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Approximate Continuous Review Inventory Models with the Consideration of Purchase Dependence

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구매종속성을 고려한 근사적 연속검토 재고모형

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This paper introduces the existence of purchase dependence that was identified during the analysis of inventory operations practice at a sales agency of dealing with spare parts for ship engines and generators. Purchase dependence is an important factor in designing an inventory replenishment policy. However, it has remained mostly unaddressed. Purchase dependence is different from demand dependence. Purchase dependence deals with the purchase behavior of customers, whereas demand dependence deals with the relationship between item-demands. In order to deal with purchase dependence in inventory operations practice, this paper proposes (Q, r) models with the consideration of purchase dependence. Through a computer simulation experiment, this paper compares performance of the proposed (Q, r) models to that of a (Q, r) model ignoring purchase dependence. The simulation experiment is conducted for two cases : a case of using a lost sale cost and a case of using a service level. For a case of using a lost sale cost, this paper calculates an order quantity, Q and a reorder point, r using the iterative procedure. However, for a case of using a service level, it is not an easy task to find Q and r. The complexity stems from the interactions among inventory replenishment policies for items. Thus, this paper considers the genetic algorithm (GA) as an optimization method. The simulation results demonstrates that the proposed (Q, r) models incur less inventory operations cost (satisfies better service levels) than a (Q, r) model ignoring purchase dependence. As a result, the simulation results supports that it is important to consider purchase dependence in the inventory operations practice.

Keywords : Purchase Dependence, Demand Dependence, Inventory Model, Association Rule, Simulation

1. Introduction

This paper introduces a new type of dependence that we have faced during the analysis of inventory operations practice at a sale agency. The sale agency deals with spare parts for ship engines and generators. Shipping companies put several spare parts in one order and inquire whether a sales agency can deliver the required spare parts all at once within a short delivery window. If the sales agency cannot satisfy these requirements, the shipping companies contact other sales agencies. In spite of considerably large capital investments in stocking spare parts, the sale agency has experienced lower order fulfillment rates than expected. With the help of inventory operations managers, we have analyzed the historical transaction data to figure out the root causes and develop alternatives for improving the order fulfillment rates.

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In the course of scrutinizing the historical transaction data, we have found an interesting fact: considerably many orders have been cancelled due to a few other stock-out spare parts. In other words, if an order includes at least one stock-out spare part, the order itself is cancelled, even though other several spare parts have large quantities of stocks. Based on this experience, we have acknowledged that there exists a new type of dependence in the customer's purchase behavior of spare parts. In many orders, shipping companies buy spare parts only when all spare parts are in stock, because the shipping companies want to purchase all spare parts together. Under this purchase pattern, if one of the spare parts is not in stock, even though the other spare parts are in stock, the case is as good as they are out-of-stock.

In this paper, we define such kind of dependency as 'purchase dependence.' Purchase dependence is different from demand dependence. Purchase dependence deals with the purchase behavior of customers, whereas demand dependence deals with demand correlation between items [4, 10, 12], between locations [3, 5, 7, 18], or over time [6, 9, 11, 14-16]. In literature, Bala [1] and Bala et al. [2] mentioned purchase dependence for the first time as follows. In a store, item B is in stock and item A is out of stock. One customer is interested in purchasing item B, provided item A is also available in the store, so that he can purchase both items A and B. As the purchase for item B depends on the purchase of item A, he will not purchase item B, if item A is not available. Under this situation, a stock of item B is as good as a stock-out for that customer. Hence, if item A is not in stock, there will be no sale of item B either in many cases. This example depicts a case of purchase dependence.

In fact, however, the notion of Bala [1] and Bala et al. [2] is a kind of cross-selling effect considered by Wong et al. [17], Zhang et al. [20, 21], and Zhang [19]. They defined cross-selling as that the purchase of one item is related to the purchase of another item. In other words, sales of the major item may lead to additional demand for minor items. Thus, the minor items can be either sold independently or promoted by joint sales with the major item, which means that demand for the minor items will decrease while the major item is stocked out.

The definition of purchase dependence used by this paper is different from the notion used by the previous literature in that our definition does not discern a major item and a minor item. Additionally, we do not limit a directional dependence by one purchase on another. In other words, according to our definition, an order from customers can be cancelled if any items in the order are out of stock. Thus, all of items in the order will not be sold, even though some items in the order are available.

Knowledge of purchase dependence can be used in designing an inventory replenishment policy for better profitability and lesser cost of inventory operations. Given additional information of purchase dependence, however, finding an optimal inventory replenishment policy can become a complex problem [2]. The complexity of inventory problem requires a reliable methodology for determining the operating policy that optimizes the inventory control.

