Effect of Outdated Channel Estimates on Multiple Antennas Multiple Relaying Networks

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Abstract

In this paper, we propose an intergraded unified imperfect CSI model and investigate the joined effects of feedback delay and channel estimation errors (CEE) for two-hop relaying systems with transmit beamforming and relay selection. We derived closed-form expressions for important performance measures including the exact analysis and lower bounds of outage probability as well as error performance. The ergodic capacity is also included with closed-form results. Furthermore, diversity and coding gains based on the asymptotic analysis at high SNRs are also presented, which are simple and concise and provide new analytical insights into the corresponding power allocation scheme. The analysis indicates that delay effect results in the coding gain loss and the diversity order loss, while CEE will merely cause the coding gain loss. Numerical results verify the theoretical analysis and illustrate the system is more sensitive to transmit beamforming delay compared with relay selection delay and also verify the superiority of optimum power allocation. We further investigate the outage loss due to the CEE and feedback delays, which indicates that the effect of the CEE is more influential at low-to-medium SNR, and then it will hand over the dominate role to the feedback delay.

Keywords: Transmit beamforming and relay selection; delay and channel estimation errors; diversity and coding gains; power allocation

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1. Introduction

Cooperative communications using low complexity relay terminals in wireless networks, offers a variety of significant performance benefits, including combating wireless impairments, hotspot throughput improvements and cellular signal coverage enhancements [1]. Recently significant work has been reported towards deploying multiple antennas or multiple relays in relaying systems, where transmit beamforming (TB), and relay selection (RS), have been intensively researched to achieve full spatial diversity and better system performance.

Transmit beamforming has been shown to be as an effective fading countermeasure technique in multiple antenna systems [2]. To achieve coherent beamforming for a maximal ratio transmission, channel state information (CSI) is required at the transmitter, provided by feedback of CSI from the receiver. There have been extensive studies on the dual-hop beamforming and its equivalent systems by using various antenna configurations [3]. Relay selection for multi-relay systems is another highly active topic in the literature [4, 5], which requires fewer orthogonal resources, and is generally classified into two categories, i.e., the opportunistic relay selection (ORS) [4] and the partial relay selection(PRS) [5]. Nevertheless, an indispensable assumption of the techniques is the perfect knowledge of the CSI, which is probably unavailable because of practical limitations such as feedback delay and channel estimation errors (CEE). These limitations degrade the beamforming or relay selection performance.

Effect of CSI delay on the TB performance in [6-8], and selection relaying systems [9-15] have been extensively studied, where the beamforming vector is computed based on the outdated CSI or the selected relay may not be optimal for actual data transmission in time-varying channels. In particular, the effect of feedback delay on the performance of a practical mobile downlink scenario was studied in [8], in which a multi-antenna equipped source employing TB was communicating to a destination via a relay. Accordingly, the impact of the corresponding outdated relay selection on the system performance was investigated for the ORS [9, 10] and PRS [11] over various channel environments. More specifically, the outage and error rate performance for both the two variations of amplify-and-forward (AF) strategies with outdated CSI was well investigated in [12]. Recently, these works of outdated RS have been extended to two-way relaying [13], coded cooperation [14], and underlay cognitive networks [15]. However, these available results have assumed perfect channel estimation which may not be valid in practice because CEE also always occurs as a result of imperfect estimation algorithm or the instability of the channel. Although outdated and imprecise CSI corresponds to most possible realistic scenarios, to the best of our knowledge, few papers have analytically investigated both of the two issues in TB and RS systems.

The joint effects of delayed and receiver CEE on the capacity of TB were studied in [16], but only for point-to-point systems. Outdated maximum ratio transmission and maximum ratio combining with CEE in a single relay beamforming network were considered in [17]. Furthermore, considering the decode-and-forward (DF) relaying strategy, the average symbol error rate (SER) and asymptotic diversity order for distributed beamforming and relay selection at the relay-destination link in the presence of CEE and feedback delay were investigated in [18] and [19], respectively. However, most of the prior works always include an impractical CEE model that the estimation error is independent from the data transmit power, which leads to zero diversity order. And the imperfect CSI model needs to be reconsidered. Moreover, prior works on TB/RS have limited analysis to either single-relay or single-antenna systems, while the multiple-antennas and multiple-relays assisted networks employing TB and RS simultaneously has not been treated before and no existing results can be directly referred to.

In light of the aforementioned researches, we incorporate multiple relays into a practical mobile downlink network and employing the RS technique. TB at the source and AF relay selection at the destination are employed. This model can be regarded as a general case of the mobile downlink network as described in [3] and [10], in which a set of mobile stations are used to act as relays for communications between a multi-antenna equipped base station and a single-antenna mobile station. Comparing with the prior related works, multi-antenna with TB and multi-relaying with RS are both incorporated, and the joint impact of feedback delay and CEE are jointly considered. Specifically, the main contribution of this paper can be summarized as:

- We firstly propose an intergraded unified CSI model taking both the effects of noisy and outdated channel estimates into account. We introduce a new concept namely channel estimation quality order for making the imperfect CSI model more straightforward and practical.
- We address the primary performance metrics comprehensively, including closed-form expressions for the outage probability, average SER, ergodic capacity, diversity and array gains. Lower bounds and asymptotic analysis are also presented.
- We determine the optimized power allocation between the source and the relay to achieve profound coding gains and improve the SER performance.
- We further investigated the outage loss due to the CEE and feedback delays, which indicates that the effect of the CEE is more influential at low-to-medium SNR, and then it will hand over the dominate role to the feedback delay.

