (|u_n|^\alpha + |v_n|^\alpha) dx - C_2 \\
&\geq \left(\frac{\alpha}{2} - 1\right) b_1 (\|u_n\|_{L^\alpha}^\alpha + \|v_n\|_{L^\alpha}^\alpha) - C_2.
\end{aligned} \tag{4}$$

Combining (3), (4), we obtain

$$\begin{aligned}
C_1 &+ \frac{1}{2}o(1)\left(\|u_n\| + \|v_n\|\right) \\
&\geq \int_\Omega \left(\frac{\delta_1(p_1 - 1)}{2(p_1 + 1)}({u_n}^+)^{p_1+1} + \frac{\delta_2(p_2 - 1)}{2(p_2 + 1)}({v_n}^+)^{p_2+1} \right) dx \\
&\quad + \left(\frac{\alpha}{2} - 1\right) b_1 (\|u_n\|_{L^\alpha}^\alpha + \|v_n\|_{L^\alpha}^\alpha) - C_2
\end{aligned} \tag{5}$$

Since $\alpha > 2$ and $b_1 > 0$, we get

$$\begin{aligned}\frac{\delta_1(p_1-1)}{2(p_1+1)}\int_\Omega ({u_n}^+)^{p_1+1}dx \quad &+ \quad \frac{\delta_2(p_2-1)}{2(p_2+1)}\int_\Omega ({v_n}^+)^{p_2+1}dx \\ &\le \quad C_3 + \frac{1}{2}o(1)\left(\|u_n\| + \|v_n\|\right).\end{aligned}$$

By observing that each term in the expression above is nonnegative, we conclude that the estimate from above holds for each of them, and then

$$(6)\ \frac{1}{\|(u_n,v_n)\|_E}\int_\Omega ({u_n}^+)^{p_1+1}dx \to 0, \frac{1}{\|(u_n,v_n)\|_E}\int_\Omega ({v_n}^+)^{p_2+1}dx \to 0.$$

On the other hand

$$\begin{aligned}o(1)\|u_n\| \quad &\ge \quad I'(u_n,v_n)\cdot(u_n,0) \\ &= \quad \|u_n\|^2 - \int_\Omega (a{u_n}^2 + bu_nv_n)dx \\ &\quad\ - \int_\Omega \left(\delta_1({u_n}^+)^{p_1+1} - \eta_1({u_n}^-)^2\right)dx - \int_\Omega u_nf_1(x,u_n,v_n)dx, \\ o(1)\|v_n\| \quad &\ge \quad I'(u_n,v_n)\cdot(0,v_n) \\ &= \quad \|v_n\|^2 - \int_\Omega (bu_nv_n + c{v_n}^2)dx \\ &\quad\ - \int_\Omega \left(\delta_2({v_n}^+)^{p_2+1} - \eta_2({v_n}^-)^2\right)dx - \int_\Omega v_nf_2(x,u_n,v_n)dx.\end{aligned}$$

We know that

$$\int_\Omega (u^-)^2dx \le \int_\Omega u^2dx \le \frac{1}{\lambda_1}\|u\|^2$$

for any $u \in H$. Using (F2), we obtain

$$
\begin{aligned}
\|u_n\|^2 + \|v_n\|^2 &\leq o(1)(\|u_n\| + \|v_n\|) + \int_\Omega (a{u_n}^2 + 2bu_nv_n + c{v_n}^2)dx \\
&\quad + \int_\Omega \left(\delta_1({u_n}^+)^{p_1+1} + \delta_2({v_n}^+)^{p_2+1}\right) dx \\
&\quad - \int_\Omega \left(\eta_1({u_n}^-)^2 + \eta_2({v_n}^-)^2\right) dx \\
&\quad + \int_\Omega (u_nf_1(x,u_n,v_n) + v_nf_2(x,u_n,v_n))dx \qquad (7)\\
&\leq o(1)(\|u_n\| + \|v_n\|) + \frac{a+b+\eta_1}{\lambda_1}\|u_n\|^2 + \frac{b+c+\eta_2}{\lambda_1}\|v_n\|^2 \\
&\quad + \int_\Omega \left(\delta_1({u_n}^+)^{p_1+1} + \delta_2({v_n}^+)^{p_2+1}\right) dx \\
&\quad + C_4\int_\Omega (|u_n|^{r+1} + |v_n|^{r+1})dx + C_5.
\end{aligned}
$$

(7) imply that if $a+b+\eta_1 < \lambda_1$ and $b+c+\eta_2 < \lambda_1$ then

$$
\begin{aligned}
\|u_n\|^2 + \|v_n\|^2 &\leq o(1)C_6(\|u_n\| + \|v_n\|) \\
&\quad + \int_\Omega \left(\delta_1({u_n}^+)^{p_1+1} + \delta_2({v_n}^+)^{p_2+1}\right) dx \qquad (8)\\
&\quad + C_7\int_\Omega (|u_n|^{r+1} + |v_n|^{r+1})dx + C_8.
\end{aligned}
$$

