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The Study about the Influence of Mathematics Language on Mathematics Reading[1]

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The study is about the influence of literal, symbolic and graphics languages on mathematics reading. The results show that the scores of symbolic language volume are significantly lower than that of literal language volume. The abstractness of the mathematical symbols will not have a significant impact on the students with excellent mathematical academic, but as for the medium and poor students, abstract mathematics symbols will cause their cognitive impairment. Due to picture-superiority-effect, the test scores of the graphics language volume are significantly higher than that of the symbolic language volume. Graphics language will have a significant impact on the excellent and medium students, but has no impact on the poor students.

Keywords: mathematics reading, literal language, symbolic language, graphics language
MESC Classification: C70
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1. DESCRIPTION OF PROBLEM

Reading is an important activity of human social life and it's also the principle means to acquire knowledge and know about the world. Mathematics learning can't do without

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reading. Mathematics reading is a basic skill in mathematics learning. Mathematics reading is an essential skill to learn mathematics. “Mathematics curriculum standards of compulsory education” and “mathematics curriculum standards of regular high school” have emphasized that we should advocate reading by oneself and other diverse learning styles, and teachers should guide students reading by oneself. In recent years, in the college entrance examination to students’ Mathematics reading ability also put forward higher requirements.

The psychological research shows that mathematics reading is a positive cognitive mental process to get meanings from mathematics text; and character encoding, language translation, and comprehensive understanding are the core elements of mathematics reading. They are also the three levels of mathematics reading comprehension. Character is the generic terms of words, symbols and graphics. Encoding refers to information conversion received by senses. The original idea of encoding is to transform the source object into another format by standards, and here it refers as readers processing, arranging and organizing new information with their prior knowledge and interpreting the new information on the basis of prior knowledge. The language translation mainly refers to translating or converting among mathematics languages (literal language, symbolic language and graphics language); comprehensive understanding refers to integrating of mathematics materials as well as the perception comprehension and application of mathematics text.

In fact, many students have learning difficulties or make errors, which are derived from misuse and misunderstanding of mathematics literal language, symbolic language and graphics language. As mathematics meaning must be conveyed by words, symbols, graphics, and so on. If methods, laws and conventions of mathematics language-expression are not clear, familiar or unaccustomed to students, they will not be able to understand their meanings, which will cause understanding problems in the reading process (Li, 2001, p. 176).

Mathematics reading survey (cf. Yang & Yu, 2014) show that students’ reading obstacles caused by mathematics language are mainly as follows:

Giving up after seeing too many words; language words with ambiguity; no relevant mathematics graphs and charts are offered to help them understand; not good at the method of symbolic-graphic combination and combination of words and diagrams; application questions cannot be understood; when they meet such questions with too many words, most of them will choose to give up or hastily finish them; waiting for teacher’s explanation; too many mathematics symbols; symbols and diagrams cannot be understood; frustrated at the sight of too many mathematics symbols; mathematics diagrams cannot be understood; diagrams are complex; charts and graphics are difficult to understand and so on.

Therefore, this study focuses on the influence of mathematics language on mathematics reading.

2. RESEARCH METHODS

2.1. Participants

Participants come from two parallel classes in S middle school, Shanxi province. We select three math grades which are from mid-term, final-term and a monthly exam of this term from each class, and then we conduct variance analysis to average value. If there is difference, we will give up and then re-elect, at last two parallel classes, Class 26 and Class 27, are conformed to the subjects.

2.2. Materials

According to the above definition to the mathematics reading and characteristics of mathematics language, “test value about the influence of symbolic language on the mathematics reading” and “text value about the influence of graphics language on mathematics reading” are complied. Test materials are made up of two groups, two in each group, and there are four questionnaires. The two questionnaires of each group are the same topics, but different ways of language representation. “The test value about the influence of symbolic language on the mathematics reading” is used to test the influence of symbolic language on reading, so one questionnaire is characterized by literal language; the other questionnaire is characterized by symbolic language. “Text value about the influence of graphics language on mathematics reading” is used to test the influence of graphics language on reading, one questionnaire is characterized by literal language, and the other is added to graphics on the basis of literal language.