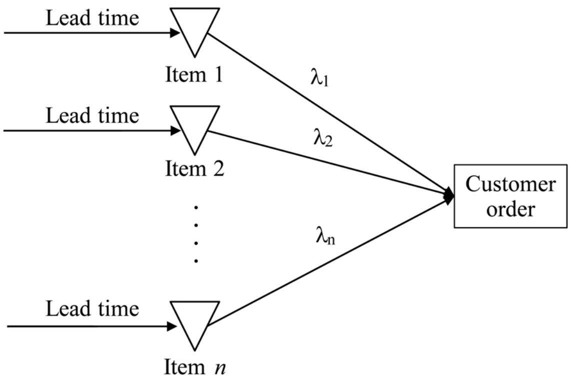
In this paper, we propose new inventory models utilizing knowledge of purchase dependence. With the proposed inventory models, we also demonstrate the impact of purchase dependence on inventory operations costs. Since the concept of purchase dependence is relatively new and the design of an inventory model with the consideration of purchase dependence is complex, we first consider approximate continuous review models that replenish each item separately. However, we will extend knowledge of purchase dependence to the joint replenishment problem in the future.

This paper is organized as follows. In the next section, we propose inventory models that consider purchase dependence. Then we conduct simulation experiments to justify performance of the proposed inventory models and illustrate the importance of purchase dependence in section 3. In section 4, this paper is closed by presenting a conclusion.

2. Model Formulation

We consider an inventory system with n items as shown in <Figure 1> under the environment in which purchase dependence exists. Customers request several items in one order. Each item in a customer order can be required by any quantity. All items requested in the order should be delivered immediately all together in one shipment. Otherwise, the customer cancels the order and leaves for other sources.

In order to help the reader understand easily the proposed inventory models, first we explain briefly an existing continuous review model that does not consider purchase dependence. Then we describe the proposed inventory models using knowledge of purchase dependence.



<Figure 1> A Multi-item Inventory System

2.1 An Existing (Q, r) Model

If it is assumed that purchase dependence does not exist, we can easily manage the inventory system with n items using the approximate approach for the lost sale case proposed by Hadley and Whitin [8]. In this section, we summarize briefly the approximate approach. Since each item can be controlled separately, we suppress subscripts. First, we list notations that appear in this section.

Notations

- λ Average annual demand
- π Cost of a lost sale including the lost profit
- μ Expected lead time demand
- x Random variable of lead time demand having a probability density function of $h(x)$
- $H(x)$ Complementary cumulative of $h(x)$
- $\bar{\eta}(r)$ Expected number of lost sales per cycle
- A Cost of placing an order
- I Inventory carrying charge
- C Unit cost of the item
- $\hat{T}$ Average length of time per cycle for which the system is out of stock
- K Average annual cost
- f The upper limit to the average fraction of time which the system is out of stock
- Q Order quantity
- r reorder point

2.1.1 The Approach of Using a Lost Sale Cost

Hadley and Whitin [8] showed that the minimization of the average annual cost is equivalent to the maximization of the average annual profit if in the cost expression the cost of a lost sale includes the lost profit. The average annual cost is composed of the ordering costs, the cost of carrying inventory, and the cost of a lost sale. Then,

$$K = \frac{\lambda}{Q}A + IC\left[\frac{Q}{2} + r - \mu\right] + \left(IC + \frac{\pi\lambda}{Q}\right) \left[\int_r^\infty x h(x)dx - rH(r)\right] \quad (1)$$

In general, the average number of cycles per year is not λ/Q but is instead $\lambda/(Q+\lambda\hat{T})$. Since $\hat{T}$ is usually a very small fraction of the total length of the cycle in the real world, it can be assumed that the value of $\hat{T}$ is small enough to be neglected, so that the average number of cycles per year is λ/Q.

The values of Q and r which minimize the K of equation (1) must satisfy the equations.

$$Q = \sqrt{\frac{2\lambda[A + \pi\bar{\eta}(r)]}{IC}} \quad (2)$$

$$H(r) = \frac{QIC}{\lambda\pi + QIC} \quad (3)$$

where

$$\bar{\eta}(r) = \int_r^\infty x h(x)dx - rH(r) \quad (4)$$

An iterative procedure for numerically solving Q and r is proposed by Hadley and Whitin [8].

2.1.2 The Approach of using a Service Level

When it is very difficult to assign numerical values to the cost of a lost sale, we can use other alternative (e.g., service level). This procedure is to minimize the average annual costs of ordering and carrying inventory, subject to the constraint that the average fraction of the time out of stock is not greater than a fixed value. The average fraction of time during which the system is out of stock is the same as the expected number of lost sales incurred per year divided by the average rate of demand. That is,

$$\left(\frac{\lambda}{Q}\bar{\eta}(r)\right)\frac{1}{\lambda} = \frac{\bar{\eta}(r)}{Q} \quad (5)$$

Thus we wish to minimize

$$K = \frac{\lambda}{Q}A + IC\left[\frac{Q}{2} + r - \mu + \bar{\eta}(r)\right] \quad (6)$$

subject to the constraint

$$\frac{\bar{\eta}(r)}{Q} \le f \quad (7)$$

2.2 The Proposed (Q, r) Models

In this section, in order to achieve better profitability and lesser cost of inventory operations, we extend the existing (Q, r) model described in section 2.1 to incorporate knowledge of purchase dependence. We first describe the approach of using of a lost sale cost and then describe the approach of using a service level.