Combining (5), (8) and using $\alpha = r+1$, one infers that

$$
\begin{aligned}
\|u_n\|^2 + \|v_n\|^2 &\leq o(1)C_8(\|u_n\| + \|v_n\|) + C_9 \\
&\quad + C_{10}\int_\Omega \left(\delta_1({u_n}^+)^{p_1+1} + \delta_2({v_n}^+)^{p_2+1}\right) dx.
\end{aligned}
$$

We get

$$
\begin{aligned}
\|(u_n,v_n)\|_E &\leq \frac{o(1)C_8(\|u_n\| + \|v_n\|) + C_9}{\|(u_n,v_n)\|_E} \\
&\quad + \frac{C_{10}}{\|(u_n,v_n)\|_E}\int_\Omega \left(\delta_1({u_n}^+)^{p_1+1} + \delta_2({v_n}^+)^{p_2+1}\right) dx \to 0
\end{aligned}
$$

which, by using (6), imply that $\|(u_n,v_n)\|_E \to 0$ This gives rise to a contradiction to the assumtion of $\{(u_n,v_n)\}$. We conclude that $\{(u_n,v_n)\}$ is bounded. □

Lemma 3.2. *Assume F satisfies (F1) and (F2) with $\alpha = r+1$. If $a+b+\eta_1 < \lambda_1$ and $b+c+\eta_2 < \lambda_1$, then the functional I satisfies the $(PS)_c^*$ condition with respect to E_n.*

Proof. By Lemma 3.1, any $(PS)_c^*$ sequence $\{(u_n, v_n)\}$ in E is bounded and hence $\{(u_n, v_n)\}$ has a weakly convergent subsequence. That is there exist a subsequence $\{(u_{n_j}, v_{n_j})\}$ and $(u,v) \in E$, with $u_{n_j} \rightharpoonup u$ and $v_{n_j} \rightharpoonup v$. Since $\{u_{n_j}\}$ and $\{v_{n_j}\}$ are bounded, by Remark of Rellich-Kondrachov compactness theorem [4], $u_{n_j} \to u$, $v_{n_j} \to v$ and thus I satisfies $(PS)_c^*$ condition. □

3.3. Proof of main theorem.

Lemma 3.3. *Assume F satisfies (F3). If $c < \lambda_1$, then there exists $\rho_1 > 0$ such that*

$$\inf_{\partial B_{\rho_1}(H_2)} I > 0.$$

Proof. By (F3), for any $\varepsilon > 0$, there exists $\rho > 0$ such that

$$0 < \|v\| < \rho \quad \Rightarrow \quad |F(x,0,v)| < \varepsilon |v|^2.$$

Then

$$\left| \int_\Omega F(x,0,v)dx \right| < \int_\Omega |F(x,0,v)|dx < \int_\Omega \varepsilon |v|^2 dx < \frac{\varepsilon}{\lambda_1} \|v\|^2.$$

By the continuous embedding of H in L^{p_2+1}, we get

$$\int_\Omega \frac{(v^+)^{p_2+1}}{p_2+1} dx \le \int_\Omega \frac{|v|^{p_2+1}}{p_2+1} dx \le \beta \|v\|^{p_2+1},$$

where β is a positive constant.

and hence

$$\begin{aligned} I(0,v) &= \frac{1}{2}\int_\Omega |\nabla v|^2 dx - \frac{c}{2}\int_\Omega v^2 dx - \frac{\delta_2}{p_2+1}\int_\Omega (v^+)^{p_2+1} dx \\ &\quad + \frac{\eta_2}{2}\int_\Omega (v^-)^2 dx - \int_\Omega F(x,0,v)dx \\ &> \frac{1}{2}\|v\|^2 - \frac{c}{2\lambda_1}\|v\|^2 - \beta\delta_2 \|v\|^{p_2+1} - \frac{\varepsilon}{\lambda_1}\|v\|^2 \\ &> \frac{1}{2}\left(1 - \frac{c+2\varepsilon}{\lambda_1} - 2\beta\delta_2\rho^{p_2-1}\right)\|v\|^2 > 0 \end{aligned}$$

which gives the result for sufficiently small ε and ρ. Therefore we can choose $0 < \rho_1 < \rho$ such that $I(0, v) > 0$ for any $\|v\| = \rho_1$. □