All the test values are demonstrated by education experts, special and advanced math teachers in middle school, and then modifying, forecasting, and re-modifying according to the demonstrated opinions, finally each questionnaire is composed by three questions.

2.3. Process

The test is produced during the self-study time, and a set of questionnaires are completed in two classes during the same time. In the first day, Class 26 completes symbolic language test volume and Class 27 completes the corresponding literal language test value. In the fourth day, Class 26 completes the literal language test volume, and Class 27 completes the corresponding diagram language test volume. During specific test, the headlines of questionnaires of the two groups are modified: learning questionnaire 1, learning questionnaire 2, learning questionnaire 3, and learning questionnaire 4, among which learning questionnaire 1 and learning questionnaire 2 are a group, and learning questionnaire 3 and learning questionnaire 4 are the other group, so as to exclude the interference of irrelevant factors. The test time is 45 minutes.

3. STUDY RESULTS

3.1. The situation of two classes' mathematics academic groups

According to the previous calculating results, the mathematics academic average scores of Class 26 and Class 27 are respectively 76.86 and 73.88. According this, we divide each of the two classes into three groups: below 69 points is a low-score group, between 70-80 points is a middle-score group, and above 81 points is a high-score group. The specific situations are listed in Table 1.

Table 1. The group-situation of the two classes

Class	Class size	Below 69 points	70–80 points	Above 81 points
Class 26	44	10	21	13
Class 27	43	8	23	12

3.2. Compare between symbolic and literal language

First, test results of the two classes are overall analyzed, and the results are shown in Table 2.

Table 2. Overall compare to the test results between symbolic and literal language

Class	Class size	Average value	Standard deviation	Standard error	95% confidence interval	
					lower limit	upper limit
Class 26	44	10.7045	5.72867	0.86363	8.9629	12.4462
Class 27	43	15.5581	5.13764	0.78348	13.9770	17.1393
Total	87	13.1034	5.93773	0.63659	11.8379	14.3689

Table 3. Variance analysis of the test results between symbolic and literal language

	Quadratic sum	Degree of freedom	Mean square	F ratio	P
Between groups	512.305	1	512.305	17.282	0.000
Within group	2519.764	85	29.644		
Total	3032.069	86			

Table 2 and Table 3 show that the average score of Class 27 (literal language) is 15.56, which is higher than Class 26 (symbolic language), whose average score is 10.70, $P = 0.000 < 0.01$, and the scores of the two classes have significant differences at 0.01

level. Therefore, on the whole, the overall scores of the questionnaire represented by words language are significantly higher than the scores of questionnaire represented by symbolic language.

2×3 variance analysis is used to further analysis of the test results, and the results are shown in table 4.

Table 4. Variance analysis of different groups test scores

Different groups	Class	Number of people	Average scores	Standard deviation	P
High-score groups	26	13	13.1538	6.34883	0.208
	27	12	16.2500	5.51238	
Middle-score groups	26	21	10.5238	4.62189	0.003
	27	23	15.3913	5.56563	
Low-score groups	26	10	7.9000	6.17252	0.010
	27	8	15.0000	3.46410	

Table 4 shows that the mathematics academic high-score groups of the two classes P = 0.208 > 0.05, and there is no significant differences on test scores of symbolic and literal language; the mathematics academic middle-score groups of the two classes P = 0.003 < 0.01, and there exists significant differences at 0.01 level of the test scores of symbolic and literal language; the mathematics academic low-score groups of the two classes P = 0.010, and there exists significant differences at 0.01 level of the test scores of symbolic and literal language.

3.3. Compare between graphics and literal language

First, the test results of the two classes are overall analyzed, and the results are shown in Table 5.

