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Certain Fractional Integral Operators and Extended Generalized Gauss Hypergeometric Functions

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Abstract. Several interesting and useful extensions of some familiar special functions such as Beta and Gauss hypergeometric functions and their properties have, recently, been investigated by many authors. Motivated mainly by those earlier works, we establish some fractional integral formulas involving the extended generalized Gauss hypergeometric function by using certain general pair of fractional integral operators involving Gauss hypergeometric function ${}_2F_1$, Some interesting special cases of our main results are also considered.

1. Introduction and preliminaries

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Throughout this paper, $\mathbb{N}$, $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{Z}_0^-$ denote the sets of positive integers, real numbers, complex numbers, and nonpositive integers, respectively, and $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$. In recent years, several interesting and useful extensions of some familiar special functions such as Beta and Gauss hypergeometric functions and their properties have been investigated by many authors (see, *e.g.*, [7, 8, 9, 14, 16, 17] and see also, very recent work, [15]). Motivated mainly by those earlier works, we establish some fractional integral formulas involving the extended generalized Gauss hypergeometric function (1.10) by using certain general pair of fractional integral operators involving Gauss hypergeometric function ${}_2F_1$, which are given in Section 2 below. Some interesting special cases of our main results are also considered.

For our purpose, we begin by recalling some known functions and earlier works. In 1997, Chaudhry *et al.* [9] presented the following extension of Euler's Beta function $B(\alpha, \beta)$:

$$B_p(x,y) := \int_0^1 t^{x-1}(1-t)^{y-1} \exp\left\{\frac{-p}{t(1-t)}\right\} dt \quad (\Re(p) > 0), \tag{1.1}$$

where the Beta function $B(\alpha, \beta)$ is a function of two complex variables α and β defined by

$$B(\alpha, \beta) = \begin{cases} \displaystyle\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt & (\Re(\alpha) > 0;\ \Re(\beta) > 0), \\ \dfrac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)} & (\alpha, \beta \in \mathbb{C} \setminus \mathbb{Z}_0^-) \end{cases} \tag{1.2}$$

and Γ is the familiar Gamma function. In the sequel, in 2004, by making use of $B_p(x,y)$, Chaudhary *et al.* [9] extended the Gauss's hypergeometric function as follows:

$$F_p(a,b;c;z) := \sum_{n=0}^{\infty} (a)_n \frac{B_p(b+n, c-b)}{B(b, c-b)} \frac{z^n}{n!} \tag{1.3}$$
$$(\Re(p) \geq 0;\ |z| < 1;\ \Re(c) > \Re(b) > 0),$$

where $(\lambda)_n$ is the Pochhammer symbol defined (for $\lambda \in \mathbb{C}$) by (see [26, p. 2 and p. 5]):

$$(\lambda)_n := \begin{cases} 1 & (n = 0) \\ \lambda(\lambda+1)\ldots(\lambda+n-1) & (n \in \mathbb{N}) \end{cases} \tag{1.4}$$
$$= \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} \quad (\lambda \in \mathbb{C} \setminus \mathbb{Z}_0^-).$$

The generalized hypergeometric series ${}_pF_q$ is defined by (see [19, p. 73]):

$${}_pF_q\left[\begin{matrix} \alpha_1, \ldots, \alpha_p; \\ \beta_1, \ldots, \beta_q; \end{matrix}\ z\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \cdots (\alpha_p)_n}{(\beta_1)_n \cdots (\beta_q)_n} \frac{z^n}{n!} \tag{1.5}$$
$$= {}_pF_q(\alpha_1, \ldots, \alpha_p; \beta_1, \ldots, \beta_q; z).$$

Here p and q are positive integers or zero (interpreting an empty product as 1), and we assume (for simplicity) that the variable z, the numerator parameters $\alpha_1, \ldots, \alpha_p$, and the denominator parameters $\beta_1, \ldots, \beta_q$ take on complex values, provided that no zeros appear in the denominator of (1.5), that is, that

$$(\beta_j \in \mathbb{C} \setminus \mathbb{Z}_0^-;\ j = 1, \ldots, q). \tag{1.6}$$

The special case ${}_2F_1(.)$ of (1.5) is called (Gauss) hypergeometric series.

