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An algorithm for estimating surface normal from its boundary curves

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Abstract

Recently, along with the improvements of geometry modeling methods using sketch-based interface, there have been a lot of developments in research about generating surface model from 3D curves. However, surfacing a 3D curve network remains an ambiguous problem due to the lack of geometric information. In this paper, we propose a new algorithm for estimating the normal vectors of the 3D curves which accord closely with user intent. Bending energy is defined by utilizing RMF(Rotation-Minimizing Frame) of 3D curve, and we estimated this minimal energy frame as the one that accords design intent. The proposed algorithm is demonstrated with surface model creation of various curve networks. The algorithm of estimating geometric information in 3D curves which is proposed in this paper can be utilized to extract new information in the sketch-based modeling process. Also, a new framework of 3D modeling can be expected through the fusion between curve network and surface creating algorithm.

Keywords: Computer-aided design; Curve network; Surface modeling; Surface normal estimation

1. Introduction

Despite decades of research and development, three-dimensional (3D) modeling remains a significant challenge. Computer-aided design (CAD), notwithstanding its enormous progress from the initial-phase Boolean operations on basic diagrams to a variety of geometric operations, still requires a lengthy training period and a high degree of technical expertise. Recent years have seen the emergence of novel modeling technologies for building 3D curve networks from designers' two-dimensional (2D) sketches in an effort to lower the hurdle for using professional 3D modeling tools [1–3]. Such modeling methods are widely used for concept design in the initial phases of design because they facilitate the intuitive expression of the designer's ideas and allow room for additional creative elements.

Despite the advantages of sketch-based modeling tools, 3D curve networks alone cannot provide sufficient visual feedbacks. 3D curve networks give us only the feature lines, which are drawn by designer, not the complete rendered model. In the strict sense, they are not the models, just set of lines. They do not have any surface information so that it is hard to imagine detailed shape of the model. Therefore, constructing the surface model from its drawn curve networks is essential. Besides providing more precise visual feedback, the constructed surface model can be used in a wide variety of application fields, such as analysis, simulation, and 3D printing.

However, constructing a model with only position information and topological relationships on the curve has an inherent problem of uncertainty that provides room for numerous analytical approaches rather than an optimal solution.

This fundamental problem has long been treated in the field of CAD, and is referred to as lofting or skinning. This is a problem arising from the lack of geometric information on the generated model. In order to address this problem, an optimization model that can precisely reflect additional constraint conditions or design intentions needs to be defined.

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The difficulties related to solving these problems are as follows: (i) whether additional geometric information can be obtained from the position information and the topological relationships of curves (possibility issue), and (ii) whether the geometric information obtained in one manner or another reflects the design intention correctly (validation issue).

Against this background, this study aims to achieve an automatic implementation of curved surface modeling by determining the geometric features of 3D curve networks. For the determination of such geometric features, a new frame for points on the target curve was determined using a rotation-minimizing frame (RMF).

RMF is one kind of moving orthogonal frame defined on curves that has no rotation about the instantaneous tangent of the curve. Due to its minimal twist, RMF is widely used in computer graphics, motion design and sweep surface modeling in CAD. Though it is harder to compute than Frenet frame, it does not have discontinuity that is unacceptable in surface modeling. With these features, RMF is suitable for estimating geometric information of curves.

Using this newly defined frame, we determined the normal vector to the curve that matches the design intention on the basis of sketch tutorials and previous cognition-related studies.

The method proposed in this paper is different from the methods presented in existing papers in that the proposed method allows curve modeling under the condition of a lack of geometric information by establishing an optimization equation reflecting the design intention and determining the normal vector. The methods to create surface model in most previous studies are based on patch generation algorithm with known geometric information. In this paper, we proposed new method that estimating surface normal vector from its boundary curve. It has significance for not only constructing 3D model from curve network with better result but also estimating unknown geometric information.