Additional notations

- $\hat{\pi}_i$ The (lost) profit of item i
- p_i Percentage of order type i in the total annual orders
- J_i Set of items that are requested with an item i, where item i is not included
- O_j Set of order types that include an item j
- p_{ij} Percentage of order type that contains both item i and item j
- α_i Additional cost of a lost sale incurred by other items when item i is out of stock
- β_i Additional cost of a lost sale incurred by item i when other items are out of stock

2.2.1 The Approach of using a Lost Sale Cost

Basically, the proposed (Q, r) models also follow the same approach as the inventory model described in section 2.1.1. In other words, the average annual cost of the proposed inventory models is also composed of the ordering costs, the cost of carrying inventory, and the cost of a lost sale. However, we need to append an additional term in the average annual cost of equation (1) because an additional cost of a lost sale incurs when purchase dependence exists. The key question is how to calculate the additional cost of a lost sale.

We propose two methods of calculating the additional cost of a lost sale. The first method is to consider additional costs of a lost sale incurred by other items when item i is out of stock. On the other hand, the second method is to consider additional costs of a lost sale incurred by item i when other items are out of stock.

(1) The First Method

We explain how to calculate the additional cost of a lost sale incurred by other items when item i is out of stock using an example of inventory system with 3 items shown in <Table 1>. The order type shows which items are required by the order. For example, if an order requires both item 1 and item 2, then the order is the order type of 4 in Table 1. Percentage p_i shows the portion of order type i in the total annual orders. The last column in Table 1 shows the average annual profit by the order type i.

<Table 1> Example of Inventory System with 3 Items

Order Type	Item			Percentage	Average Annual Profit
	1	2	3		
1	√			p_1	$\frac{p_1}{p_1+p_4}\lambda_1\hat{\pi}_1$
2		√		p_2	$\frac{p_2}{p_2+p_4+p_5}\lambda_2\hat{\pi}_2$
3			√	p_3	$\frac{p_3}{p_3+p_5}\lambda_3\hat{\pi}_3$
4	√	√		p_4	$\frac{p_4}{p_1+p_4}\lambda_1\hat{\pi}_1 \frac{p_4}{p_2+p_4+p_5}\lambda_2\hat{\pi}_2$
5		√	√	p_5	$\frac{p_5}{p_2+p_4+p_5}\lambda_2\hat{\pi}_2 \frac{p_5}{p_3+p_5}\lambda_3\hat{\pi}_3$

If we sum average annual profits of order types that include item i and divide it by average annual demand, Λ_i, we obtain the lost profit of item i and the additional cost of a lost sale incurred by other items when item i is out of stock. Then, by subtracting the lost profit of item i, we can obtain the additional cost of a lost sale incurred by other items as follows.

$$\alpha_i = \frac{1}{\lambda_i}\sum_{j\in J_i}\frac{p_{ij}}{\sum_{k\in O_j} p_k}\lambda_j\hat{\pi}_j \quad (8)$$

The average annual cost of item i for the proposed (Q,r) model is expressed as follows.

$$K_i = \frac{\lambda_i}{Q_i}A_i + I_iC_i\left[\frac{Q_i}{2} + r_i - \mu_i\right] + \left(I_iC_i + \frac{\pi_i\lambda_i}{Q_i}\right) \quad (9)$$

$$\bar{\eta}_i(r_i) + \frac{\lambda_i}{Q_i}\alpha_i\bar{\eta}_i(r_i)$$

We can determine the values of Q_i and r_i which minimize the $\sum_i K_i$ of equation (9) by finding the values of Q_i and r_i that satisfy the equations.

$$\frac{\partial \sum_i K_i}{\partial Q_i} = \frac{\lambda_i}{Q_i^2}A_i + \frac{I_i C_i}{2} - \frac{\lambda_i \pi_i}{Q_i^2}\overline{\eta}_i(r_i) - \frac{\lambda_i}{Q_i^2}\alpha_i \overline{\eta}_i(r_i) = 0 \quad (10)$$

$$\frac{\partial \sum_i K_i}{\partial r_i} = I_i C_i - \left(I_i C_i + \frac{\pi_i \lambda_i}{Q_i}\right)H_i(r_i) - \frac{\lambda_i}{Q_i}\alpha_i H_i(r_i) = 0 \quad (11)$$

The equivalents of equations (2) and (3) then become

$$Q_i = \sqrt{\frac{2\lambda_i [A_i + (\pi_i + \alpha_i)\overline{\eta}_i(r_i)]}{I_i C_i}} \quad (12)$$

$$H_i(r_i) = \frac{Q_i I_i C_i}{\lambda_i(\pi_i + \alpha_i) + Q_i I_i C_i} \quad (13)$$

To solve Q_i and r_i, we can use the iterative procedure proposed by Hadley and Whitin [6].