Section 2 introduces the system model. In Section 3, we present a set of new analytical expressions for the key performance measures. In Section 4, the high SNR asymptotic analysis of outage and average SER are provided. Numerical results and discussions are provided in Section 5. Finally, Section 6 concludes the paper.

2. System Model

2.1 System Description

Consider a multiple antennas multiple relaying network where a source, S, equipped with N_t antennas communicates with a destination, D, assisted by a set of N_r relays R_i, $i=1,\cdots,N_r$. Both the relay and the destination are equipped with a single antenna respectively. This scenario can be directly applied to current cellular networks where the use of multiple antennas in a base station is reasonable, but the use of multiple antennas at mobile terminals and or relays may be prohibitive due to the terminal size and power constraints. The $S \rightarrow D$ direct communication link is assumed to be unavailable due to heavy shadowing between the source and the destination.

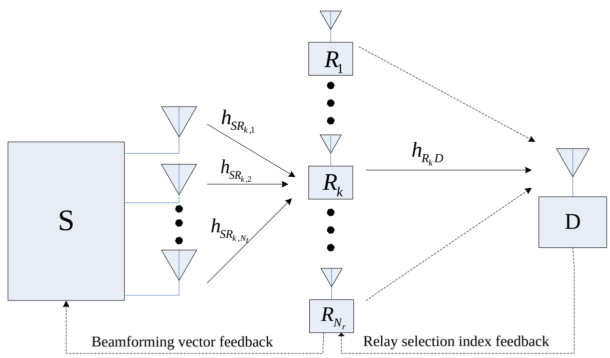

Fig. 1. System model of the transmit beamforming and AF relay selection

All relays operate in the half-duplex AF mode, and only a single relay is selected through a procedure based on the highest instantaneous signal-to-noise-ratio (SNR) of link $R \rightarrow D$ by the destination. Then, S beamforms its signal to the selected relay based on the information feedback from the relay, which is supposed to forward the signal transmitted by source to destination. It should be noted that relay selection and conveying transmit beamforming vector may cause a time difference between the actual channel value and its estimate. The $S \rightarrow R$ and $R \rightarrow D$ links are assumed independent and identically circularly symmetric complex Gaussian distributed as $h_{SR_{i,j}} \sim \mathcal{CN}(0,\sigma_1^2)$ and $h_{R_iD} \sim \mathcal{CN}(0,\sigma_2^2)$, $j=1,\cdots,N_t$. according to Rayleigh distribution interfered by zero-mean additive white Gaussian noise (AWGN) with a fixed variance N_0. In addition, the transmit power of the source and relays are denoted by P_s and P_r, respectively.

2.2 Outdated Channel Estimation Model

We assume that both the TB and relay selection processes are based on outdated and imperfect estimated channel state information. To simplify the problem formulation in this subsection we will omit the subscript of the channel gains and the time delays in this subsection. Let $\hat{h}(t)$ and $\hat{h}(t-T_d)$ represent the actual (used for data transmission) and the outdated (used for relay selection or beamforming vector calculation) channel estimates with a time delay T_d. Generally speaking, on one hand, the outdated CSI is commonly modeled as

$$h(t) = \rho_d h(t - T_d) + \omega(t) \tag{1}$$

where $\rho_d = J_0(2\pi f_d T_d)$ is the normalized delay correlation coefficient according to Jakes' autocorrelation model [6]. $J_0(\cdot)$ is the zero order Bessel function of the first kind, f_d is the Doppler frequency, and $\omega(t)$ has the Gaussian distribution with variance $\sqrt{1-\rho_d^2}\sigma_h^2$.

On the other hand, the CEE model valid for minimum mean-squared error (MMSE) channel estimation [19, 20], is formulated as

$$h(t) = \hat{h}(t) + e(t) \tag{2}$$

which can also be further rewritten by

$$\hat{h}(t) = \rho_e h(t) + v(t) \tag{3}$$

where $e(t)$ and $v(t)$ are modeled as zero mean Gaussian random variables (RVs) with variances σ_e^2, σ_v^2 and $\sigma_{\hat{h}}^2$, ρ_e are the CEE correlation coefficient. Furthermore, in contrast to the impractical assumption that the estimation error is independent from the data transmit power, which leads to zero diversity order, we introduce the channel estimation quality order $\delta = P_{pilot}/P$ which is adjustable and maintain a scale model to the data transmit power. Thus, following a similar way in [20], the variance of $e(t)$ can be described as $\sigma_e^2 = \sigma_h^2 / (1 + \delta\eta\sigma_h^2)$, where η is the data transmission SNR, δ is determined by the cost of obtaining CSI in terms of the training pilots' power consumption and reflects the quality of channel estimation. Correspondingly, the correlation coefficient $\rho_e = \sigma_{\hat{h}}^2 / \sigma_h^2$ can be modeled as an increasing function of the training symbols' SNR, rewritten as $\rho_e = \delta\eta\sigma_h^2 / (1 + \delta\eta\sigma_h^2)$.