The remainder of this paper is organized as follows: Section 2 presents the problem-solving approaches adopted by existing papers in relevant fields. Section 3 describes the entire procedure for the proposed curved surface modeling. Section 4 contains an elaborated description of the algorithm used for the determination of geometric features and the modeling process based on these features. Section 5 provides an overview of the modeling results for various curve networks. Finally, Section 6 contains conclusions, limitations, and the future research direction.

2. Literature review

Since the introduction of the modeling method using a sketch-based interface over a decade ago, it has been the object of intense research. Teddy, a system developed by Igarashi et al. [4] converts a 2D silhouette into a 3D surface by adding an inflated appearance to the surface surrounded by the silhouette, thus enabling intuitive modeling from a simple sketch. Fibermesh developed by Nealen et al. [5] added functional features allowing free transformations and corrections of Teddy-based surfaces. Rivers et al. [6] presented a new algorithm for 3D modeling by integrating multifactorial 2D silhouette data.

In parallel to this stream of research, there has been considerable research focused on the methodologies for the construction of 3D curve networks. ILoveSketch presented by Bae et al. [1] is a system for constructing 3D curve networks allowing direct sketching on gesture-based interfaces. With JustDrawIt!, Grimm and Joshi [2] presented a 2D-to-3D conversion method by analyzing a newly drawn 2D sketch and defining its relationships to an existing sketch. Schmidt et al. [3] implemented a mathematical model by analyzing the sketching process and reconstructing a 2D sketch as a 3D sketch.

The paper by Schaefer et al. [7] is representative of the studies presenting surface modeling based on the 3D curve networks thus constructed. This paper presented a method of improving a model towards satisfying C1 continuity after generating rectangular patches from the adjacent curves within the constructed curve networks and segmenting them using the Catmull–Clark subdivision method. Although this method shows satisfactory results provided that curve networks describe the model exactly, it has the limitation of model entanglement when dealing with complicated curve sketches.

Abbasinejad et al. [8] defined problems through linearization by using a Laplacian method instead of an optimization approach. In this method, any given circle is rendered in triangular meshes, followed by model construction by joining meshes on the silhouette of each curve patch and generating a model similar to a soap film by using linear interpolation. Although it has the advantage of rapid linear algebraic computation, it is limited by the inferior quality of a thus-yielded model because of the insufficient exploitation of geometric data.

Bessmeltsev et al. [9] conducted a study on a design-based modeling method, wherein a model is generated from rectangular meshes constructed with the curve pairs that best reflect the design after iteratively extracting them from the curves on a surface. This method has the advantage of the superior quality of the resulting model because it uses an optimization method considering the design, but has the drawback of requiring an enormous amount of time for extracting all curve pairs from the targeted curves.

The latest method known thus far is the patch decomposition method proposed by Abbasinejad et al. [10]. In this method, the problem of patch generation on a complicated curve is addressed by decomposing patches into quasi-planar ones. The main limitation of this new approach is the unsmooth continuity because patches are generated through decomposition.

Although these studies also convert curve network to 3D surface model, the results are different with our aim which is providing visual feedback to designer. It should compute quickly enough to show the result model in response to designer's sketch and also provide precise visual feedback. Schaefer et al. [7] does not get the consistent results when used in complex curve network. Bessmeltsev et al. [9] takes too much time so it cannot show its result in real-time. Abbasinejad et al. [8,10] possibly generate the surface model in real-