(2) The Second Method

The additional cost of item i lost sales can also be calculated by multiplying the lost profit of item i by the expected number of item i lost sales per year due to other stock-out items. Then, the average annual cost for the proposed (Q, r) model is expressed as follows.

$$\sum_i K_i = \sum_i \frac{\lambda_i}{Q_i}A_i + \sum_i I_i C_i\left[\frac{Q_i}{2} + r_i - \mu_i\right] + \sum_i \left(I_i C_i + \frac{\pi_i \lambda_i}{Q_i}\right)\overline{\eta}_i(r_i) + \sum_i \hat{\pi}_i \lambda_i \sum_{j \in J_i} \frac{\overline{\eta}_j(r_j)}{Q_j} \frac{p_{ij}}{\sum_{k \in O_i} p_k} \quad (14)$$

We can determine the values of Q_i and r_i which minimize the $\sum_i K_i$ of equation (14) by finding the values of Q_i and r_i that satisfy the equations.

$$\frac{\partial \sum_i K_i}{\partial Q_i} = -\frac{\lambda_i}{Q_i^2}A_i + \frac{I_i C_i}{2} - \frac{\lambda_i \pi_i}{Q_i^2}\overline{\eta}_i(r_i) - \lambda_i \beta_i \frac{\overline{\eta}_i(r_i)}{Q_i^2} = 0 \quad (15)$$

$$\frac{\partial \sum_i K_i}{\partial r_i} = I_i C_i - \left(I_i C_i + \frac{\pi_i \lambda_i}{Q_i}\right)H_i(r_i) - \frac{\lambda_i}{Q_i}\beta_i H_i(r_i) = 0 \quad (16)$$

where

$$\beta_i = \hat{\pi}_i \sum_{j \in J_i} \frac{p_{ji}}{\sum_{k \in O_i} p_k} \quad (17)$$

The equivalents of equations (12) and (13) then become

$$Q_i = \sqrt{\frac{2\lambda_i [A_i + (\pi_i + \beta_i)\overline{\eta}_i(r_i)]}{I_i C_i}} \quad (18)$$

$$H_i(r_i) = \frac{Q_i I_i C_i}{\lambda_i(\pi_i + \beta_i) + Q_i I_i C_i} \quad (19)$$

We can also use the iterative procedure proposed by Hadley and Whitin [6] to solve Q_i and r_i.

2.2.2 The Approach of Using a Service Level

This approach is to minimize the total average annual costs of ordering and carrying inventory, subject to the constraint that for each item *i*, the expected number of lost sales incurred per year divided by the average rate of demand is not greater than a fixed value, f_i. In order to incorporate purchase dependence, the proposed (Q, r) model need to modify the constraint. In other words, the expected number of lost sales for item i should include the expected number of lost sales incurred by item i itself and the expected number of lost sales incurred by other stock-out items. That is,

$$\left(\frac{\lambda_i}{Q_i}\overline{\eta}_i(r_i) + \sum_{j \in J_i} \frac{\overline{\eta}_j(r_j)}{Q_j} \frac{p_{ij}}{\sum_{k \in O_i} p_k}\lambda_i\right)\frac{1}{\lambda_i} = \frac{\overline{\eta}_i(r_i)}{Q_i} + \sum_{j \in J_i} \frac{\overline{\eta}_j(r_j)}{Q_j} \frac{p_{ij}}{\sum_{k \in O_i} p_k} \quad (20)$$

Thus we wish to minimize

$$\sum_i K_i = \sum_i \frac{\lambda_i}{Q_i}A_i + \sum_i I_i C_i\left[\frac{Q_i}{2} + r_i - \mu_i + \overline{\eta}_i(r_i)\right] \quad (21)$$

subject to the constraints

$$\frac{\overline{\eta}_i(r_i)}{Q_i} + \sum_{j \in J_i} \frac{\overline{\eta}_j(r_j)}{Q_j} \frac{p_{ij}}{\sum_{k \in O_i} p_k} < f_i \quad \text{for all } i \quad (22)$$

3. Simulation Experiment

This section describes a computer simulation experiment that is conducted to justify performance of the proposed inventory models by comparing to that of an inventory model ignoring purchase dependence when purchase dependence exists. The results will illustrate how important it is to consider purchase dependence in the inventory operations practice.

The simulation experimental setting is designed as follows. We consider an inventory system with 3 items among which purchase dependence exists. Customer orders arrive at the system by following a homogeneous Poison process. In other words, the inter-arrival times of customer orders are set to follow the exponential distribution with a mean time of 2 (days). Customer orders take one of 5 order types shown in <Table 1>. Percentages of order types in the total annual orders are determined by the degree of purchase dependence.

In this paper, we measure the degree of purchase dependence by an associated confidence. The associated confidence is defined in an association rule that is a type of data mining technique [13]. The confidence of an association rule that item 1 associates with item 2 is defined as the percentage of customer orders containing both items 1 and 2 out of all the customer orders containing item 1. That is,

$$Confidence = \frac{number\_of\_customer\_orders\_containing\_items\_1\_and\_2}{number\_of\_customer\_orders\_containing\_item\_1}$$

We consider several confidence levels for the generality of experimental results. <Table 2> shows the percentages of order types corresponding to confidence levels in this simulation experimental setting.