Table 5. Overall compare to the test results between graphics and literal language

Class	Class size	Average value	Standard deviation	Standard error	95% confidence interval	
					lower limit	upper limit
Class 26	44	10.2045	3.16654	0.47737	9.2418	11.1673
Class 27	43	14.6744	5.13031	0.78236	13.0955	16.2533
Total	87	12.4138	4.78746	0.51327	11.3934	13.4341

Table 6. Variance analysis of the test results between graphics and literal language

	Quadratic sum	Degree of freedom	Mean square	F ratio	P
Between groups	434.502	1	434.502	24.035	0.000
Within groups	1536.601	85	18.078		
Total	1971.103	86			

Table 5 and Table 6 show that the average test value of Class 26 is 10.2, which is lower than Class 27, whose average test value is 14.7, $P = 0.000 < 0.01$, and there exists significant differences between the two classes at 0.01 level. Therefore, on the whole, the overall test scores of graphics language volume are significantly higher than that of literal language volume.

2×3 variance analysis is used to further analysis of the test results, and the results are shown in table 7.

Table 7. Variance analysis of test scores of different groups

Different groups	Class	Number of people	Average scores	Standard deviation	P
High-score groups	326	13	10.3077	4.26975	0.013
	327	12	15.5833	5.50138	
Middle-score groups	326	21	10.6667	2.92119	0.000
	327	23	15.4783	4.69884	
Low-score groups	326	10	9.1000	1.66333	0.248
	327	8	11.0000	4.69042	

Table 7 shows that the mathematics-academic high-score groups of the two classes $P = 0.013 < 0.05$, and there exits significant differences at 0.05 level of test scores among the graphics and the literal language; the middle-score groups of the two classes $P = 0.000<0.01$, and there exists significant differences at 0.01 level of test scores among the graphics and literal language; The low-score groups of the two classes $P = 0.248 > 0.05$, and there exists no significant differences of test scores among the graphics and literal language.

4. RESULTS ANALYSIS

4.1. The effect of symbols to mathematics reading

Seeing from the whole two classes, the test scores of the literal language volume are significantly higher than that of the volume of symbolic language volume. As for the impact of symbolic language to mathematics reading, there exists no significant differences between the high-score groups of the two classes, but there exits significant differences at 0.01 levels among middle-score groups and low-score groups.

The reason for this phenomenon is due to the abstractness of mathematics symbols. The mathematical symbols are used to express mathematical abstractions, but there are differences on the level of them. For example, a collection of different bases, and operational relationships among different spaces of dimensions and so on. Therefore, the system of mathematical symbols not only reflect the differences among the different mathematical abstractions on the same level, but also reflect the differences among the different levels of abstractions, which shows different degrees of abstractions.

It is just the abstractness of mathematical symbols that caused students' obstacles in reading. A survey shows that people have some psychological obstacles in the process of learning mathematical symbols more or less. The psychological obstacles in learning mathematical symbols mainly manifest the following aspects: negative transfer and emotional disorder in learning (Liu, 1993, p. 192–202). In the aspect of learning mathematical symbols, sometimes the form-structure of mathematical symbols disjoints with the semantic content, which will also cause the phenomenon of negative transfer. For example, students are familiar with the distributive formula

$$a(b+c)=ab+ac$$

of multiplication to addition, if they ignore the semantic content of

$$\sin(\alpha+\beta),$$

they will apply the distributive formula, and then they will get the wrong conclusion of

$$\sin(\alpha+\beta)=\sin\alpha+\sin\beta.$$

The emotion of students to mathematical symbols will directly affect their learning. For some students, after learning mathematics, they become not only indifferent to mathematical symbols, but also feel terrible about them. The odd and strange mathematics symbols that like "mumbo-jumbo" make them daunting. Furthermore, some inappropriate and exaggerated propaganda distorts the image of mathematical symbols, which make students be afraid of difficulty.

Because of the above reasons, the scores of symbolic language volume are significantly lower than that of literal language volume. Students with excellent mathematical academic have strong abstract thinking ability and perfect mathematics cognitive structure in signed or unsigned text-reading process and they can achieve a comprehensive understanding of the text. Therefore, the symbols will not have a significant impact to the excellent students. But for common students and the students with learning difficulties, the abstractness of the mathematical symbols will bring obstacles for them. Being lack of perfect cognitive structure of mathematics and reading self-monitoring capability, these students cannot understand the true meaning of the mathematical symbols and the ideas within it in learning. When they remember something, they just use the simply mechanical memory which is in accordance with the requirement of the teachers, and they only remember a few abstract symbols. In the process of mathematics reading, they will disjoint content and form that the simplified mathematical symbols disjoint from the represented mathematical content. Therefore, for these groups of students, the scores of symbolic language volume are significantly lower than the scores of literal language volume.

