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FUZZY ALMOST STRONGLY (r, s)-SEMIOPEN AND SEMICLOSED MAPPINGS

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Abstract. In this paper, we introduce the concepts of fuzzy almost strongly (r, s)-semiopen and semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and investigate some of their characteristic properties.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [13]. Chang [3] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [12], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [4], and by Ramadan [11].

As a generalization of fuzzy sets, Atanassov [1] introduced the concept of intuitionistic fuzzy sets, and Çoker [5] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [6] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. Shi-Zhong Bai [2] introduced the concepts of fuzzy almost

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strongly semiopen and semiclosed mappings on Chang's fuzzy topological spaces.

In this paper, we introduce the concepts of fuzzy almost strongly (r, s)-semiopen and semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and investigate some of their characteristic properties.

2. Preliminaries

For the nonstandard definitions and notations we refer to [7–10].

Let $I(X)$ be a family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

DEFINITION 2.1. ([6]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.
(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.
(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

DEFINITION 2.2. ([7, 8, 10]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy* (r, s)*-semiopen* if $\mathrm{cl}(\mathrm{int}(A, r, s), r, s) \supseteq A$,
(2) *fuzzy* (r, s)*-semiclosed* if $\mathrm{int}(\mathrm{cl}(A, r, s), r, s) \subseteq A$,
(3) *fuzzy* (r, s)*-regular open* if $\mathrm{int}(\mathrm{cl}(A, r, s), r, s) = A$,
(4) *fuzzy* (r, s)*-regular closed* if $\mathrm{cl}(\mathrm{int}(A, r, s), r, s) = A$,
(5) *fuzzy strongly* (r, s)*-semiopen* if $A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A, r, s), r, s), r, s)$,
(6) *fuzzy strongly* (r, s)*-semiclosed* if $A \supseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A, r, s), r, s), r, s)$.

DEFINITION 2.3. ([10]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy strongly* (r, s)*-semiinterior* is defined by

$$\mathrm{ssint}(A, r, s) = \bigcup \{B \in I(X) \mid B \subseteq A,\ B \text{ is fuzzy strongly } (r, s)\text{-semiopen}\}$$

and the *fuzzy strongly* (r,s)*-semiclosure* is defined by

$$\text{sscl}(A,r,s) = \bigcap\{B \in I(X) \mid A \subseteq B,\ B \text{ is fuzzy strongly } (r,s)\text{-semiclosed}\}.$$

THEOREM 2.4. ([8]) *(1) The fuzzy* (r,s)*-closure of a fuzzy* (r,s)*-open set is fuzzy* (r,s)*-regular closed for each* $(r,s) \in I \otimes I$.
(2) The fuzzy (r,s)*-interior of a fuzzy* (r,s)*-closed set is fuzzy* (r,s)*-regular open for each* $(r,s) \in I \otimes I$.

DEFINITION 2.5. ([2]) Let $f : (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy topological space X_1 to another fuzzy topological space X_2. Then f is called

(1) a *fuzzy almost strongly semiopen* mapping if $f(A)$ is a fuzzy strongly semiopen set of X_2 for each fuzzy regular open set A of X_1,
(2) a *fuzzy almost strongly semiclosed* mapping if $f(A)$ is a fuzzy strongly semiclosed set of X_2 for each fuzzy regular closed set A of X_1.

3. Fuzzy almost strongly (r,s)-semiopen and semiclosed mappings

Now, we define the notions of fuzzy almost strongly (r,s)-semiopen and semiclosed mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their properties.

DEFINITION 3.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is called

(1) a *fuzzy almost strongly* (r,s)*-semiopen* mapping if $f(A)$ is a fuzzy strongly (r,s)-semiopen set in Y for each fuzzy (r,s)-regular open set A in X,
(2) a *fuzzy almost strongly* (r,s)*-semiclosed* mapping if $f(A)$ is a fuzzy strongly (r,s)-semiclosed set in Y for each fuzzy (r,s)-regular closed set A in X.

THEOREM 3.2. *Let* $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ *be a mapping from a SoIFTS* X *to a SoIFTS* Y *and* $(r,s) \in I \otimes I$. *Then the following statements are equivalent:*

(1) f *is fuzzy almost strongly* (r,s)*-semiopen.*

(2) *For each fuzzy (r,s)-open set A in X,*

$$f(A) \subseteq ssint(f(int(cl(A,r,s),r,s)),r,s).$$

(3) *For each fuzzy (r,s)-semiclosed set A in X,*

$$f(int(A,r,s)) \subseteq ssint(f(A),r,s).$$

(4) *For each intuitionistic fuzzy set B in Y and each fuzzy (r,s)-regular closed set A in X with $f^{-1}(B) \subseteq A$, there is a fuzzy strongly (r,s)-semiclosed set C in Y such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.*