In a similar manner, in 2011, Özergin *et al.* [17] introduced the following generalizations of (1.1) (see, *e.g.*, [17, p. 4602, Eq.(4)]; see also [16, p. 32, Chapter 4.]):

$$B_p^{(\alpha,\beta)}(x,y) := \int_0^1 t^{x-1}(1-t)^{y-1}\,{}_1F_1\left(\alpha;\beta;\frac{-p}{t(1-t)}\right)dt \tag{1.7}$$
$$(\Re(p) > 0;\ \min(\Re(x), \Re(y), \Re(\alpha), \Re(\beta)) > 0).$$

In this sequel, by applying to $B_p^{(\alpha,\alpha)}(x,y)$, Özergin *et al.* introduced and studied a further extension of Gauss's hypergeometric functions as follows (see, *e.g.*, [17, p. 4606, Section 3]; see also [16, p. 39, Chapter 4]):

$$F_p^{(\alpha,\beta)}(a,b;c;z) := \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\alpha,\beta)}(b+n,c-b)}{B(b,c-b)}\frac{z^n}{n!} \tag{1.8}$$
$$(\Re(p) \geq 0;\ |z| < 1;\ \min\{\Re(\alpha), \Re(\beta)\} > 0;\ \Re(c) > \Re(b) > 0).$$

Very recently, Srivastava *et al.* [25] introduced a further natural generalization of the (1.7) and (1.8), respectively, in terms of the function $\Theta(\kappa_l;z)$ defined as follows (see [25, p. 243, Eqs. (2.3) and (2.4)]):

$$B_p^{(\kappa_l)}(x,y;p) := \int_0^1 t^{x-1}(1-t)^{y-1}\,\Theta\left(\kappa_l;-\frac{p}{t(1-t)}\right)dt \tag{1.9}$$
$$(\Re(p) \geq 0;\ \min\{\Re(x), \Re(y)\} > 0)$$

and

$$F_p^{(\kappa_l)}(a,b;c;z) := \sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\kappa_l)}(b+n,c-b;p)}{B(b,c-b)}\frac{z^n}{n!} \tag{1.10}$$
$$(|z| < 1;\ \Re(c) > \Re(b) > 0 \ \text{and}\ \Re(p) \geq 0),$$

where $\Theta(\kappa_l;z)$ is given by the following definition:

Definition.(see [25, p. 243, Eq. (2.1)]) Let a function $\Theta(\kappa_l;z)$ be a analytic within the disk $|z| < R\,(0 < R < \infty)$ and let its Taylor-Maclaurin coefficient be explicitly denoted by the sequence $\{\kappa_l\}_{l\in\mathbb{N}_0}$. Suppose also that the function $\Theta(\kappa_l;z)$ can be continued in the right half-plane $\Re(z) > 0$ with the asymptotic property given as follows:

$$\begin{aligned}\Theta(\kappa_l; z) &\equiv \Theta(\{\kappa_l\}_{l\in\mathbb{N}_0}; z)\\ &:= \begin{cases} \sum_{l=0}^{\infty} \kappa_l \frac{z^l}{l!} & (|z| < R;\ 0 < R < \infty;\ \kappa_0 = 1)\\ M_0\, z^{\omega}\, \exp(z)[1 + o(\frac{1}{z})] & (\Re(z) \to \infty;\ M_0 > 0;\ \omega \in \mathbb{C}).\end{cases}\end{aligned} \tag{1.11}$$

Remark 1. It is easy to that, by special choices of the sequence $\{\kappa_l\}_{l\in\mathbb{N}_0}$ and taking $p = 0$, the definitions (1.9) and (1.10) reduce to Beta function $B(x, y)$ and hypergeometric series ${}_2F_1(\cdot)$, respectively.

The outlined above-mentioned detailed and systematic investigation was indeed motivated largely by the demonstrated potential for applications of the more extended generalized Gauss hypergeometric function and their special cases in many diverse areas of mathematical, physical, engineering and statistical sciences (see, for details, [25] and the references cited therein).