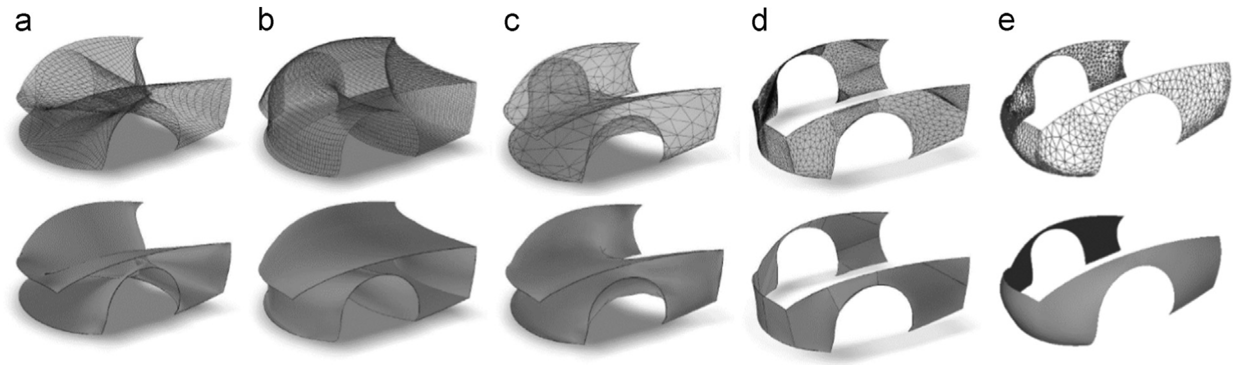

Fig. 1. Five methods to create a surface approximation for a single, complex patch boundary: (a) the lofting curve networks of Schaefer et al. [7], (b) the design-driven quadrangulation of Bessmeltsev et al. [9], (c) linearized Laplacian smoothing of Abbasinejad et al. [8], (d) minimal ruled surface of Abbasinejad et al. [10], and (e) our method, which estimates surface normal that accord design intent.

time but have weakness that cannot provide sufficient visual feedback. Our study compensates these defect by estimating new geometric information reflect design intent and constructing surface model based on them.

Fig. 1 illustrates the results obtained using one of the existing methods and the method proposed in this paper for modeling a closed sketch.

3. Overview of curved surface modeling

Fig. 2 shows the entire process of curved surface modeling as presented in this paper. The process of curved surface modeling is divided into two subprocesses: determination of the normal vector, as shown in Fig. 2(d), and generation of initial meshes, as shown in Fig. 2(b) and (c), for the target curve corresponding to the input shown in Fig. 2(a). These data are used for obtaining the target normal vector of each vertices of the meshes as shown in Fig. 2(e), leading to the results shown in Fig. 2(f).

The subprocess of obtaining the normal vector involves the RMF-based definition of a new frame. RMF is determined from the initial vectors and the vectors tangential to the curve, thereby changing the initial vectors and iteratively seeking values for the frame that minimizes the RMF-defined computational cost. The normal vector to the curve is then determined from the defined frame.

The cost of selecting a frame capable of reflecting the design intention is defined in Sections 4.1 and 4.2 contains the process of setting the new frame of the curve and the determination of the normal vector to the curve.

The second subprocess of curved surface modeling involves the generation of triangular meshes representing the overall shape (mesh generation) and an improvement of their quality (mesh fairing). Initial mesh generation applying the hole-filling algorithm used in the field of CAD is explained in Section 4.3.

Mesh fairing is the final step of the modeling process, wherein the target normal vectors of the vertices of all meshes are obtained from the normal vector to the curve, and then, they are applied to the initial mesh. Section 4.4 contains the methodological explication of this process.

4. Algorithm

4.1. Curved surface bending energy

According to the result of the sketch perception study conducted by Mamassian and Landy [11], curves with smaller curvatures are preferably perceived in a simple sketch. The study of developable surface by Rose et al. [12] sets similar hypothesis to this and experimentally validated it. The theory is also supported by studies conducted by Shao et al. [13] and Zhuang et al. [14].

To quantify curvature, we defined the curved surface bending energy as follows: the curvature is considered to be small if the change in the normal vector to the curve is small in the vicinity of the sharply bending part of a curve constituting a patch. In the normal vectors marked in Fig. 3(a) and (b), the adjacent normal vector in Fig. 3(b) has less change and smaller curvature as compared to that in Fig. 3(a). This can be expressed by the following equation:

$$\sum_{v \in V_c} \text{arc}\, cos\langle n_{v1}, n_{v2}\rangle$$

$$V_c = \{v|\text{intersection of the curves}\} \quad (1)$$

The minimization of the curved surface bending energy is tantamount to the minimization of the surface area. In other words, the definition of the curved surface bending energy postulates a minimal surface and is directly associated with curved surface modeling.