Proof. (1) ⇒ (2) Let A be a fuzzy (r,s)-open set in X. By Theorem 2.4, $\text{int}(\text{cl}(A,r,s),r,s)$ is fuzzy (r,s)-regular open in X. Since f is a fuzzy almost strongly (r,s)-semiopen mapping, $f(\text{int}(\text{cl}(A,r,s),r,s))$ is fuzzy strongly (r,s)-semiopen in Y. Hence we have

$$\begin{aligned} f(A) = f(\text{int}(A,r,s)) &\subseteq f(\text{int}(\text{cl}(A,r,s),r,s)) \\ &= \text{ssint}(f(\text{int}(\text{cl}(A,r,s),r,s)),r,s). \end{aligned}$$

(2) ⇒ (3) Let A be a fuzzy (r,s)-semiclosed set in X. Then $\text{int}(A,r,s)$ is fuzzy (r,s)-open in X. Thus by (2), we have

$$\begin{aligned} f(\text{int}(A,r,s)) &\subseteq \text{ssint}(f(\text{int}(\text{cl}(\text{int}(A,r,s),r,s),r,s)),r,s) \\ &\subseteq \text{ssint}(f(\text{int}(\text{cl}(A,r,s),r,s)),r,s) \\ &\subseteq \text{ssint}(f(A),r,s). \end{aligned}$$

(3) ⇒ (1) Let A be a fuzzy (r,s)-regular open set in X. Then A is fuzzy (r,s)-open and also fuzzy (r,s)-semiclosed in X. Hence by (3), we obtain

$$f(A) = f(\text{int}(A,r,s)) \subseteq \text{ssint}(f(A),r,s) \subseteq f(A).$$

Thus $f(A) = \text{ssint}(f(A),r,s)$, which is a fuzzy strongly (r,s)-semiopen set in Y. Therefore f is fuzzy almost strongly (r,s)-semiopen.

(1) ⇒ (4) Let B be any intuitionistic fuzzy set in Y and A a fuzzy (r,s)-regular closed set in X with $f^{-1}(B) \subseteq A$. Then $A^c \subseteq f^{-1}(B^c)$, and hence $f(A^c) \subseteq f(f^{-1}(B^c)) \subseteq B^c$. Since f is fuzzy almost strongly (r,s)-semiopen and A^c is fuzzy (r,s)-regular open, we have $f(A^c) \subseteq \text{ssint}(B^c,r,s)$. Thus $A^c \subseteq f^{-1}(f(A^c)) \subseteq f^{-1}(\text{ssint}(B^c,r,s))$. Hence $A \supseteq f^{-1}(\text{sscl}(B,r,s))$. Let $C = \text{sscl}(B,r,s)$. Then C is fuzzy strongly (r,s)-semiclosed such that $B \subseteq C$ and $f^{-1}(C) \subseteq A$.

(4) $\Rightarrow$ (1) Let A be a fuzzy (r,s)-regular open set in X. Then A^c is fuzzy (r,s)-regular closed. Note that $A^c \supseteq (f^{-1}(f(A)))^c = f^{-1}(f(A)^c)$. According to the assumption, there is a fuzzy strongly (r,s)-semiclosed set B in Y such that $f(A)^c \subseteq B$ and $f^{-1}(B) \subseteq A^c$. From $f(A)^c \subseteq B$ we have $\text{sscl}(f(A)^c, r, s) \subseteq B$, and hence $B^c \subseteq \text{sscl}(f(A)^c, r, s)^c = \text{ssint}(f(A), r, s)$. Since $f^{-1}(B) \subseteq A^c$, we obtain $f^{-1}(B^c) \supseteq A$, and thus $B^c \supseteq f(f^{-1}(B^c)) \supseteq f(A)$. Hence $f(A) = \text{ssint}(f(A), r, s)$. Thus $f(A)$ is a fuzzy strongly (r,s)-semiopen set in Y. Therefore f is almost strongly (r,s)-semiopen. □

THEOREM 3.3. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is fuzzy almost strongly (r,s)-semiopen if and only if $f(int(A,r,s)) \subseteq ssint(f(A),r,s)$ for each fuzzy (r,s)-semiclosed set A in X.*

Proof. ($\Rightarrow$) Let f be fuzzy almost strongly (r,s)-semiopen and A a fuzzy (r,s)-semiclosed set in X. Then

$$\text{int}(A,r,s) \subseteq \text{int}(\text{cl}(A,r,s),r,s) \subseteq A.$$

By Theorem 2.4, $\text{int}(\text{cl}(A,r,s),r,s)$ is a fuzzy (r,s)-regular open set in X. Since f is fuzzy almost strongly (r,s)-semiopen, $f(\text{int}(\text{cl}(A,r,s),r,s))$ is fuzzy strongly (r,s)-semiopen in Y. Hence we obtain

$$\begin{aligned} f(\text{int}(A,r,s)) &\subseteq f(\text{int}(\text{cl}(A,r,s),r,s)) \\ &= \text{ssint}(f(\text{int}(\text{cl}(A,r,s),r,s)),r,s) \\ &\subseteq \text{ssint}(f(A),r,s). \end{aligned}$$