<Table 2> Percentages of Order Types Corresponding to Confidence Levels

Confidence Level	P_1	P_2	P_3	P_4	P_5
60%	20	0	20	30	30
70%	15	0	15	35	35
80%	10	0	10	40	40
90%	5	0	5	45	45

Quantities of items required by customer orders are randomly generated using uniform distributions, UNIF (1, 10), UNIF (1, 5), and UNIF (1, 5) for items 1, 2, and 3, respectively. We generate 5 data sets for each confidence level. <Table 3> shows average annual demands for each item corresponding to confidence levels.

<Table 3> Average Annual Demands Corresponding to Confidence Levels

Confidence Level	λ_1	λ_2	λ_3
60%	512	328	278
70%	473	357	276
80%	512	424	272
90%	488	479	273

Since purchase dependence exists, all items requested by the customer order should be delivered all together from the stock. Otherwise, the system faces a lost sale. We conduct the simulation experiment for two cases : a case of using a lost sale cost and a case of using a service level.

3.1 A Case of using a Lost Sale Cost

For each confidence level, we first calculate an order quantity, Q_i and a reorder point, r_i for each item using a (Q,r) model ignoring purchase dependence. With equations (2) and (3), we utilize the iterative procedure proposed by Hadley and Whitin [8]. The parameter values for the inventory model are as follows.

$$A_1 = A_2 = A_3 = 100$$
$$I_1 = I_2 = I_3 = 0.2$$
$$C_1 = 100,\ C_2 = 150,\ C_3 = 200$$
$$\hat{\pi}_1 = 30,\ \hat{\pi}_2 = 45,\ \hat{\pi}_3 = 60$$
$$\pi_1 = 60,\ \pi_2 = 75,\ \pi_3 = 90$$

For 5 data sets per each confidence level, we simulate the inventory system using the values of Q_i and r_i obtained above. The inventory system is run under the environment in which purchase dependence exists. The simulation results are shown in <Table 4>.

The above process is repeated again for the proposed (Q, r) models with the consideration of purchase dependence. Additional cost of a lost sale incurred by other items when item i is out of stock, α_i is shown in <Table 5>. The values of β_i are shown in <Table 6>. For the iterative procedure of numerically solving Q_i and r_i, equations (12) and (13) are used for the first method and equations (18) and (19) are used for the second method. The simulation results are also shown in <Table 4>.

<Table 4> Simulation Results of (Q, r) Models for a Case of using a Lost Sale Cost

Confidence Level	Purchase Dependence	Inventory Policy				Ordering Cost	Inventory Carrying Cost	Lost profit	Average total cost
			Item 1	Item 2	Item 3				
60%	Ignore	Q	86	65	45	1540.00	3740.92	381.00	5661.92
		r	31	18	16				
	Consider(1st)	Q	83	57	47	1480.00	3846.35	69.00	5395.35
		r	35	20	16				
	Consider(2nd)	Q	83	57	47	1480.00	3882.56	69.00	5431.56
		r	35	20	17				
70%	Ignore	Q	85	61	48	1420.00	3871.06	558.00	5849.06
		r	32	23	17				
	Consider(1st)	Q	80	58	50	1580.00	3823.66	384.00	5787.66
		r	33	24	17				
	Consider(2nd)	Q	80	58	50	1580.00	3823.66	384.00	5787.66
		r	33	24	17				
80%	Ignore	Q	91	68	50	1520.00	4089.79	672.00	6281.79
		r	31	23	16				
	Consider(1st)	Q	88	74	44	1420.00	4124.13	498.00	6042.13
		r	32	27	17				
	Consider(2nd)	Q	87	74	44	1480.00	4124.16	453.00	6057.16
		r	32	27	17				
90%	Ignore	Q	91	71	46	1340.00	3975.52	1239.00	6554.52
		r	27	26	15				
	Consider(1st)	Q	89	78	48	1420.00	4346.76	312.00	6078.76
		r	34	30	16				
	Consider(2nd)	Q	88	81	47	1380.00	4429.63	363.00	6172.63
		r	34	30	16				

<Table 5> Additional Cost of a Lost Sale Incurred by Other Items When Item i Is Out of Stock

Confidence Level	α_1	α_2	α_3
60%	14.41	58.61	26.55
70%	16.96	60.36	29.07
80%	18.65	59.8	35.02
90%	22.09	58.25	39.52

<Table 6> The Values of β_i

Confidence Level	β_1	β_2	β_3
60%	15	59	30
70%	15	63	30
80%	15	72	30
90%	15	81	30

3.2 A Case of using a Service Level

For a (Q, r) model ignoring purchase dependence, it is not so much difficult to find Q and r that minimize the objective function of equation (6) while satisfying the constraint of equation (7). We can use the Microsoft Excel software. First, we calculate the expected number of lost sales per cycle using equation (4) for each reorder point r. Secondly, for each reorder point r, we search the order quantity Q that minimizes the objective function of equation (6) while satisfying the constraint of equation (7), and search the corresponding K value. Then, we select the order quantity Q and the reorder point r that minimize the K value.

