Journal for History of Mathematics
Vol. 28 No. 4 (Aug. 2015), 181–190
http://dx.doi.org/10.14477/jhm.2015.28.4.181

Space-Time Symmetry and Space-Time Ontology

시공간 대칭성과 시공간 존재론

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In spite of various attempts to characterize the ontological status of space-time, Newtonian substantivalism and Leibnizian relationism, what is really at issue in the controversy between the two parties is by no means clear. This essay argues that from the perspective of space-time symmetries, classical space-time can be unambiguously classified as substantival space-time and relational space-time. The symmetries of space-time theories distinguish the invariant geometric relationships between events. The essential difference between the two space-times stems from whether or not there exists the affine structure that distinguishes the inertial trajectories of a given body.

Keywords: the ontological status of space-time, substantivalism, relationalism, space-time symmetries; 시공간의 존재론, 실체론, 관계론, 시공간의 대칭성.

MSC: 01A05 ZDM: A30

1 Introduction

The dispute over the ontological status of classical space-time seems to be essentially about the *mode of existence of space-time*. The debates over the ontology of space-time between two classic rivals, Newton and Leibniz, have been interpreted as a part of the *substance-relation* controversy. On the one hand, Newtonian substantivalism holds a view that the existence of a part of space is analogous to that of material substance. On the other hand, the rival Leibnizian relationalism, while denying that a part of space has an independent existence, views space as a set of relations among material bodies or events. In spite of various attempts to characterize the ontology of space-time, what is at issue in the controversy between the two parties is far from clear. After summarizing failed attempts to provide the clear-cut borderline between substantivalism and relationalism, this essay offers a bet-

This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government(NRF-2013S1A5A8026239)

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Received on July 17, 2015, revised on Aug. 20, 2015, accepted on Aug. 25, 2015.

ter account based on the symmetry of space-time geometry. This essay argues that from the perspective of space-time symmetries, classical space-time can be unambiguously classified as either substantivalism or relationalism. Given that the symmetry groups of space-time geometry enable us to sort out the essential elements of classical physics, this mathematical structure provides a well-defined distinction between the ideas of Newton and Leibniz. After historical analysis and formal consideration, philosophical consideration of emphasizing the symmetry group is also provided in the concluding section.

2 Failed Attempts to Characterize the Ontology of Space-Time[1)]

At first, it seems that what is concerned with the debates over the ontological status of classical space-time is the specification of the categorical properties that space exhibits as an existent. Both substantivalism and relationism could be viewed as contending rival attempts to specify the mode of existence of space, that is, whether it is essentially substance or relation. Newton, in his Scholium, captures the ontology of space from its "essence or nature" in that space is a collections of "the primary places of things." [8] From his reading of Newton's Scholium in *Principia*, Earman views Newtonian space as "a substance in that it forms a substratum that underlies physical events and processes, and spatiotemporal relations among such events and processes are parasitic on inhere in a substratum of space or space-time points." [5] In contrast, Leibniz, denying the conceptual foundation of space as a substance, provided an alternative interpretation of the ontology of space; "[Newtonians] maintain, therefore, that space is *a real absolute being* … As for my opinion, … I hold space to be something merely relative, as time is; that I hold it to be an order of coexistences … ." [1] Accordingly, Leibniz found fault with Newton's ontological account of space as substance, and viewed space as the relationships between material bodies or events. Hence, from the above accounts, both Newton and Leibniz seem to consider the debates over the ontological status of space as being concerned with the metaphysical nature of space as an existent being.

Newton, however, was not consistent in characterizing the mode of the existence of space in that "it has its own manner of existence which fits neither substance nor accident." [7] As space is by no means active, the property of space is different from that of a typical case of substances, such as mind and body. Furthermore, the mode of existence of space is not genuine substance given that its existence depends on something else, that is, God. Newton interprets space as substance *analogically*,

1) The section summarizes and modifies Yang [10] which provides a much more detailed account of the failures of characterizing the ontology of space-time.

rather than literally, considering his remark that "[space] *approaches more nearly to* the nature of substance." [7] Hence, the category of space as substance should be read as a *metaphor or analogy*, rather than an exact metaphysical characterization.