Based on the outdated and CEE model described above, we can conduct an intergraded unified model taking both the effects of noisy and outdated channel estimates into account, expressed as

$$\hat{h}(t) = \rho \hat{h}(t - T_d) + \varepsilon(t) \tag{4}$$

where

$$\rho = \begin{cases} \rho_e \rho_d, & \text{if } \rho_d < 1 \\ 1, & \text{if } \rho_d = 1 \end{cases} \tag{5}$$

It is clear that when the CSI is not outdated, i.e. $\rho_d = 1$, we have $\hat{h}(t) = \hat{h}(t - T_d)$, while for the case of $\rho_d < 1$, substituting (1) and (2) into (3), we have

$$\begin{aligned} \hat{h}(t) &= \rho_e \left[\rho_d h(t - T_d) + \omega(t)\right] + v(t) \\ &= \rho_e \rho_d \hat{h}(t - T_d) + \rho_e \rho_d e(t) + \rho_e \omega(t - T_d) + v(t) \end{aligned} \tag{6}$$

where $\rho_e \rho_d e(t) + \rho_e \omega(t - T_d) + v(t) = \varepsilon(t)$ can be simplified into a zero mean complex-Gaussian RV, with variance of $\sqrt{1-\rho^2}\sigma_{\hat{h}}^2$ by the relationship of variances. In the following text, the corresponding parameters $(\sigma_e^2, \rho_e, \rho_d, \rho)$ of the outdated CSI model for the $S \to R$ and $R \to D$ link will be annotated with subscript $\imath = 1, 2$, respectively.

2.3 Effective Output SNR

As mentioned above, before data transmission, a partial relay selection process is performed based on the highest instantaneous SNR of the second hop that $k = \arg\max_i \left\{ \tilde{\gamma}_{R_iD} = P_r \left| \hat{h}_{R_iD}(t - T_d) \right|^2 \Big/ (P_r \sigma_{e_2}^2 + N_0) \right\}$, where $\tilde{\gamma}_{R_iD}$ is the estimated instantaneous SNR in the relay selection process. We employ the relay-destination link based partial relay selection (PRS) in respect that PRS alleviates the task of acquiring global CSI over opportunistic relay selection (ORS) and reduces the cooperation overhead [5, 11]. Moreover, because the first hop corresponds to a MISO channel enhanced with multiple antennas which is more likely better than the second hop, the relay-destination link probably plays the dominate role in determining the received SNR of the two-hop system. Therefore, we assume that the destination node is in charge of the relay selection process and feedback the index of the selected relay, k. While in the data transmission process, the actual instantaneous SNR is a time delay version $\gamma_{R_iD} = P_r \left| \hat{h}_{R_iD}(t) \right|^2 \Big/ (P_r \sigma_{e_2}^2 + N_0)$. Noted that $\tilde{\gamma}_{R_iD}$ and γ_{R_iD} are correlated exponential distributions, whose joint probability density function (PDF) is [11]

$$f_{\gamma_{R_iD}, \tilde{\gamma}_{R_iD}}(x, y) = \frac{e^{-\frac{x+y}{(1-\rho_2^2)\bar{\gamma}_2}}}{(1-\rho_2^2)\bar{\gamma}_2} I_0\left(\frac{2\sqrt{\rho_2^2 xy}}{(1-\rho_2^2)\bar{\gamma}_2} \right) \tag{7}$$

where $I_0(\cdot)$ is the modified Bessel function of the first kind [21, Eq. (8.447.1)], $\rho_2 = \rho_{e_2} J_0(2\pi f_{d_2} T_{d_2})$ and $\bar{\gamma}_2 = P_r \sigma_2^2 \big/ (P_r \sigma_{e_2}^2 + N_0)$ are correlation coefficient and average SNR of the $R \to D$ link, respectively.

After relay selection, the chosen relay estimates the CSI of $S \to R_k$ link $\mathbf{h}_{SR_k}(t) = \left[h_{SR_k,1}(t), \ldots, h_{SR_k,N_t}(t) \right]^T$ and conveys the transmit beamforming information vector to the source. The subsequent data transmission process can be divided into two phases. During the first phase, S beamforms its signal $s(t)$ to R_k, and the beamforming vector is calculated from the outdated channel estimates $\hat{\mathbf{h}}_{SR_k}(t) = \left[\hat{h}_{SR_k,1}(t), \ldots, \hat{h}_{SR_k,N_t}(t) \right]^T$ and given by $\mathbf{w}(t|T_{d_1}) = \hat{\mathbf{h}}_{SR_k}^H (t - T_{d_1}) \Big/ \left| \hat{\mathbf{h}}_{SR_k}(t - T_{d_1}) \right|$.

During the second phase, the received signal $y_R(t)$ at the relay is multiplied by a variable-gain G, written as

$$G = \sqrt{\frac{P_r}{P_s \left| \mathbf{w}(t|T_{d_1}) \hat{\mathbf{h}}_{SR_k}(t) \right|^2 + P_s \sigma_{e_1}^2 + N_0}} \tag{8}$$

Then, the relay will retransmit the scaled signal to the destination. The received signal at the destination is given by

$$\begin{aligned} y_D(t) &= h_{R_kD}(t) G y_R(t) + n_{R_kD}(t) \\ &= \sqrt{P_s} G \mathbf{w}(t|T_{d_1}) \hat{\mathbf{h}}_{SR_k}(t) \left(\hat{h}_{R_kD}(t) + e_{R_kD}(t) \right) s(t) \\ &+ G \left(\sqrt{P_s} \mathbf{w}(t|T_{d_1}) \mathbf{e}_{SR_k}(t) s(t) + n_{SR_k}(t) \right) \left(\hat{h}_{R_kD}(t) + e_{R_kD}(t) \right) + n_{R_kD}(t) \end{aligned} \tag{9}$$