LEMMA 3.4. *Assume F satisfies (F1). If a, b, c, δ_1, δ_2, η_1, and η_2 are positive, then there exists an $R > 0$ such that for any $R_1 > R$*

$$\sup_{\partial Q_{R_1}(H_1, e_1^2)} I < 0.$$

Proof. In the following we denote different constants by C_1, C_2 etc. Remark 1.1 implies that

$$\begin{aligned}
I(u, \beta e_1) &= \frac{1}{2}\int_\Omega |\nabla u|^2 dx + \frac{\lambda_1 \beta^2}{2} - \frac{a}{2}\int_\Omega u^2 dx - b\beta \int_\Omega u e_1 dx - \frac{c\beta^2}{2} \\
&\quad - \frac{\delta_1}{p_1+1}\int_\Omega (u^+)^{p_1+1} dx - \frac{\delta_2}{p_2+1}\int_\Omega ((\beta e_1)^+)^{p_2+1} dx \\
&\quad + \frac{\eta_1}{2}\int_\Omega (u^-)^2 dx + \frac{\eta_2}{2}\int_\Omega ((\beta e_1)^-)^2 dx - \int_\Omega F(x, u, \beta e_1) dx \\
&\le \frac{1}{2}\|u\|^2 + \frac{\lambda_1\beta^2}{2} - \frac{b\beta}{2}\|u\|^2 - \frac{b\beta}{2} \\
&\quad + \frac{\eta_1}{2}\int_\Omega (u^-)^2 dx + \frac{\eta_2}{2}\int_\Omega ((\beta e_1)^-)^2 dx - \int_\Omega F(x, u, \beta e_1) dx \\
&\le \frac{1}{2}\|u\|^2 + \frac{\lambda_1\beta^2}{2} - \frac{b\beta}{2}\|u\|^2 - \frac{b\beta}{2} + \frac{\eta_1}{2\lambda_1}\|u\|^2 + \frac{\eta_2\beta^2}{2\lambda_1} \\
&\quad - b_1 \int_\Omega (|u|^\alpha + |\beta e_1|^\alpha) dx + C_1 \\
&\le \frac{\lambda_1 - b\beta\lambda_1 + \eta_1}{2\lambda_1}\|u\|^2 + \frac{(\lambda_1^2 + \eta_2)\beta^2}{2\lambda_1} - \frac{b\beta}{2} \\
&\quad - C_2\|u\|^\alpha - C_3|\beta|^\alpha + C_4,
\end{aligned}$$

for any $(u, 0) \in H_1$ and any constant β. Since $\alpha > 2$, $I(u, \beta e_1) \to -\infty$ for $\|u\| \to \infty$ or $|\beta| \to \infty$. Therefore we can choose $0 < R_1 < \infty$ such that $I(u, \beta e_1) < 0$ for any $\|(u, \beta e_1)\|_E = R_1$. □

Proof of Theorem 1.1. By Lemma 3.3 and 3.4, there exists $0 < \rho_1 < R_1$ such that

$$\sup_{\partial Q_{R_1}(H_1, e_1^2)} I < 0 < \inf_{\partial B_{\rho_1}(H_2)} I.$$

By Theorem 2.1, $I(u,v)$ has at least two nonzero critical values c_1, c_2

$$\inf_{B_{\rho_1}(H_2)} I \leq c_1 \leq \sup_{\partial Q_{R_1}(H_1,e_1^2)} I < \inf_{\partial B_{\rho_1}(H_2)} I \leq c_2 \leq \sup_{Q_{R_1}(H_1,e_1^2)} I.$$

Therefore, (1) has at least two nontrivial solutions.

References

[1] A.Szulkin, *Critical point theory of Ljusterni-Schnirelmann type and applications to partial differential equations,* Sem. Math. Sup., Vol. 107, pp. 35–96, Presses Univ. Montreal, Montreal, QC, 1989.

[2] D.D.Hai, *On a class of semilinear elliptic systems,* J. Math. Anal. Appl., **285** (2003), 477–486.

[3] D.Lupo and A.M.Micheletti, *Two applications of a three critical points theorem,* J. Differential Equations **132** (1996) (2), 222–238.

[4] Eugenio Massa, *Multiplicity results for a subperlinear elliptic system with partial interference with the spectrum,* Nonlinear Anal., **67** (2007), 295–306.

[5] Hyewon Nam, *Multiplicity results for the elliptic system using the minimax theorem,* Korean J. Math. **16** (2008) (4), 511–526.

[6] Jin Yinghua, *Nontrivial solutions of nonlinear elliptic equations and elliptic systems,* Inha U. 2004.

[7] T. Jung and Q.H. Choi, *Multiple solutions result for the mixed type nonlinear elliptic problem,* Korean J. Math. **19** (2011), 423–436.

[8] Yukun An, *Mountain pass solutions for the coupled systems of second and fourth order elliptic equations,* Nonlinear Anal. **63** (2005), 1034–1041.

[9] Zou and Wenming, *Multiple solutions for asymptotically linear elliptic systems,* J. Math. Anal. Appl. **255** (2001) (1), 213–229.

Hyewon Nam
Department of General Education
Namseoul University
Chonan, 330-707 Korea
E-mail: hwnam@nsu.ac.kr