Another notable attempt to characterize the debate concerning the ontology of space is the philosophical controversy over realism vs. anti-realism. In other words, what is at issue over the controversies is whether or not one admits *the existence of an entity* called 'space'. Substantivalists such as Newton are interpreted as realists about the existence of space, which interprets space as an enduring entities analogous to ordinary matter. According to Newton, "we can possibly imagine that there is nothing in space, yet we cannot think that space does not exist … ." [7] On the other hand, relationalism held by Leibniz maintains that what really exists is only material bodies, and "there is *no real space* out of material universe." [1]

However, the reality of space is not at issue within the controversy between substantivalism and relationalism. Although we admit that the reality of space is one of motivations under the controversies, it is not a criterion to distinguish substantivalism from relationalism. Earman [5] and DiSalle [4] claim that relationalism can be also viewed as a realism about certain geometric structure of space-time, such as the temporal metric embracing absolute simultaneity and the spatial metric embracing Euclidean geometry on each simultaneous time.[2)] Although having less structures than Newtonian space-time has, relationalism holds geometric structures of simultaneity which are discarded by Einstein's special relativity. Given that relationalism admits the reality of certain geometric structures, the link between the relationalism of space-time and anti-realism is not straightforward enough for the one to provide a criterion for the other.

Furthermore, the inconsistency between substantivalism and realism prohibits us from relating the two philosophical perspectives. The reality of Newtonian space can be captured by its geometric structure supporting the 'trans-temporal identity' of the points of space. However, the existence of individual points of space, although enables us to select privileged trajectories, contradicts with the principle of Galilean relativity. In his Corollary V to the laws of motion, Newton recognized this problem of the existence of individual points of space; "When bodies are enclosed in a given space, their motions in relation to one another are the same *whether that space is at rest or whether it is moving uniformly straight forward* without any circular motion."

2) Earman [5] points out that Leibniz, just like Newton, admitted the absolute simultaneity given Leibniz's own statement that "whatever exists is either simultaneous with other experiences or prior or posterior." Along the same lines, DiSalle [4] maintains that "[Leibniz's] relational theory of motion therefore presupposes absolute simultaneity and Euclidean Geometry on space at each moment of time. … Leibniz, like Newton, attributes physically objective characteristics to space and time that do not depend on time."

[8] Considering this inconsistency that Newton himself pointed out, it is doubtful that trans-temporal identity of the points of space fits well into Newtonian dynamics. This conceptual problem of the trans-temporal identity for the points of space leads us to doubt the link between the reality of space and a substantival entity. Accordingly, the substance-relation controversy over space cannot be characterized by the reality of space.

Given that the geometric structure of space-time has a theoretical role of making sense of the laws of motion including the principle of Galilean relativity, one might consider Newtonian substantival space-time as an inertia generating entity that *causes* material bodies to follow trajectories corresponding to the laws of motion. By identifying substantival space as a causally efficient entity that influences material bodies to follow inertial or non-inertial trajectories, it seems that we respect the context of physical theories that we are concerned with. According to Newton, the central goal of the *Principia* is "how we obtain true motions from their causes, effects, and apparent differences." [8] Einstein also interpreted Newton's absolute space as a causally efficient entity, which "influences but cannot be influenced by" other material bodies. [2]

However, it has been pointed out that this attempt is based on a misunderstanding of the way space-time makes sense of the laws of inertia. As pointed out earlier, Newton did not view the causal property of space as defining its essence in that he did not identify space as a typical case of substances because space is not active. According to Brown [2] and DiSalle [4], rather than identifying absolute space as an entity causing bodily motion, Newton *defined* the true absolute motion by means of cause and effect affecting bodily behaviors; "the true and absolute circular motion of the water ... becomes known, and may be measured by this endeavour." [8] In other words, Newton himself distinguished the true absolute motions from the relative ones with reference to absolute space. The way space-time explains the laws of inertia is not by means of the causal mechanism of a substantival entity, but by an assumption that posits the affine connection as the geometrical structure which allows the definition of inertial motions. In this sense, space-time geometry, by means of theoretical assumptions, is related with the behaviors of motion encapsulated by the laws of motion.