Conversely, let A be a fuzzy (r,s)-regular open set in X. Then A is fuzzy (r,s)-open in X, and hence $\text{int}(A,r,s) = A$. Since $\text{int}(\text{cl}(A,r,s),r,s) = A$, A is fuzzy (r,s)-semiclosed in X. Hence

$$f(A) = f(\text{int}(A,r,s)) \subseteq \text{ssint}(f(A),r,s) \subseteq\subseteq f(A).$$

Thus $f(A) = \text{ssint}(f(A),r,s)$, which is a fuzzy strongly (r,s)-semiopen set in Y. Therefore f is a fuzzy almost strongly (r,s)-semiopen mapping. □

THEOREM 3.4. *Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent:*

(1) *f is fuzzy almost strongly (r,s)-semiclosed.*

(2) *For each fuzzy (r,s)-closed set A in X,*

$$sscl(f(cl(int(A,r,s),r,s)),r,s) \subseteq f(A).$$

(3) *For each fuzzy (r,s)-semiopen set B in Y,*

$$sscl(f(A),r,s) \subseteq f(cl(A,r,s)).$$

Proof. (1) $\Rightarrow$ (2) Let A be a fuzzy (r,s)-closed set in X. By Theorem 2.4, $\mathrm{cl}(\mathrm{int}(A,r,s),r,s)$ is fuzzy (r,s)-regular closed in X. Since f is fuzzy almost strongly (r,s)-semiclosed, $f(\mathrm{cl}(\mathrm{int}(A,r,s),r,s))$ is fuzzy strongly (r,s)-semiclosed in Y. Hence we have

$$\begin{aligned} \mathrm{sscl}(f(\mathrm{cl}(\mathrm{int}(A,r,s),r,s)),r,s) &= f(\mathrm{cl}(\mathrm{int}(A,r,s),r,s)) \\ &\subseteq f(\mathrm{cl}(A,r,s)) = f(A). \end{aligned}$$

(2) $\Rightarrow$ (3) Let A be a fuzzy (r,s)-semiopen set in X. Then $\mathrm{cl}(A,r,s)$ is fuzzy (r,s)-closed in X. By (2), we have

$$\begin{aligned} \mathrm{sscl}(f(A),r,s) &\subseteq \mathrm{sscl}(f(\mathrm{cl}(A,r,s)),r,s) \\ &\subseteq \mathrm{sscl}(f(\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A,r,s),r,s),r,s)),r,s) \\ &\subseteq f(\mathrm{cl}(A,r,s)). \end{aligned}$$

(3) $\Rightarrow$ (1) Let A be a fuzzy (r,s)-regular closed set in X. Then A is fuzzy (r,s)-closed and fuzzy (r,s)-semiopen in X. By (3), we obtain

$$f(A) \subseteq \mathrm{sscl}(f(A),r,s) \subseteq f(\mathrm{cl}(A,r,s)) = f(A).$$

Thus we have $f(A) = \mathrm{sscl}(f(A),r,s)$, which is a fuzzy strongly (r,s)-semiclosed set in Y. Hence f is fuzzy almost strongly (r,s)-semiclosed. □

References

[1] K. T. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems **20** (1986), 87–96.

[2] Shi-Zhong Bai, *Fuzzy almost strong semicontinuous mappings*, Bulletin for Studies and Exchanges on Fuzziness and its Applications **62** (1995), 89–93.

[3] C. L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl. **24** (1968), 182–190.

[4] K. C. Chattopadhyay, R. N. Hazra, and S. K. Samanta, *Gradation of openness : Fuzzy topology*, Fuzzy Sets and Systems **49** (1992), 237–242.

[5] D. Çoker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy Sets and Systems **88** (1997), 81–89.

[6] D. Çoker and M. Demirci, *An introduction to intuitionistic fuzzy topological spaces in Sostak's sense*, BUSEFAL **67** (1996), 67–76.

[7] Eun Pyo Lee, *Semiopen sets on intuitionistic fuzzy topological spaces in Sostak's sense*, J. Fuzzy Logic and Intelligent Systems **14** (2004), 234–238.

[8] Seok Jong Lee and Jin Tae Kim, *Fuzzy* (r, s)*-irresolute maps*, International Journal of Fuzzy Logic and Intelligent Systems **7** (2007) (1), 49–57.

[9] Seok Jong Lee and Jin Tae Kim, *Fuzzy almost* (r, s)*-semicontinuous mappings*, Commun. Math. Math. Sci. **6** (2010), 1–9.

[10] Seung On Lee and Eun Pyo Lee, *Fuzzy strongly* (r, s)*-semiopen sets*, International Journal of Fuzzy Logic and Intelligent Systems **6** (2006), no. 4, 299–303.

[11] A. A. Ramadan, *Smooth topological spaces*, Fuzzy Sets and Systems **48** (1992), 371–375.

[12] A. P. Šostak, *On a fuzzy topological structure*, Suppl. Rend. Circ. Matem. Janos Palermo, Sr. II **11** (1985), 89–103.

[13] L. A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338–353.

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