2. Fractional Calculus of the Extended Generalized Hypergeometric Functions

Recently fractional integral operators involving the various special functions have been considered by many authors (see, *e.g.*, [1]-[6], [10]-[12], [18, 20, 21, 22]; see also [24]). Here, in this section, we shall establish some fractional integral formulas for the extended generalized Gauss hypergeometric type functions $F_p^{(\kappa_l)}(\cdot)$. For our purpose, we begin by recalling the following pair of Saigo hypergeometric operators of fractional integration. For $x > 0$, $\mu, \nu, \gamma \in \mathbb{C}$ and $\Re(\alpha) > 0$, we have

$$\left(I_{0,x}^{\mu,\nu,\eta} f(t)\right)(x) = \frac{x^{-\mu-\nu}}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1}\ {}_2F_1\left(\mu+\nu, -\eta; \mu; 1 - t/x\right) f(t)dt \tag{2.1}$$

and

$$\left(J_{x,\infty}^{\mu,\nu,\eta} f(t)\right)(x) = \frac{1}{\Gamma(\mu)} \int_x^{\infty} (t-x)^{\mu-1}\ t^{-\mu-\nu}{}_2F_1\left(\mu+\nu, -\eta; \mu; 1 - x/t\right) f(t)dt, \tag{2.2}$$

where ${}_2F_1(\cdot)$ is the Gauss hypergeometric series which is a special case of the generalized hypergeometric series ${}_pF_q(\cdot)$ in (1.5).

The operator $I_{0,x}^{\mu,\nu,\eta}(\cdot)$ contains both the Riemann-Liouville and the Erdélyi-Kober fractional integral operators by means of the following relationships:

$$\left(R_{0,x}^{\mu} f(t)\right)(x) = \left(I_{0,x}^{\mu,-\mu,\eta} f(t)\right)(x) = \frac{1}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1} f(t)dt \tag{2.3}$$

and

$$\left(E_{0,x}^{\mu,\eta} f(t)\right)(x) = \left(I_{0,x}^{\mu,0,\eta} f(t)\right)(x) = \frac{x^{-\mu-\eta}}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1}\ t^{\eta} f(t)dt, \tag{2.4}$$

whereas the operator (2.2) unifies the Weyl type and the Erdélyi-Kober fractional integral operators as follows:

$$\left(W^{\mu}_{x,\infty}f(t)\right)(x) = \left(J^{\mu,-\mu,\eta}_{x,\infty}f(t)\right)(x) = \frac{1}{\Gamma(\mu)}\int_x^{\infty}(t-x)^{\mu-1}f(t)dt, \tag{2.5}$$

and

$$\left(K^{\mu,\eta}_{x,\infty}f(t)\right)(x) = \left(J^{\mu,0,\eta}_{x,\infty}f(t)\right)(x) = \frac{x^{\eta}}{\Gamma(\mu)}\int_x^{\infty}(t-x)^{\mu-1}\,t^{-\mu-\eta}f(t)dt. \tag{2.6}$$

We use the following image formulas which are easy consequences of the operators (2.1) and (2.2) (see [21, 23]):

$$\left(I^{\mu,\nu,\eta}_{0,x}\,t^{\lambda-1}\right)(x) = \frac{\Gamma(\lambda)\,\Gamma(\lambda-\nu+\eta)}{\Gamma(\lambda-\nu)\,\Gamma(\lambda+\mu+\eta)}\,x^{\lambda-\nu-1} \quad (\lambda>0,\ \lambda-\nu+\eta>0) \tag{2.7}$$

and

$$\begin{aligned}\left(J^{\mu,\nu,\eta}_{x,\infty}\,t^{\lambda-1}\right)(x) &= \frac{\Gamma(\nu-\lambda+1)\,\Gamma(\eta-\lambda+1)}{\Gamma(1-\lambda)\,\Gamma(\nu+\mu-\lambda+\eta+1)}\,x^{\lambda-\nu-1}\\ &(\beta-\lambda+1>0,\ \eta-\lambda+1>0).\end{aligned} \tag{2.8}$$

Applying (1.10) to the Saigo fractional integral operator (2.1), we obtain a fractional integral formula asserted by Theorem 1.

Theorem 1. *Let* $x>0$, $\Re(c)>\Re(b)>0$ *and the parameters* $\mu,\nu,\eta,\rho,e\in\mathbb{C}$ *be such that*

$$\Re(\mu)>0,\ \Re(p)\geq 0 \ \text{ and } \ \Re(\rho)>\max\{0,\ \Re(\nu-\eta)\}.$$