Given that various space-time theories are concerned with the motions of material bodies, another interpretive scheme for space-time is to consider the substance vs. relation dispute as the controversy over *the nature of motion*. In other words, the aim of the controversy is to decide whether the true motion of bodies is 'absolute' or 'relational'. It could be said then that substantivalism is a view that the true motions are only absolute ones which associates the same effects without regard to other bodies. In contrast, relationalism claims that the true motions are relational in the sense that the motions of bodies are with respect to the other bodies. Accordingly, the aspect with which the disputes over the structure of space-time are concerned might be empirical substructure, i.e., the characterization of the true motions.

Although this interpretive scheme does value an aim of space-time theories, they are much more than providing appropriate empirical substructures. Granted that space-time theories are concerned with the phenomena of motions of bodies, it does not follow that the motions of bodies are described only by observable means. Earman [5] holds that space-time theories are not the result of only unassuming perceptual experiences of the relative changes of material bodies' positions. Over and above their empirical substructure, both substantival and relational space-time equally assume that space-time is represented as a four dimensional differentiable manifold, which can be decomposed into the temporal structure of linear time and the spatial structure of three-dimensional simultaneous spaces.

Summing up this section, the perspective of metaphysics, reality, causal structure, and empirical substructure all fail to characterize the nature of space-time. All these accounts fail as they do not respect what is the main aim of space-time theories. The way that the motions of material bodies are explained by space-time is through its theoretical assumption, rather than through its metaphysics, its reality, its causal structure, and its empirical substructure. Accordingly, the difficulties of the aforementioned attempts to comprehending the ontology of space leads us to examine the essential theoretical structure of Newtonian and Leibnizian space-time. Our task of the next section is, then, to elucidate which essential contents of space-time theories explain material bodies' motions.

3 The Symmetry Group of Space-Time and the Essence of Space-Time

Space-time symmetries are expected to provide an accurate criterion to classify space-time theories, given that the essential structures of space-time theories can be characterized as space-time symmetries coordinated with dynamic symmetries. Along these lines, these symmetry groups enable us to sort out the essential elements of classical physics: coordination between the spatio-temporal relationships encoded within space-time geometry and the laws of motion encapsulating the behaviors of moving bodies. A specific symmetry, within group theoretic formulation, is defined as the invariance under a specified group of transformations. On the one hand, the symmetry group of the movements of an idealized rigid body encodes the invariance of the laws of motion. On the other hand, the invariance of the laws of motion under a group of spatio-temporal translations of material bodies in reference to specific reference frames enables us to *define* space-time geometry. Given that space-time geometry is defined by the specific reference frames invariant under groups of transformations, space-time geometry can be characterized by its symmetries. Within the coordinate system formalism of space-time, the transformation between coordinate systems, which represents reference frames, form a group. In this way, the symmetry groups of space-time encode the invariant geometric structures independent of choosing specific reference frame. And these coordinate systems are defined for the motion of a given body to satisfy the laws of motion.

From the perspective of space-time symmetries, space-time geometry can be unambiguously classified as either Newtonian or Leibnizian space-time. The symmetries of Newtonian space-time distinguish the invariant geometric relationships between events: absolute simultaneity, spatial and temporal metric of the instantaneous spaces, and affine structure specifying straight line representing inertial motion. [6] In contrast, Earman [5] claims that to be relational space-time, the structure of space-time should have at least the one of Leibnizian space-time, of which symmetries are absolute simultaneity, spatial and temporal metric of the instantaneous spaces, *without* affine structure. Substantival space-time can be then distinguished from relational one by which a set of spatio-temporal relationships between events represents each space-time. In the forthcoming section, we will see that the crucial difference between the two space-times stems from whether or not there exists the affine structure that distinguishes the inertial trajectories of a given body.