where $\mathbf{e}_{SR_k}(t) = \left[e_{SR_k,1}(t), \ldots, e_{SR_k,N_t}(t) \right]^T$ is the corresponding CEE from S to R_k, $n_{SR_k}(t)$ and $n_{R_kD}(t)$ are the AWGNs at the relay and the destination. The end-to-end SNR is the equivalent receiver SNR at the destination by treating the first term in (9) as the effective signal, and regarding the terms including $\mathbf{e}_{SR_k}(t)$ and $e_{R_kD}(t)$ as noises. We can calculate the e2e SNR as follows

$$\gamma_{eq} = \frac{P_s \left| \mathbf{w}(t|T_{d_1}) \hat{\mathbf{h}}_{SR_k}(t) \hat{h}_{R_kD}(t) \right|^2}{P_s \left| \mathbf{w}(t|T_{d_1}) \hat{\mathbf{h}}_{SR_k}(t) \right|^2 \sigma_{e_{RD}}^2 + \Delta + N_0 / G^2} \tag{10}$$

where $\Delta = \left(P_s \sigma_{e_1}^2 + N_0 \right) \left(\left| \hat{h}_{R_kD}(t) \right|^2 + \sigma_{e_2}^2 \right)$. Then, by substituting (9) into (10) and after carrying out some trivial mathematical manipulations, the effective end-to-end (e2e) SNR γ_{eq} can be derived as

$$\gamma_{eq} = \frac{\gamma_{SR_k} \gamma_{R_kD}}{\gamma_{SR_k} + \gamma_{R_kD} + c} \tag{11}$$

where $c = 1$, $\gamma_{SR_k} = P_s \left| \mathbf{w}(t|T_{d_1}) \hat{\mathbf{h}}_{SR_k}(t) \right|^2 / (P_s \sigma_{e_1}^2 + N_0)$, $\rho_1 = \rho_{e_1} J_0(2\pi f_{d_1} T_{d_1})$ and $\overline{\gamma}_1 = P_s \sigma_1^2 / (P_s \sigma_{e_1}^2 + N_0)$.

3. Performance Analysis

In this section, we derive important performance measures for the two hop system under investigation. This includes the closed-form expressions of the outage probability, average SER, and ergodic capacity.

3.1 Outage Probability

The outage probability is an important wireless system parameter of quality of service (QoS) measure defined as the probability that γ_{eq} drops below an acceptable SNR threshold γ_{th}. Therefore, to study the system's outage probability, the cumulative distribution function (CDF) of the e2e SNR is required. We have

$$P_{out} = \Pr(\gamma_{eq} < \gamma_{th}) = F_{\gamma_{eq}}(\gamma_{th}) \tag{12}$$

The CDF of γ_{eq} can be written as a single-integral expression as follows:

$$F_{\gamma_{eq}}(x) = 1 - \int_0^\infty \left(1 - F_{\gamma_{R_k D}}\left(\frac{xz + x(x+c)}{z}\right)\right) f_{\gamma_{SR_k}}(z+x)\, dz \tag{13}$$

where $f_{\gamma_{SR_k}}(\cdot)$ and $F_{\gamma_{R_k D}}(\cdot)$ denote the PDF of γ_{SR_k} and CDF of $\gamma_{R_k D}$ respectively.

According to the principles of concomitants or induced order statistics, the PDF of the instantaneous SNR from the selected relay to destination, $\gamma_{R_k D}$, is given by

$$f_{\gamma_{R_k D}}(x) = \int_0^\infty f_{\gamma_{R_k D}|\tilde{\gamma}_{R_k D}}(x, y)\, f_{\tilde{\gamma}_{R_k D}}(y) dy \tag{14}$$

where $f_{\gamma_{R_k D}|\tilde{\gamma}_{RkD}}(x, y) = f_{\gamma_{R_i D}, \tilde{\gamma}_{R_i D}}(x, y) / f_{\hat{\gamma}_{R_i D}}(y)$ is the PDF of $f_{\gamma_{R_k D}}(x)$ conditioned on $\tilde{\gamma}_{R_k D}$. Given that $\tilde{\gamma}_{R_k D} = \max_i \{\hat{\gamma}_{R_i D}\}$, we have

$$f_{\tilde{\gamma}_{R_k D}}(y) = N_r \left(F_{\hat{\gamma}_{R_i D}}(y)\right)^{N_r - 1} f_{\hat{\gamma}_{R_i D}}(y) \tag{15}$$

Recalling that $\hat{\gamma}_{R_i D}$ has the exponential distribution with average value $\bar{\gamma}_2$ and applying the binomial expansion [21, Eq.(1.111)], we have

$$f_{\tilde{\gamma}_{R_k D}}(y) = N_r \sum_{m=0}^{N_r - 1} (-1)^m \binom{N_r - 1}{m} \frac{\exp\left(-(m+1) y / \bar{\gamma}_2\right)}{\bar{\gamma}_2} \tag{16}$$

Thus, $f_{\gamma_{R_k D}}(y)$ is obtained by substituting (16) and (7) into (14), yielding

$$f_{\gamma_{R_k D}}(y) = N_r \sum_{m=0}^{N_r - 1} (-1)^m \binom{N_r - 1}{m} \frac{e^{-(m+1) y / \left(m(1-\rho_2^2)+1\right)\bar{\gamma}_2}}{(m(1-\rho_2^2)+1)\bar{\gamma}_2} \tag{17}$$