Then the following fractional integral formula holds true:

$$\begin{aligned}\left(I^{\mu,\nu,\eta}_{0,x}\left[t^{\rho-1}\,F^{(\kappa_l)}_{p}(a,b;c;et)\right]\right)(x) &= x^{\rho-\nu-1}\frac{\Gamma(\rho)\Gamma(\rho-\nu+\eta)}{\Gamma(\rho+\mu+\eta)\Gamma(\rho-\nu)}\\ &\times\ {}_2F^{(\kappa_l)}_{p+2}(a,b,\rho,\rho-\nu+\eta;c,\rho-\nu,\rho+\mu+\eta\ ;ex) \quad (|x|<1).\end{aligned} \tag{2.9}$$

Proof. For convenience and simplicity, we denote the left-hand side of the result (2.9) by $\mathfrak{I}$. Applying (1.10) to the Saigo fractional integral operator (2.1), and changing the order of integration and summation, which is valid under the condition of Theorem 1, we find that

$$\mathfrak{I} = \left(I^{\mu,\nu,\eta}_{0,t}\left[t^{\rho-1}\sum_{n=0}^{\infty}(a)_n\frac{B^{(\kappa_l)}_{p}(b+n,c-b;p)}{B(b,c-b)}\frac{et^n}{n!}\right]\right)(x)$$

$$= \sum_{n=0}^{\infty}(a)_n\frac{B^{(\kappa_l)}_{p}(b+n,c-b;p)}{B(b,c-b)}\frac{e^n}{n!}\left(I^{\mu,\nu,\eta}_{0,t}\left\{t^{\rho+n-1}\right\}\right)(x). \tag{2.9}$$

Now, making use of (2.7), we obtain

$$\begin{aligned}\mathcal{I} &= x^{\rho-\nu-1}\sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\kappa_l)}(b+n,c-b;p)}{B(b,c-b)}\frac{\Gamma(\rho+n)\Gamma(\rho-\nu+\eta+n)}{\Gamma(\rho-\nu+n)\Gamma(\rho+\mu+\eta+n)}\frac{(ex)^n}{n!}\\ &= x^{\rho-\nu-1}\frac{\Gamma(\rho)\Gamma(\rho-\nu+\eta)}{\Gamma(\rho-\nu)\Gamma(\rho+\mu+\eta)}\\ &\quad\times\sum_{n=0}^{\infty}(a)_n \frac{B_p^{(\kappa_l)}(b+n,c-b;p)}{B(b,c-b)}\frac{(\rho)_n(\rho-\nu+\eta)_n}{(\rho-\nu)_n(\rho+\mu+\eta)_n}\frac{(ex)^n}{n!},\end{aligned}$$

which, in view of (1.10), proves the required result (2.9). □

Theorem 2. *Let $x > 0$, $\Re(c) > \Re(b) > 0$ and the parameters μ, ν, η, ρ, $c \in \mathbb{C}$ satisfying the following inequalities*:

$$\Re(\mu) > 0,\ \Re(p) \geq 0 \ \text{and}\ \Re(\rho) < 1 + \min\{\Re(\eta), \Re(\nu)\}.$$

Then the following fractional integral formula holds true:

(2.10)
$$\begin{aligned}&\left(J_{x,\infty}^{\mu,\nu,\eta}\left[t^{\rho-1}F_p^{(\kappa_l)}\left(a,b;c;\frac{e}{t}\right)\right]\right)(x) = x^{\rho-\nu-1}\frac{\Gamma(1-\rho+\nu)\Gamma(1-\rho+\eta)}{\Gamma(1-\rho)\Gamma(1-\rho-\eta+\nu+\mu)}\\ &\times {}_2F_{p+2}^{(\kappa_l)}\left(a,b,1-\rho+\nu,1-\rho+\eta;c,1-\rho,1-\rho+\mu+\nu-\eta;\frac{e}{x}\right) \quad (|x|<1).\end{aligned}$$

Proof. As in the proof of Theorem 1, taking the operator (2.2) and the result (2.8) into account, one can easily prove Theorem 2. Therefore, we omit its details. □

Interestingly, on setting $\nu = 0$ and using the relations (2.4) and (2.6), Theorems 1 and 2 yield Corollaries 1 and 2, respectively.