4.2. Determination of a normal vector to a curve

The normal vector to a curve constituting a patch should have continuity; i.e., it should not have dramatic changes. This requirement can be met using RMF [13]. RMF is computed with the initial vectors and the vectors tangential to the curve; in this study, we used the doubling reflection method proposed by Wang et al. [15] for the RMF computation.

The vectors tangential to the curve can be obtained from the position information of the curve. Therefore, in order to obtain the RMF minimizing the curved surface bending energy as

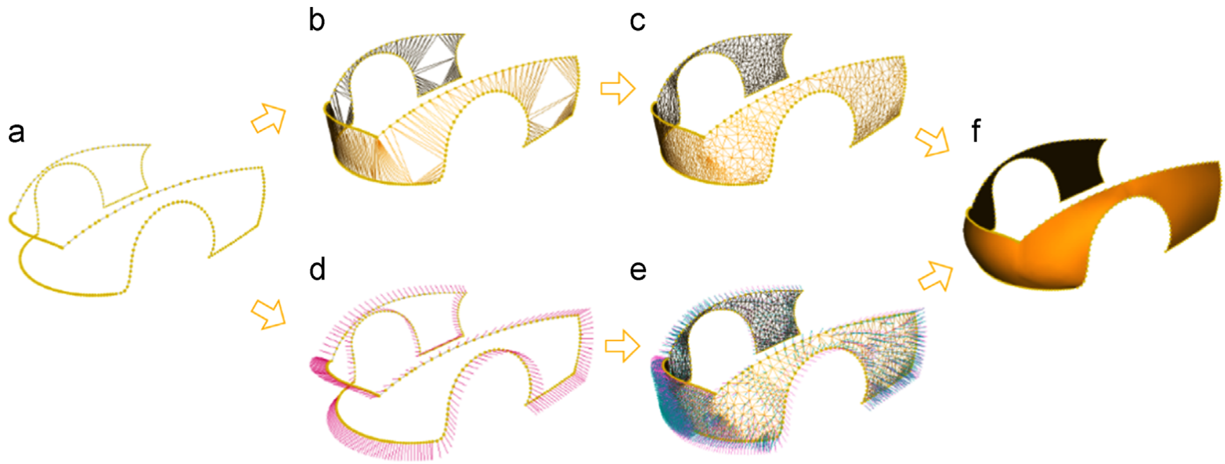

Fig. 2. Our surface model creation process (a) input 3D boundary curve, (b) initial triangular mesh, (c) mesh refine, (d) estimate normal vectors of the input curve using RMF and bending energy, (e) calculate target surface normal from the result of (d), and (f) mesh fairing with target normal.

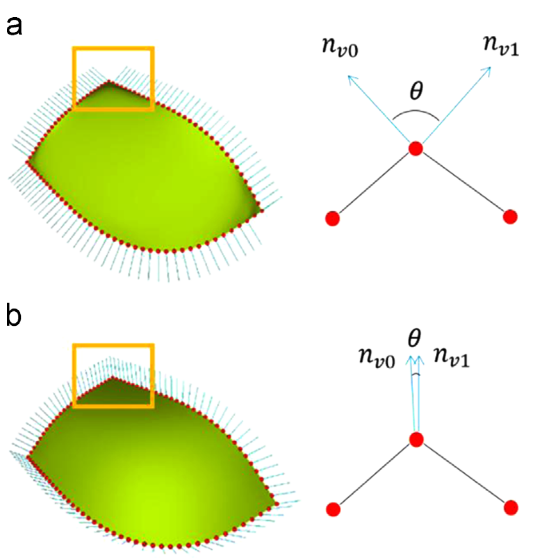

Fig. 3. Definition of surface bending energy. The difference of angle between normal vectors of adjacent points indicates the amount of surface bending. In this case, (a) is bended more than (b).

explained in Section 4.1, iterative computation should be performed, thereby varying the RMF's initial vector. One axis of the new frame is set to be the vectors perpendicular to those tangential to the curve at the starting point. After iterative computation by gradually varying the initial vector, the RMF having the lowest value of the curved surface bending energy is determined as the normal vector of the curve.