The corresponding CDF of $\gamma_{R_k D}$ can be derived as

$$F_{\gamma_{R_k D}}(y) = N_r \sum_{m=0}^{N_r - 1} (-1)^m \binom{N_r - 1}{m} \frac{1 - e^{\frac{-(m+1) y}{(m(1-\rho_2^2)+1)\bar{\gamma}_2}}}{m+1} \tag{18}$$

On the other hand, the PDF of γ_{SR_k} using [9, Eq. (15)], i.e. the case of full-rate feedback without quantization errors, can be written as

$$f_{\gamma_{SR_k}}(x) = \frac{1}{\bar{\gamma}_1^{N_t}} \sum_{n=0}^{N_t - 1} \binom{N_t - 1}{n} \frac{\rho_1^{2(N_t - 1 - n)} (\bar{\gamma}_1 (1 - \rho_1^2))^n}{(N_t - 1 - n)!} x^{N_t - 1 - n} e^{-x/\bar{\gamma}_1} \tag{19}$$

Consequently, by substituting (18) and (19) into (13), and applying the fact that $N_r \sum_{m=0}^{N_r - 1} \binom{N_t - 1}{n} (-1)^m / m + 1 = 1$ and some binomial expansions, the CDF of γ_{eq} can be rewritten as

$$\begin{aligned} F_{\gamma_{eq}}(x) = 1 - \sum_{n=0}^{N_t - 1} \sum_{m=0}^{N_r - 1} (-1)^m \binom{N_t - 1}{n} \binom{N_r - 1}{m} \frac{N_t \rho_1^{2(N_t - 1 - n)} (\bar{\gamma}_1 (1 - \rho_1^2))^n}{\bar{\gamma}_1^{N_t} (N_t - 1 - n)! (m+1)} e^{-\left(\frac{1}{\bar{\gamma}_1} + \frac{(m+1)}{(m(1-\rho_2^2)+1)\bar{\gamma}_2}\right) x} \\ \times \int_0^\infty \sum_{k=0}^{N_t - 1 - n} \binom{N_t - 1 - n}{k} x^{N_t - 1 - n - k} e^{\frac{-(m+1)}{(m(1-\rho_2^2)+1)\bar{\gamma}_2} \frac{x(x+c)}{z} - \frac{z}{\bar{\gamma}_1}} z^k dz \end{aligned} \tag{20}$$

By using [21, Eq. (3.471.9)], the integral in (20) can be solved to yield a closed-form expression for $F_{\gamma_{eq}}(x)$ as follows:

$$F_{\gamma_{eq}}(x)=1-2\sum_{n=0}^{N_t-1}\sum_{m=0}^{N_r-1}\sum_{k=0}^{N_t-1-n}(-1)^m\binom{N_t-1}{n}\binom{N_r-1}{m}\binom{N_t-1-n}{k}\frac{N_t\rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n\,\overline{\gamma}_1^{\,n-N_t}}{(N_t-1-n)!(m+1)}$$
$$\times x^{N_t-1-n-k}\left(\frac{(m+1)\overline{\gamma}_1(x^2+cx)}{(m(1-\rho_2^2)+1)\overline{\gamma}_2}\right)^{(k+1)/2}e^{-\left(\frac{1}{\overline{\gamma}_1}+\frac{(m+1)}{(m(1-\rho_2^2)+1)\overline{\gamma}_2}\right)x}K_{k+1}\left(2\sqrt{\frac{(m+1)(x^2+cx)}{(m(1-\rho_2^2)+1)\overline{\gamma}_1\overline{\gamma}_2}}\right) \quad (21)$$

Substituting (21) into (12), we can obtain P_{out}.

3.2 Average Symbol Error Rate

The average SER, which is valid for a wide range of modulation schemes can be written as [22].

$$P_{error}=aE_{\gamma}\left[Q\left(\sqrt{b\gamma}\right)\right] \quad (22)$$

where $Q(x)=1/\sqrt{2\pi}\int_x^{\infty}e^{-y^2/2}dy$ is the Gaussian Q-function, a and b represent modulation specific constants, which can be obtained from [22].

By using integration by parts, (22) can be written in a single-integral form as

$$P_{error}=\frac{a}{2}\sqrt{\frac{b}{2\pi}}\int_0^{\infty}\frac{F_{\gamma}(x)}{\sqrt{x}}e^{-\frac{b}{2}x}dx \quad (23)$$

In this subsection, the average SER is derived for the sake of analytical tractability by substituting that $c=0$, which is in line with the channel-assisted AF relays when ignoring the noise part in the amplifying gain. Note that the average SER can be approximated by substituting (21) into (23), and solving the resulting integral as follows:

$$P_{error}\approx\frac{a}{2}-a\sqrt{\frac{b}{2}}2\sum_{n=0}^{N_t-1}\sum_{m=0}^{N_r-1}\sum_{k=0}^{N_t-1-n}(-1)^m\binom{N_t-1}{n}\binom{N_r-1}{m}\binom{N_t-1-n}{k}\frac{N_t\rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n}{(N_t-1-n)!(m+1)}$$
$$\times\overline{\gamma}_1^{\,n-N_t}\left(\frac{4(m+1)}{(m(1-\rho_2^2)+1)\overline{\gamma}_2}\right)^{k+1}\frac{\Gamma\left(N-n+k+3/2\right)\Gamma\left(N-n-k-1/2\right)}{(\alpha+\beta+b/2)^{N-n+k+\frac{3}{2}}\Gamma\left(N-n+1\right)} \quad (24)$$
$$\times{}_2F_1\left(N-n+k+\frac{3}{2},k+\frac{3}{2};N-n+1;\frac{\alpha-\beta+b/2}{\alpha+\beta+b/2}\right)$$

where $\alpha=\frac{1}{\overline{\gamma}_1}+\frac{(m+1)}{(m(1-\rho_2^2)+1)\overline{\gamma}_2}$, $\beta=2\sqrt{\frac{(m+1)}{(m(1-\rho_2^2)+1)\overline{\gamma}_1\overline{\gamma}_2}}$.