4.2. The effect of graphics to mathematics reading

Seeing from the whole two classes, the scores of the diagrams language test are significantly higher than that of the literal language test. Except for the groups of low scores, the other two groups both have obvious differences,

The reason for this phenomenon is the intuition of diagrams. Descartes once said: "there is nothing that can impress your mind more easily than geometry, expressing things in this way is very beneficial." Larkin & Simon (1987) think that even if text representation contains the same information with graphics representation, but there also exists some basic differences as follows: firstly, the graphics representation can clearly show the topological relationship and geometric relationship of the various components of the problem; Secondly, the relevant information of the graphics representation is usually in an adjacent location, which makes people easily recognize patterns, search information and deploy derivation. Therefore, People say that graphics representation is a good mathematics representation, that is not because it contains more information, but it supports more effective operation and more simple ratiocination.

However, Larkin & Simon (1987) analyze the function of the graphical representation from another point of view. They think that one representation generally has two effects: characterization and operation of information. Useful representation is not its better operational effects but its effects of characterization information. Having compared the reasoning effects from the graphics representation with those from text representation whose information is equivalent to the former, they get the following conclusions: graphics representation can contribute to the sense judgment. Stenning & Oberlander (1999) also explain the operational effect of the graphical representation from the semantic perspective of the representation system.

Graphics language has an effect on Mathematical Reading. And there are distinct differences between high-score group and middle-score group in two classes, which are caused by the picture-superiority-effect. How can graphics representation improve people's ability to represent and solve problems? And why? Larkin & Simon (1987) pointed out that it cannot absolutely be guaranteed that the individual can correctly solve the problem by using graphs or charts. However, the graphs and charts can guide the subjects to systematically analyze and continuously delicate problem situation, and improve planning methods for a certain task. Besides, they can be used to explain and examine the answers and drawing graphs or charts are usually regarded as cognitive tools that solve problems usefully and effectively.

The cognitive effect of graphic representation is mainly through the positioning method and organizing information to reduce the blindness of searching information and the load of working memory. It helps to solve problems through the promotion of perceptual judgments and it can promote the extraction of information (Koedinger & Anderson, 1990). As for the subjects of low scores, because the cognitive structures of individuals are incomplete, unstable and confused, and lack the effective integration of the internalization of knowledge, and the concepts and propositions cannot be understood from multiple angles and levels. In this way, even if they are given appropriate tips in exploring process, it is hard to activate the available resources in their minds, at the same time external tips just enable participants to make limited inferences and it is difficult to produce a continuous process of reasoning (Yu, 2003; Yu, Li & Yang, 2006). Therefore, the effect of graphics tips to mathematics reading in low-score groups of the two classes cannot produce significant differences.

5. TEACHING SUGGESTIONS

5.1. Multiple external representation strategies

The abstractness of mathematical symbols will increase the difficulty of the problems, which will cause obstacles to the students. Research shows that some problems that have the same internal representation but different external expressions will cause completely different solving behaviors of individuals, that is to say, the external representation of the problem will have a great influence on solving problems. External representation refers to the components and structures of problem situations, which includes symbols, objects, dimensions and external rules, constraint or boundary conditions etc., all of which can only be perceived, analyzed and processed by the perceptual system. Zhang (1997) points out that the external representation play roles in the problem solving process by two mechanisms, one is direct awareness of the unchangeable structure within the external representation through perception, which reduces cognitive load, without activating some complicated mental models of internal memory system, and the problems are solved without the participate of inference. The other is to activate the internal complex cognitive process to establish internal mental model which generates the problem space (Zhang, 1997).

Different concept representation provides multiple specific manifestation of a concept, which may strengthen some aspects of the complex concept or may weaken the other aspects at the same time, promoting cognitive link between different representations (Keller & Hirsch, 1998), so as to deepen students’ understanding of the concept. Multiple representation of problem solving process can complement each other in content. For example, complex information often distributes parallel in multiple representation, in order to avoid the information of a single representation beyond the cognitive load of one individual; and can complement each other in representation and operation efficiency, therefore, different representations can be used to different purposes, and also play different roles in problem-solving process; one representation may limit the interpretation of another representation.