For 5 data sets per each confidence level, similarly to section 3.1, we simulate the inventory system using the values of Q_i and r_i obtained for f_i = 0.05 and f_i = 0.10. The inventory system is run under the environment in which purchase dependence exists. The simulation results are shown in <Tables 7 and 8>.

For the proposed (Q, r) model with the consideration of purchase dependence, it is not an easy task to find Q_i and r_i that minimize the objective function of equation (21) while satisfying the constraints of equation (22). The complexity stems from the interactions among inventory replenishment policies for items. In this section, we consider the genetic algorithm (GA) as an optimization method.

<Table 7> Simulation Results of (Q, r) Model for a Case of using a Service Level (f = 0.05)

Confidence Level	Purchase Dependence	Inventory Policy				Ordering Cost+ Inventory Carrying Cost	Lost Profit	Average total cost	1-Service Level ($\le f$)			
			Item 1	Item 2	Item 3				Item 1	Item 2	Item 3	Order
60%	Ignore	Q	82	47	39	4340.71	2511.00	6851.71	0.072	0.063	0.029	0.049
		r	14	10	11							
	Consider	Q	75	47	47	4627.92	1575.00	6202.92	0.032	0.030	0.038	0.029
		r	20	16	9							
70%	Ignore	Q	75	49	47	4354.14	3630.00	7984.14	0.084	0.076	0.073	0.068
		r	16	13	9							
	Consider	Q	77	57	40	4682.27	1959.00	6641.27	0.048	0.041	0.038	0.040
		r	21	16	13							
80%	Ignore	Q	80	65	37	4229.51	5076.00	9305.51	0.108	0.099	0.093	0.092
		r	14	11	9							
	Consider	Q	76	58	41	4736.76	2631.00	7367.76	0.049	0.044	0.064	0.046
		r	18	22	9							
90%	Ignore	Q	73	59	40	4462.06	3162.00	7624.06	0.057	0.062	0.061	0.058
		r	14	14	9							
	Consider	Q	74	57	40	5337.70	696.00	6033.70	0.015	0.008	0.018	0.017
		r	31	33	9							

<Table 8> Simulation Results of (Q, r) Model for a Case of using a Service Level (f = 0.10)

Confidence Level	Purchase Dependence	Inventory Policy				Ordering Cost+ Inventory Carrying Cost	Lost Profit	Average Total Cost	1-Service Level ($\le f$)			
			Item 1	Item 2	Item 3				Item 1	Item 2	Item 3	Order
60%	Ignore	Q	72	49	37	4038.80	5988.00	10026.80	0.147	0.152	0.088	0.107
		r	8	5	7							
	Consider	Q	74	50	38	4336.48	3414.00	7750.48	0.083	0.077	0.060	0.063
		r	14	11	7							
70%	Ignore	Q	73	59	38	3929.96	7074.00	11003.96	0.144	0.176	0.133	0.128
		r	9	6	6							
	Consider	Q	76	56	40	4323.14	3543.00	7866.14	0.061	0.090	0.074	0.066
		r	18	12	9							
80%	Ignore	Q	74	55	43	3903.91	8880.00	12783.91	0.163	0.188	0.171	0.171
		r	8	7	4							
	Consider	Q	75	55	42	4442.61	3942.00	8384.61	0.089	0.068	0.079	0.072
		r	12	17	6							
90%	Ignore	Q	72	59	37	3871.23	8817.00	12688.23	0.198	0.156	0.156	0.159
		r	7	8	5							
	Consider	Q	71	66	49	4500.81	4431.00	8931.81	0.091	0.081	0.082	0.077
		r	13	15	6							

The GA is designed as follows. First the chromosome is designed by utilizing the inventory replenishment policy, a set of Q_i and r_i, and then is encoded into binary strings. The number of populations in each generation is set to 30 and the initial populations are randomly generated by assigning a binary integer to each gene of the chromosome. Crossover operates on two chromosomes at a time and generates offspring by recombining the current genes. We employ the one-cut-point method. Mutation introduces random changes to the chromosomes by altering the value to a gene with a user-specified probability called the mutation rate. For the selection procedure, we adopt an enlarged sampling space that has the size of *pop_size* (the size of population)+*off_size* (the size of offspring) and contains whole parents and offspring. When selection is performed on the enlarged sampling space, both parents and offspring have the same chance of competing for survival. We adopt a truncation selection method that ranks all chromosomes according to their fitness and selects the best ones as parents.

All GA runs in this section have the following standard characteristics:

- Crossover rate (= *off_size*/*pop_size*) : 0.3
- Mutation rate (= % of the total # of genes) : 0.1
- Population size : 30
- Number of generation in each run : 1,000

The above simulation process is repeated again with the values of Q_i and r_i found by the GA. The simulation results are shown in <Table 7>, <Table 8>.