3.3 Ergodic Capacity

The ergodic capacity, in the Shannon sense, is an important performance metric since it provides insight on the maximum achievable transmission rate under which the errors are recoverable. It is well known that the ergodic capacity of the system can be expressed by

$$C = \frac{1}{2}\int_0^\infty \log_2\left(1+x\right) f_{\gamma_{eq}}\left(x\right) dx \tag{25}$$

where the reason for the one-half factor is that we need two time slots (or orthogonal channels) to transmit the data, and the PDF of γ_{eq} can be derived by ignoring the undesirable factor c and differentiating (21) with respect to x as

$$\begin{aligned} f_{\gamma_{eq}}\left(x\right) = 2\sum_{n=0}^{N_t-1}\sum_{m=0}^{N_r-1}\sum_{k=0}^{N_t-1-n}\binom{N_t-1}{n}\binom{N_r-1}{m}\binom{N_t-1-n}{k}\frac{(-1)^m N_r \rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n \overline{\gamma}_1^{n-N_t}}{(N_t-1-n)!(m+1)} \\ \times\left(\frac{(m+1)\overline{\gamma}_1}{(m(1-\rho_2^2)+1)\overline{\gamma}_2}\right)^{(k+1)/2} x^{N_t-1-n}e^{-\alpha x}\left[\left(\alpha x-(N_t-n-k-1)\right)K_{k+1}(\beta x)+\beta x K_k(\beta x)\right] \end{aligned} \tag{26}$$

By substituting (26) into (25) and utilizing the integral result in [23], we have

$$\begin{aligned} C = \frac{\sqrt{\pi}}{\ln 2}\sum_{n=0}^{N_t-1}\sum_{m=0}^{N_r-1}\sum_{k=0}^{N_t-1-n}\binom{N_t-1}{n}\binom{N_r-1}{m}\binom{N_t-1-n}{k}\frac{(-1)^m N_r \rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n \overline{\gamma}_1^{n-N_t}}{(N_t-1-n)!(m+1)\left(\alpha-\beta\right)^{N_t-n+1}} \\ \times\left(\frac{(m+1)\overline{\gamma}_1}{(m(1-\rho_2^2)+1)\overline{\gamma}_2}\right)^{(k+1)/2}\left[\left(\alpha-(N_t-n-k-1)\left(\alpha-\beta\right)\right)H_{temp1}+\beta H_{temp2}\right] \end{aligned} \tag{27}$$

where

$$H_{temp1} = H^{1,2,0,1,2}_{1,[2;1],0,[2;2]}\left[\begin{array}{c|c} \dfrac{1}{\alpha-\beta} & \begin{array}{c}(2,1)\\ (1,1),(1,1);(0.5,1)\\ --\\ (1,1),(0,1);(k+1,1),(-k-1,1)\end{array} \\ \dfrac{2\beta}{\alpha-\beta} & \end{array}\right] \tag{28}$$

$$H_{temp2} = H^{1,2,0,1,2}_{1,[2;1],0,[2;2]}\left[\begin{array}{c|c} \dfrac{1}{\alpha-\beta} & \begin{array}{c}(2,1)\\ (1,1),(1,1);(0.5,1)\\ --\\ (1,1),(0,1);(k,1),(-k,1)\end{array} \\ \dfrac{2\beta}{\alpha-\beta} & \end{array}\right] \tag{29}$$

where $H^{K,N,N',M,M'}_{E,[A;C],F,[B;D]}\left[\cdot\right]$ is the generalized Fox's H-function [23].

Remark 1: It should be noted that the CDF based approach as in [24, Eq. (4)] could also be regarded as a more straightforward way for the capacity derivation, although a mathematically intractable integration will be involved. As to the two-variables Fox-H-function included in (27) which is defined in terms of multiple Mellin–Barnes type contour integral, the details can be found in many related literatures as [25]. However, since the evaluation of the generalized Fox's H-function is difficult to be directly realized in popular mathematical software, one may have to resort to an integral-based approach as in [23].

4. High SNR Analysis and Power Allocation

4.1 Lower Bounds

In order to simplify the performance analysis, (11) should be expressed in a more mathematically tractable form for systematic system optimizations. To achieve this, a commonly used tight upper bound of γ_{eq} is proposed:

$$\gamma_{eq} \le \gamma_{eq}^{up} = \min\left\{\gamma_{SR_k}, \gamma_{R_k D}\right\} \tag{30}$$

Then the corresponding CDF of γ_{eq}^{up} can be expressed as

$$F_{\gamma_{eq}^{up}}(x) = 1 - (1 - F_{\gamma_{SR_k}}(x))(1 - F_{\gamma_{R_k D}}(x)) \tag{31}$$

where $F_{\gamma_{R_k D}}(x)$ has been derived in (18), and $F_{\gamma_{SR_k}}(x)$ can also been obtained from (19), given by

$$F_{\gamma_{SR_k}}(x) = 1 - \sum_{n=0}^{N_t-1} \sum_{k=0}^{N_t-1-n} \binom{N_t-1}{n} \frac{\rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n}{k!\,\bar{\gamma}_1^k} x^k e^{-x/\bar{\gamma}_1} \tag{32}$$