We need to solve non-linear equation to get the solution for minimizing Eq. (1). But we can simplify this equation with respect to initial normal vector of curve by virtue of RMF.

4.3. Initial mesh generation

In order to generate smooth meshes without twisting on a closed curve drawn on a 3D surface, it is crucial to consider the overall curve shape. Numerous hole-filling algorithms based on CAD serve this purpose. In a study by Barequet and Sharir [16], all vertices constituting a closed curve are explored in search of a solution that yields the least value of the sum of the triangular surface areas of the mesh.

Fig. 2(b) shows the initial mesh thus constituted. The mesh quality is improved through the addition of vertices to the mesh, as shown in Fig. 2(c), in order to apply the normal vector obtained in Section 4.2.

4.4. Mesh fairing algorithm

After obtaining all normal vectors to the curve, those to the vertices within the mesh are interpolated using harmonic fields. f of the harmonic function f should satisfy the Laplace equation.

$$\nabla^2 f = 0 \tag{2}$$

Using a discrete Laplace operator, we can write this as follows:

$$\sum_{v_j \in N_i} \omega_{i,j}\left(f\left(v_j\right) - f(v_i)\right) = 0, \; v_i \in V_I$$

$$\omega_{i,j} = \cot \alpha_{i,j} + \cot \beta_{i,j}$$
$$N_i = \{v|1-\text{ring vertices of vertex } v_i\} \tag{3}$$

Fig. 4 lists the equation symbols.

Let f be the normal vectors to the vertices on the curve; then, the normal vectors to the vertices with the mesh can be obtained with a linear equation.

In order to implement the transformation into a model meeting the user's intended purpose, we applied the normal vectors to all vertices of the mesh, thereby using the discrete spring model suggested by Yamada et al. [17].

5. Results

As shown by the results illustrated in Fig. 5, the proposed algorithm enabled the transformation of the initial mesh generated under the condition of lacking geometric information into a natural-looking model approximating the user's intended purpose. We used the test data obtained from

ILoveSketch [1] and JustDrawIt! [2] as the curve networks for input information.

We dealt with constructing surface model from curve networks, which are the boundary curves, and extracting each patch from the curve networks is done manually. Abbasinejad et al. [8]; Zhuang et al. [14] handles method for extracting patch cycles from the curve network. With these studies, we can automate the patch finding process.

Using the algorithm proposed in this paper, we succeeded in generating patches of various types of curves, be it a plane curve clearly showing the user's intention or a curve with a highly complex structure. This result could be obtained since the algorithm estimates geometric features which are reflecting design intent.

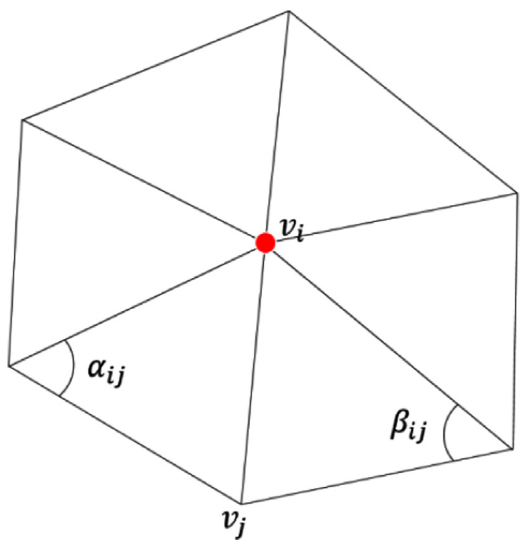

Fig. 4. Notation for Laplace equation.

The comparison of result with existing methods is illustrated in Fig. 1. The model shown in this figure is ‘roadster’ which is drawn with ILoveSketch [1].