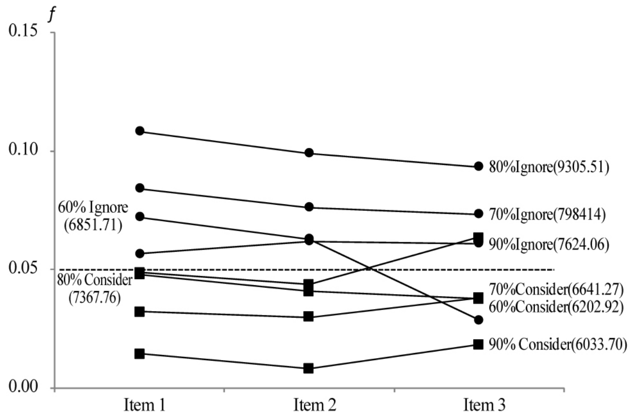
3.3 Findings

For a case of using a lost sale cost, <Table 4> shows the comparison between a (*Q, r*) model ignoring purchase dependence and the proposed (*Q, r*) models with the consideration of purchase dependence. For a fair comparison, we do not use the average annual costs defined by equations (1), (9), and (14), but instead the average total costs that substitutes the cost of a lost sale by the lost profit. The inventory operations costs in <Table 4> (<Table 7>, <Table 8>) are the average values of 5 data sets per each confidence level.

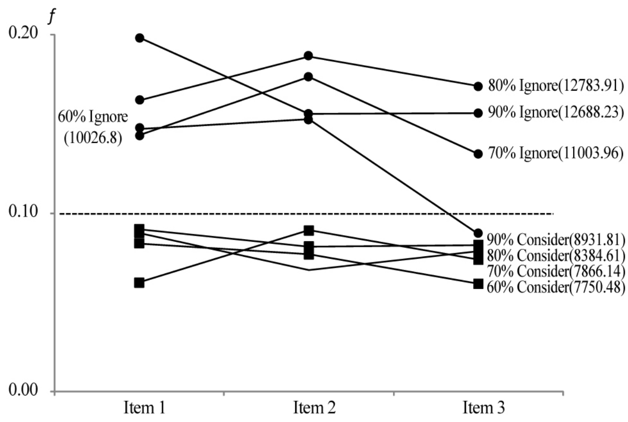
From <Table 4>, we can notice that the proposed (*Q, r*) models incur less inventory operations cost (i.e., average total cost) than a (*Q, r*) model ignoring purchase dependence. In general, the proposed (*Q, r*) models reduce the lost profit by carrying inventories a little more, resulted in less average total cost. These simulation results support that it is important to consider purchase dependence in the inventory operations practice.

It is worth noting here that there exist tiny differences in the average total costs by the proposed first and second methods of calculating the additional cost of a lost sale. The differences stem from that the two methods are approximate approaches. In general, the first method shows the preferable results.

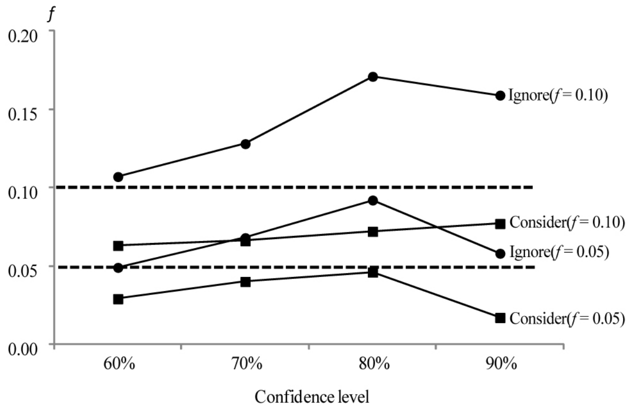
On the other hand, <Table 7>, <Table 8> show the comparison between a (*Q, r*) model ignoring purchase dependence and the proposed (*Q, r*) model with the consideration of purchase dependence for a case of using a service level. We graph the comparisons of (1-service level) in <Figure 2> and <Figure 3> for f = 0.05 and f = 0.10, respectively. From <Figure 2>, <Figure 3>, we can notice that the proposed (*Q, r*) model satisfies better service levels than a (*Q, r*) model ignoring purchase dependence. In addition, <Table 7>, <Table 8> show that the proposed (*Q, r*) model incurs less inventory operations cost than a (*Q, r*) model ignoring purchase dependence.



<Figure 2> Comparison of (1-Service Level) for (*Q, r*) Models When f = 0.05



<Figure 3> Comparison of (1-Service Level) for (*Q, r*) Models When f = 0.10



<Figure 4> Comparison of Order Unfulfillment Rates for (Q, r) Models

The last column in <Table 7>, <Table 8> shows the order unfulfillment rate. The graph shown in <Figure 4> demonstrates that the proposed (Q, r) model incurs less order unfulfillment rates than a (Q, r) model ignoring purchase dependence. These simulation results of the service level case also support that it is important to consider purchase dependence in the inventory operations practice.

4. Conclusion

We introduced the existence of purchase dependence that was identified during the analysis of inventory operations practice at a sales agency of dealing with spare parts for ship engines and generators. Even though the existence of purchase dependence was reported by a few studies, no inventory models have been developed to consider purchase dependence. This paper designed the inventory model reflecting purchase dependence.