Thus, the lower bound of the outage probability is

$$P_{out}^{low}(\gamma_{th}) = 1 - \sum_{n=0}^{N_t-1} \sum_{m=0}^{N_r-1} \sum_{k=0}^{N_t-1-n} (-1)^m N_r \binom{N_r-1}{m} \binom{N_t-1}{n} \frac{\rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n}{k!} \left(\frac{\gamma_{th}}{\bar{\gamma}_1}\right)^k \frac{e^{-\alpha\gamma_{th}}}{m+1} \tag{33}$$

By substituting (33) into (23), the lower bound of the SER can be evaluated as

$$P_{error}^{low} = \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{2\pi}} \sum_{n=0}^{N_t-1} \sum_{m=0}^{N_r-1} \sum_{k=0}^{N_t-1-n} (-1)^m N_r \binom{N_r-1}{m} \binom{N_t-1}{n} \frac{\rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n \Gamma(k+1/2)}{(m+1)k!\,\bar{\gamma}_1^k \left(\alpha+b/2\right)^{k+1/2}} \tag{34}$$

4.2 Diversity and Coding Gains

Diversity and coding gains are useful metric since such studies provide valuable insights that are useful to describe the asymptotic performance of SER to design engineers. For coherent detection with perfect receiver CSI the average error rate at high SNRs may be closely approximated by [6] $P_{error} \approx (G_c \cdot \eta)^{-G_d}$, where $\eta = P/N_0 = (N_t P_s + P_r)/N_0$, denotes the transmit SNR, G_c is termed the coding gain and defines the slope of the average SER against η in a log-log scale, G_d is referred to as the diversity gain and determines the shift of the curve with respect to the average SER curve. Here, we derive the asymptotic diversity and coding gains of the system with both outdated and error estimated CSI feedback based on the asymptotic analysis of outage probability and SER performance at high SNRs. We assume that $P_s = \lambda P/N_t$, $P_r = (1-\lambda)P$, and when $\eta \to \infty$, by substituting the related parameters we may have

$$\mu = \frac{\overline{\gamma}_2}{\overline{\gamma}_1} = \frac{P_r\, \sigma_2^2 \big/ (P_r \sigma_{e_2}^2 + N_0)}{P_s \sigma_1^2 \big/ (P_s \sigma_{e_1}^2 + N_0)} \to \frac{N_t (1-\lambda)\sigma_2^2}{\lambda \sigma_1^2} \tag{35}$$

and $\overline{\gamma}_1 \to \delta\lambda\sigma_1^2 \eta \big/ N_t \left(1+\delta\right)$.

We now analyze the system's e2e asymptotic outage probability when $\eta \to \infty$. At the case of $\rho_{d_1} = \rho_{d_2} = 1$, i.e. the CSI is not outdated, using the McLaurin series representation for the exponential function in the outage lower bound in (33) yields

$$P_{out}^{\infty}\left(x\right) = 1 - \sum_{m=0}^{N_r-1} \frac{N_r(-1)^m}{m+1} \binom{N_r - 1}{m} e^{-\left(\frac{m+1}{\mu\overline{\gamma}_1}\right)x} \sum_{k=0}^{N_t-1} \frac{\left(x/\overline{\gamma}_1\right)^k}{k!} e^{-\frac{x}{\overline{\gamma}_1}} \tag{36}$$

By applying that $(1-\exp(-x))^N = x^N + O\,(x^N)$ for $x \to 0$, we have

$$\begin{aligned} P_{out}^{\infty}\left(x\right) &= 1 - \sum_{m=0}^{N_r-1} (-1)^m \binom{N_r}{m+1} e^{-\left(\frac{m+1}{\mu\overline{\gamma}_1}\right)x} \sum_{k=0}^{N_t-1} \frac{\left(x/\overline{\gamma}_1\right)^k}{k!} e^{-\frac{x}{\overline{\gamma}_1}} \\ &= 1 - \left(1 - (1 - e^{-x/\mu\overline{\gamma}_1})^{N_r}\right)\left(1 - \sum_{k=N_t}^{\infty} \frac{\left(x/\overline{\gamma}_1\right)^k}{k!} e^{-\frac{x}{\overline{\gamma}_1}}\right) \\ &= 1 - \left(1 - \left(\frac{x}{\mu\overline{\gamma}_1}\right)^{N_r} + O\,\left(x^{N_r}\right)\right)\left(1 - \frac{\left(x/\overline{\gamma}_1\right)^{N_t}}{N_t\,!} + O\,\left(x^{N_t}\right)\right) \end{aligned} \tag{37}$$

where $O\,(x)$ denotes the high-order infinitesimal. Collecting the smallest order terms yields

$$P_{out}^{\infty}\left(x\right) \to \begin{cases} \left(\dfrac{N_t\left(1+\delta\right)}{\delta\lambda\sigma_1^2}\dfrac{x}{\eta}\right)^{N_t} \Bigg/ N_t\,!\,, & if\ N_t < N_r \\ \left(\dfrac{N_t\left(1+\delta\right)}{\mu\delta\lambda\sigma_1^2}\dfrac{x}{\eta}\right)^{N_r}\,, & if\ N_t > N_r \\ \left(\dfrac{1}{N_t\,!} + \dfrac{1}{\mu^{N_r}}\right)\left(\dfrac{N_t\left(1+\delta\right)}{\delta\lambda\sigma_1^2}\dfrac{x}{\eta}\right)^{N_t}, & if\ N_t = N_r \end{cases} \tag{38}$$

The no-delay CSI case is regarded as a reference and used for a comparison with the outdated CSI case. It is evident that the diversity order is determined by the number of the transmit antennas and relays, and is entirely independent of CEE as long as the CSI is not outdated. Besides, larger CEE will lead to larger array gain loss.