Corollary 1. *Let $x > 0$, $\Re(p) \geq 0$ and the parameters μ, η, $\rho, e \in \mathbb{C}$ be such that $\Re(\mu) > 0, \Re(\rho) > 0$ and $\Re(\rho) > \Re(-\eta)$. Then the right-side Erdélyi-Kober fractional integrals of the generalized Gauss hypergeometric type functions are given by*

$$\begin{aligned}&\left(E_{0,x}^{\mu,\eta}\left[t^{\rho-1}F_p^{(\kappa_l)}(a,b;c;et)\right]\right)(x) = x^{\rho-1}\frac{\Gamma(\rho-\eta)}{\Gamma(\rho+\mu+\eta)}\\ &\quad\times {}_1F_{p+1}^{(\kappa_l)}(a,b,\rho-\eta;c,\rho+\mu+\eta;ex) \quad (|x|<1).\end{aligned} \tag{2.11}$$

Corollary 2. *Let $x > 0$, $\Re(p) \geq 0$ and the parameters $\mu, \eta, \rho, e \in \mathbb{C}$ satisfying the inequalities $\Re(\mu) > 0$, $\Re(\rho) > 0$, $\Re(\rho) < 1 + \Re(\eta)$. Then we have*

$$\begin{aligned}&\left(K_{x,\infty}^{\mu,\eta}\left[t^{\rho-1}F_p^{(\kappa_l)}\left(a,b;c;\frac{e}{t}\right)\right]\right)(x) = x^{\rho-1}\frac{\Gamma(1-\rho+\eta)}{\Gamma(1-\rho-\eta+\mu)}\\ &\quad\times {}_1F_{p+1}^{(\kappa_l)}\left(a,b,1-\rho+\eta;c,1-\rho+\mu-\eta;\frac{e}{x}\right) \quad (|x|<1).\end{aligned} \tag{2.12}$$

Further, if we replace ν by $-\mu$ and make use of the relations (2.3) and (2.5), in Theorems 1 and 2, we obtain yet another corollaries providing Riemann-Liouville and Weyl fractional integrals of the generalized Gauss hypergeometric type function ${}_pF_q^{(\kappa_l)}(\cdot)$ asserted by Corollaries 3 and 4.

Corollary 3. *Let $x > 0$, $\Re(p) \geq 0$ and the parameters μ, $\rho \in \mathbb{C}$ satisfying $\Re(\mu) > 0$, $\Re(\rho) > 0$. Then we get*

$$\begin{aligned}\left(R^{\mu}_{0,x}\left[t^{\rho-1}\, F_p^{(\kappa_l)}\,(a,b;c;et)\right]\right)(x) &= x^{\rho+\mu-1}\frac{\Gamma(\rho)}{\Gamma(\rho+\mu)}\\ &\times\, {}_1F_{p+1}^{(\kappa_l)}\,(a,b,\rho;c,\rho+\mu\ ;ex) \quad (|x|<1).\end{aligned} \tag{2.13}$$

Corollary 4. *Let $x > 0$, $\Re(p) \geq 0$ and the parameters μ, $\rho \in \mathbb{C}$ satisfying the inequalities $\Re(\mu) > 0$, $\Re(\rho) > 0$. Then we obtain*

$$\begin{aligned}\left(W^{\mu}_{x,\infty}\left[t^{\rho-1}\, F_p^{(\kappa_l)}\left(a,b;c;\frac{e}{t}\right)\right]\right)(x) &= x^{\rho+\mu-1}\frac{\Gamma(1-\rho-\mu)}{\Gamma(1-\rho)}\\ &\times {}_1F_{p+1}^{(\kappa_l)}\left(a,b,1-\rho-\mu;c,1-\rho;\frac{e}{x}\right) \quad (|x|<1).\end{aligned} \tag{2.14}$$

Remark 2. It is noted that the results obtained here are useful in deriving various fractional integral formulas for each of the families of the extended generalized hypergeometric functions defined by (1.10). If we apply the asymptotic behavior of Kummer's confluent hypergeometric function at infinity:

$$\Phi(a;c;z) = {}_1F_1(a;c;z) = \frac{\Gamma(c)}{\Gamma(a)}\, e^z\, z^{a-c}\left[1+o\left(\frac{1}{z}\right)\right] \quad (\Re(z)\to\infty), \tag{2.15}$$

Theorems 1 and 2 provide, respectively, the known fractional integral formulas due to Agarwal [1]. It is clear that $\Phi(a;c;z)$ is a special case of $\Theta(\kappa_l;z)$ (1.11) with the sequence $\left\{\frac{(a)_l}{(c)_l}\right\}_{l\in\mathbb{N}_0}$. If we use this observation in Theorems 1 and 2 and set $p = 0$ in the resulting results, after a little simplification, we may obtain various fractional integral formulas for the hypergeometric function ${}_2F_1$.

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