4 Substantivalism vs. Relationalism from the Space-Time Symmetries

The importance of symmetries in characterizing Newtonian space-time can be traced from the phrases from Newton's own work. The uniformity of space, which can be interpreted as the symmetry of spatial displacement, is declared in *De Gravitatione*; "[space] is eternal, infinite, uncreated, *uniform throughout*, not in the least mobile or capable of inducing change of motion in bodies." [7] In addition, the "similar and immovable" feature of space over time can be interpreted as the symmetry which sets a relation of occurring at the same place. It seems that Newton also recognize the significance of the symmetry group of spatial translations, which is coordinated with dynamical symmetries: "whole space of the planetary heavens either rests or moves uniformly in a straight line, and hence the communal centre of gravity of the planets either rest or moves along with it. In both case the relative motions of the planets are the same ... ." [7]

In rectangular Cartesian coordinate systems, these spatial symmetries are written as $\mathbf{x} \to \mathbf{x}' = \mathbf{R}\mathbf{x} + \mathrm{C}_1$, where $\mathbf{x}$ consists of x, y, z components, $\mathbf{R}$ is a constant rotation matrix in SO(3) and C_1 are real numbers. [5] The temporal structure can also be characterized as the symmetry under a group of temporal translations: "absolute, true, and mathematical time, of itself, and from its own nature, flows *equably* without relation to anything external." [8] This can be written as $t = t + \mathrm{C}_2$, where t and C_2 are real numbers. These spatial and temporal translations display the symmetric properties of spatial and temporal metrics; the spatial and temporal distance between two events remains invariant under the aforementioned coordinate transformations. In addition, the space-time symmetries allows mapping of space-time onto itself preserving absolute simultaneity and affine connections. In a nutshell, the symmetry transformations, which conserve the aforementioned invariant structures, are spatial rotations $\mathbf{R}$, spatial translations C_1, and temporal translations C_2. And the spatial and temporal symmetries under rotations generated by $\mathbf{R}$ and under translations generated by C_1 and C_2 envision the structures of Newtonian space-time, i.e., isotropy and homogeneity.

In contrast, the symmetry of Leibnizian space-time in rectangular Cartesian coordinates system, can be written as $\mathbf{x} \rightarrow \mathbf{x}' = \mathbf{R}(t)\mathbf{x} + \mathbf{a}(t)$ and $t = t + \mathrm{C}$, where $\mathbf{x}$ consists of x, y, z components, $\mathbf{R}(t)$ is a time dependent rotation matrix in $\mathrm{SO}(3)$, t and C are real, and $\mathbf{a}(t)$ is an arbitrary smooth function of time. [5] Given that the rotations are time dependent, each instantaneous spatial plane can have different degrees of rotation with respect to one other. Another time dependent function $\mathbf{a}(t)$ is added in order to envision the symmetries of spatial translation. The symmetries then allow only maps of Leibnizian space-time onto itself preserving simultaneity, and the ratio of time intervals, relative distance between instantaneous spaces.

5 Inertial Structure: Newtonian Space-Time vs. Leibnizian Space-Time

At this point, we can clearly see the difference between Newtonian space-time and Leibnizian space-time. It is whether or not these space-time symmetries have the affine structure. Without this geometric structure, the inertial trajectory of a given body is not an invariant conception. In case of Newtonian space-time, the "similar and immovable" feature of space over time enables us to set a relationship of occurring at the same place. This lacks in Leibnizian space-time. So, Newton criticized Leibnizian space-time because "there is no basis from which we can at present pick out a place which was in the past, or say that such a place is any longer discoverable in nature." [7] This is because there does not exist the relationship of occurring at the same place in Leibnizian space-time. Accordingly, an inertial trajectory cannot be coherently decided since a straight inertial trajectory turns out to be a curved one depending on the choice of reference frame. The consequence is that within Leibnizian space-time either inertial trajectories or non-inertial trajectories cannot be coordinated with frame independent dynamics. Within this framework, only allowed physically meaningful relations are about relative motions of a given body with respect to the other body, such as the change of relative distance between two bodies, and the change of relative velocity of one body with respect to the other.[3)]