In order to deal with purchase dependence in inventory operations practice, we proposed (Q, r) models with the consideration of purchase dependence. Through a computer simulation experiment, we compared performance of the proposed (Q, r) models to that of a (Q, r) model ignoring purchase dependence. The simulation results demonstrated that the proposed (Q, r) models incur less inventory operations cost (satisfies better service levels) than a (Q, r) model ignoring purchase dependence. As a result, the simulation results supported that it is important to consider purchase dependence in the inventory operations practice.

Since it is a complex task to design an inventory model utilizing knowledge of purchase dependence, we considered a continuous review model that replenishes each item separately. Thus, the future study is required to extend knowledge of purchase dependence to the joint replenishment problem.

References

[1] Bala, P.K., Identification of purchase dependencies in retail sale. *International Journal of Strategic Management*, 2008, Vol. 8, No. 2, pp. 75-83.

[2] Bala, P.K., Sural, S., and Banerjee, R.N., Association rule for purchase dependence in multi-item inventory. *Production Planning and Control*, 2010, Vol. 21, No. 3, pp. 274-285.

[3] Chang, S.H., An integrated production and inventory model in a single-vendor multi-buyer supply chain. *Journal of Society of Korea Industrial and Systems Engineering*, 2015, Vol. 38, No. 3, pp. 117-126.

[4] Choi, J.Y. and Kim, M.S., A continuous review (S-1, S) inventory policy in which depletion is due to demand and loss of units. *Journal of the Korea Society for Quality Management*, Vol. 15, No. 1, 1987, pp. 55-62.

[5] Corbett, C.J. and Rajaram, K., A generalization of the inventory pooling effect to nonnormal dependent demand. *Manufacturing and Service Operations Management*, 2006, Vol. 8, No. 4, pp. 351-358.

[6] Dong, L. and Lee, H.L., Optimal policies and approximations for a serial multiechelon inventory system with time-correlated demand. *Operations Research*, 2003, Vol. 51, No. 6, pp. 969-980.

[7] Eppen, G.D., Effects of centralization on expected costs in a multi-location newsboy problem. *Management Science*, 1979, Vol. 25, No. 5, pp. 498-501.

[8] Hadly, G. and Whitin, T.M., *Analysis of Inventory Systems*. Englewood Cliffs, N.J. : Prentice-Hall, 1963.

[9] Johnson, G.D. and Thompson, H.E., Optimality of myopic inventory policies for certain dependent demand processes. *Management Science*, 1975, Vol. 21, No. 11, pp. 1303-1307.

[10] Larsen, C., The Q(s, S) control policy for the joint replenishment problem extended to the case of correlation among item-demands. *International Journal of Production Economics*, 2009, Vol. 118, No. 1, pp. 292-297.

[11] Lee, L.H. and Chew, E.P., A dynamic joint replenishment policy with auto-correlated demand. *European Journal of Operational Research*, 2005, Vol. 165, No. 3,

pp. 729-747.

[12] Liu, L. and Yuan, X.M., Coordinated replenishments in inventory systems with correlated demands. *European Journal of Operational Research*, 2000, Vol. 123, No. 3, pp. 490-503.

[13] Roiger, R.J. and Geatz, M.W., *Data Mining : A Tutorial-Based Primer*. New York, N.Y. : Addison Wesley, 2003.

[14] Urban, T.L., A periodic-review model with serially- correlated, inventory-level-dependent demand. *International Journal of Production Economics*, 2005, Vol. 95, No. 3, pp. 287-295.

[15] Urban, T.L., Reorder level determination with serially correlated demand. *Journal of the Operational Research Society*, 2000, Vol. 51, No. 6, pp. 762-768.

[16] Veinott, A.F., Optimal policy for a multi-product, dynamic, non-stationary inventory problem. *Management Science*, 1965, Vol. 12, No. 3, pp. 206-222.

[17] Wong, R.C., Fu, A.W., and Wang, K., Data mining for inventory item selection with cross-selling considerations. *Data Mining and Knowledge Discovery*, 2005, Vol. 11, No. 1, pp. 81-112.

[18] Yan, X.S., Robb, D.J., and Silver, E.A., Inventory performance under pack size constraints and spatially-correlated demand. *International Journal of Production Economics*, 2009, Vol. 117, No. 2, pp. 330-337.

[19] Zhang, R., An extension of partial backordering EOQ with correlated demand caused by cross-selling considering multiple minor items. *European Journal of Operational Research*, 2012, Vol. 220, pp. 876-881.

[20] Zhang, R., Kaku, I., and Xiao, Y., Deterministic EOQ with partial backordering and correlated demand caused by cross-selling. *European Journal of Operational Research*, 2010, Vol. 210, pp. 537-551.

[21] Zhang, R., Kaku, I., and Xiao, Y., Model and heuristic algorithm of the joint replenishment problem with complete backordering and correlated demand. *International Journal of Production Economics*, 2012, Vol. 139, No. 1, pp. 33-41.

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