Therefore, each problem will have different representation in teaching, each representation has its own pros and cons. What's more, each student has his or her own way of thinking; they have different preferences for different representation. The same title should be presented in different styles in teaching, guiding students to grasp the essential content of the problems and eliminates interference of non-essential content, so as to promote the understanding of mathematical symbols and improve reading comprehension ability.

5.2. Visualized teaching strategies

The mathematical language increases the difficulty of reading due to the abstraction, simplicity and generality of its own. A survey shows that increasing the graphics will improve the reading effectiveness of students. Teachers can reduce the abstract degree of the mathematical language by using the visualized teaching strategy. Visualization of mathematics is a cognitive method which treats mathematical visual symbols (either internal-presented or external-presented) as the basic elements, and regards visualization in information processing as its form (Fu, 2009, p. 167). Teachers can use visualized method in teaching through the following ways.

(1) Explanation of geometric meaning

The geometric meaning interpretation of mathematical concepts and mathematical theorems can help students form a mental image and promote the students' memory of knowledge. Meanwhile, students accumulate mental image construction experience during the process; the increasing of the experience promotes the possibility of various types of visualization, as well as provides a potential possibility for mental image migration in the problem-solving process. The existence of a considerable part of concepts in middle school mathematics is accompanied by the existence of their geometric meanings, collections, basic elementary functions (exponential function, logarithmic function, power function, and trigonometric function), plane vector, triangle identical transformation. Most mathematical concepts among them contain rich intuition. A general collection often contacts with Wayne map while a set of numbers often associated with the axis, and each basic elementary function has an image in correspondence with it (Fu, 2009, p. 133).

(2) The combination of numbers and shapes

Steen, an American mathematician, says that if a particular problem can be transformed into a graph, then thought has grasped problems overall, and can think how to solve the problem creatively. Graphics representation cannot only clearly show the topological relationship and geometrical relationship of each component of the problem, and the related information of the graphical representation is usually in a nearby location, which makes people identify patterns, search for information and expand derivation easily (cf. Larkin & Simon, 1987). Research shows that the text information can be profoundly remembered in a graphic description situation than in non-graphic one. The cognitive effects of graphic representation mainly lie in reducing information searching blindness and working memory load via the strategy of positioning way of organizing information that is through the promotion of perception judgment way to help to solve problems and to promote the information extraction (Luo, 2005, p, 104 & p. 106).

5.3. Concept-map construction strategies

By constructing a concept-map can improve the cognitive structure of individuals, and promote readers' understanding of the symbols and graphics. Concept maps are tools for organizing and representing knowledge. It usually puts a concept or a proposition which relates to a subject in different levels in boxes or circles, and then connects the related concepts and propositions in a variety of links thus the concept or the network-proposition about the topic is formed. Characterizing the knowledge structure of the learners in such a visualized way which can effectively help students identify the concept, know their relationship, and then improve the cognitive structure effectively.

The concept-map-making does not have strict procedures and norms, and can generally be achieved in the following steps (Yu, 2010, p. 72):

Step 1, list concepts. Read the textbook, mark the important and key points, identify the key concepts and make a list;

Step 2, starting from the key concepts, determine the relationship among the concepts, top the broadest, the most inclusive and the most abstract concept of the map;

Step 3, continue adding more specific concepts, and bottom the most specific and inclusive concepts;

Step 4, establish connection between concepts, use the simple and clear connecting words to indicate the relationship between the two in a line;

Step 5, when the concepts are connected, it is necessary to use the arrow to denote the relationship between concepts;

Step 6, present the specific examples next to the concepts.

It is necessary to reflect and improve the concept map constantly after it is formed. Generally speaking, teachers should introduce the concept-map to students, including what the map is, which function it has, what the components of the concept-map are; analysis the construction of the concept-map; practice constructing concept-map; check and discuss the concept-map of students; students exchange their concept-maps and so on.

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