The result of Abbasinejad et al. [8]; Schaefer et al. [7] (Fig. 1(a) and (c)) seems like minimal surface but it is hard to say that they reflects design intent. The method of Bessmeltsev et al. [9] can get the high quality surface model as shown in Fig. 1(c) since they consider design intent in their algorithm but computing time is still a problem. The result of Abbasinejad et al. [10] (Fig. 1(d)) looks closer to its design intent than previous three results. Its decomposing process simplify the problem but un-smooth continuity occurred where the patch connecting piece.

Our method (Fig. 1(e)) shows visually superior result than others and it proves that estimated geometric information plays important roles in constructing surface model. With the mesh generation across the entire curve comprising patches and the determination of the normal vectors tailored to the user's intention, superior results could be obtained.

6. Conclusions

In this paper, we proposed an algorithm that can automatically generate a model tailored to the user's intention from a sketch-based 3D curve. To obtain the geometrical information necessary for the curved surface modeling, we established an equation that describes the model for obtaining normal vectors to the curve by using the RMF-based minimal twisting condition and the concave preference condition. We also obtained the

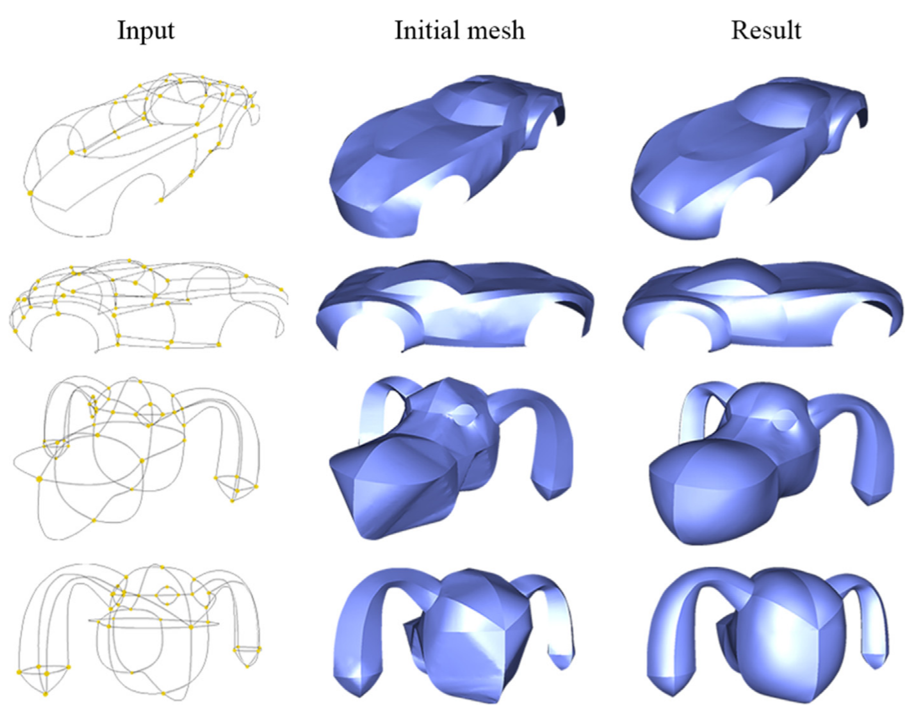

Fig. 5. Result of proposed algorithm which is applied to various curve network input. First column is input curve from ILoveSketch [1] and JustDrawIt [2]. Second and third columns show the resultant mesh without and with target normal fairing respectively.

normal vectors to the curve matching the user's intention by performing iterative computations and implemented a curved surface model by transforming the mesh accordingly.

The proposed algorithm is significant in that it renders the sketch intention in the form of an equation under the condition of a lack of geometric information and thus, obtains new geometric information using a method for obtaining the optimal solution for the normal vector. A mesh that is closer to the user's intention can be obtained by adding the normal vector data to adjacent patches to the normal vector to the boundary curve obtained for each patch. This method presents a new framework for 3D modeling in combination with the curve networking technology.

Conflict of interest

The authors have declared that no competing financial interests exist.

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