The varying symmetries of each space-time can also be witnessed in the geometric entities employed to construct space-time structures. The construction of space-time structures begins with geometric objects, a collection of points, i.e., a topologically R^4 differentiable manifold M without any additional structure. This structure represents a set of events. A class of simultaneous relations are then identified by the partitioning M into a family of hypersurface, which represents the simultaneous planes. In order to define the spatial distance between two simultaneous events and the temporal distance between two non-simultaneous events, the spatial and temporal metric that satisfy the laws of Euclidean geometry are introduced. Up to this point, Newtonian and Leibnizian space-time have the identical geometric structures. Yet, in addition to these common structures, Newtonian space-time is supplemented by the structure of 'occurring-at-same-place' at different times, which identifies the individual points on non-simultaneous spaces. This rigging of points across different simultaneous spaces enables us to define the affine structure that specifies straight lines representing the inertial trajectories of bodies. Associating with the characterization of Newtonian space-time from the perspective of symmetries, the spatial and temporal metrics determine the invariant geometric relations between events. These include the invariant spatial distance between two simultaneous or non-simultaneous events, and the invariant temporal interval whether or not two events occurs at the same location. And the affine structure enables us to specify the invariant straight lines regardless of a specific reference frame, which correspond with inertial trajectories of a given body.

The construction of Leibnizian space-time begins in a similar way to the one of Newtonian space-time. The events are defined as a collection of points, i.e., a topologically R^4 differentiable manifold M. The points are then partitioned into a class of simultaneous and non-simultaneous events. A class of simultaneous events forms a simultaneous plane i.e., hypersurface. In order to define the spatial and the temporal distance between two simultaneous events, the spatial and temporal metrics which satisfy the laws of Euclidean geometry is introduced. Comparing with Newtonian space-time, there exists no rigging between non-simultaneous points. Accordingly, Leibnizian space-time has no preferred way to connect different locations through time. In this way, the structures of Leibnizian space-time cannot maintain and trans-

3) Cartesian physics [3] which endorses relationalism decides their location only respect to other immovable bodies without kinematical properties of moving bodies with respect to immovable space: "to determine the position, we have to look at various other bodies which we regard as immovable." The motion within Cartesian system is not considered as "the action by which a body passes from one place to another," but a body's "transference from the vicinity of those bodies contiguous to it to the vicinity of others."

mit the information about specific relations between events over time. Since rotations and translations of one spatial plane does not transfers to other spatial plane on different instantaneous space, they can vary in other instant. This is what is pointed out by Newton who maintained that such structure fails to provide proper kinematics which can explain the laws of inertia. In this way, we can see that substantivalism and relationism can be clearly distinguished by means of the symmetries of space-time.

6 Concluding Remarks

Given the characterization of space-time ontology from its geometric perspective, we can see that the existence of space-time as a substance-like entity or a causally efficient entity was out of context of space-time theories. Instead, the structure of space-time theory can be characterized by the coordination between the space-time and the dynamical symmetries. In other words, the laws of motion encodes space-time geometry. In this context, we can ask what is space-time. Space-time is a theory about the invariant spatio-temporal relations between events, and the geometric structures supply its building blocks. So, the reality of space-time should be reconsidered from a perspective which brings to light the reality of the invariant geometric relations between events. In this way, we can talk about the reality of space-time geometry within the context of space-time theory.

From the importance of the role of geometry in characterizing the ontological status of space-time, we can appreciate the possibility of structuralism, which considers the nature of space-time from the perspective of its structural properties. The viewpoint endorsing the structural properties of space-time as being primary seems to be plausible since space-time is characterized by the invariant geometric relations between the events without considering their constitutions. And this characterization of the reality of space-time can be strengthened within the structuralists' view in mathematics, which claims that mathematics is "highlighting the interrelationships among the objects and ignoring any features that do not affect how they relate to each other object in the system." [9] Although the structuralism in mathematics may not be applied to all branch of mathematics, it seems to be an appropriate view for geometry, with which space-time theory is constructed. For geometry is the study of the structural properties of geometric objects, such as point, line and surface. The relations between physical events are then naturally expressed by geometric languages which emphasize the relations of mathematical objects.

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