For the case of $\rho_{d_1} < 1$ or $\rho_{d_2} < 1$, similarly, we can re write (33) when $\eta \to \infty$ as

$$\begin{aligned} P_{out}^{\infty}\left(x\right) &= 1 - \sum_{m=0}^{N_r-1} (-1)^m \binom{N_r}{m+1} \sum_{n=0}^{N_t-1} \binom{N_t - 1}{n} \rho_1^{2(N_t-1-n)} (1-\rho_1^2)^n \\ &\times \sum_{k=0}^{N_t-1-n} \left(\frac{x}{\overline{\gamma}_1}\right)^k \frac{1}{k!}\left(1 - \left(1 + \frac{m+1}{(m(1-\rho_2^2)+1)\mu}\right)\frac{x}{\overline{\gamma}_1} + O\,\left(x\right)\right) \end{aligned} \tag{39}$$

By closer examination on (39), we can obtain

$$P_{out}^{\infty}\left(x\right)=1-\sum_{m=0}^{N_r-1}(-1)^m\binom{N_r}{m+1}\left[1-\left(1+\frac{m+1}{(m(1-\rho_2^2)+1)\mu}\right)\frac{x}{\overline{\gamma}_1}+O\left(x\right)\right]$$
$$\times\left[\sum_{n=0}^{N_t-1}\binom{N_t-1}{n}\rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n+\sum_{n=0}^{N_t-2}\binom{N_t-1}{n}\rho_1^{2(N_t-1-n)}(1-\rho_1^2)^n\frac{x}{\overline{\gamma}_1}\right] \quad (40)$$

Simplifying (40), we get

$$P_{out}^{\infty}\left(x\right)\to\left[\left(1-\rho_1^2\right)^{N_t-1}+\sum_{m=0}^{N_r-1}(-1)^m\binom{N_r}{m+1}\left(\frac{m+1}{(m(1-\rho_2^2)+1)\mu}\right)\right]\frac{N_t\left(1+\delta\right)}{\delta\lambda\sigma_1^2\eta}x+O\left(x\right) \quad (41)$$

Following [26, Prop. 1], and substituting $P_{out}^{\infty}\left(x\right)\to\kappa\left(x/\eta\right)^{G_d}+O\left(x^{G_d}\right)$ into (18), the average BER at high SNRs can be closely approximated as

$$P_{error}^{\infty}\to\frac{2^{G_d}a\Gamma\left(G_d+1/2\right)}{2\sqrt{\pi}b^{G_d}}\kappa\eta^{-G_d}+O\left(\eta^{-G_d}\right) \quad (42)$$

Thus we may conclude that the diversity order and coding gain of the system can be obtained as

$$G_d=\begin{cases}\min\left\{N_t,N_r\right\}, & if\ \rho_{d_1}=\rho_{d_2}=1\\ 1, & if\ \rho_{d_1}<1\ or\ \rho_{d_2}<1\end{cases} \quad (43)$$

and

$$G_c=\frac{b}{2}\left(\frac{a\Gamma\left(G_d+1/2\right)}{2\sqrt{\pi}}\kappa\right)^{-G_d} \quad (44)$$

when $\rho_{d_1}=\rho_{d_2}=1$,

$$\kappa=\begin{cases}\left(\frac{N_t\left(1+\delta\right)}{\delta\lambda\sigma_1^2}\right)^{N_t}\Big/N_t!\,, & if\ N_t<N_r\\ \left(\frac{N_t\left(1+\delta\right)}{\mu\delta\lambda\sigma_1^2}\right)^{N_r}, & if\ N_t>N_r\\ \left(\frac{1}{N_t!}+\frac{1}{\mu^N}\right)\left(\frac{N_t\left(1+\delta\right)}{\delta\lambda\sigma_1^2}\right)^{N}, & if\ N_t=N_r=N\end{cases} \quad (45)$$

when $\rho_{d_1}<1$ or $\rho_{d_2}<1$,

$$\kappa=\left[\left(1-\rho_1^2\right)^{N_t-1}+\sum_{m=0}^{N_r-1}(-1)^m\binom{N_r}{m+1}\left(\frac{m+1}{(m(1-\rho_2^2)+1)\mu}\right)\right]\frac{N_t\left(1+\delta\right)}{\delta\lambda\sigma_1^2} \quad (46)$$

(43) reveals that the diversity order is $\min\left\{N_t,N_r\right\}$ if and only if the CSI is not outdated. Once the CSI is outdated, i.e., the delay exists, the diversity order reduces to 1, whereas CEE has no impact on the performance loss of diversity order. However, both delay effect and CEE can reduce the coding gain, which is the shift of SER curve, e.g., different delay coefficients ρ_d (determined by f_dT_d) and CEE coefficients ρ_e (determined by the

channel estimation quality order δ) will result in different ρ , and thus the coding gain is different. Therefore, it is safe to conclude that the feedback delay plays the dominate role in degrading the system performance through the transmit SNR region, while the effect of the CEE may be more observable only at low SNRs. More discussions on the comparison between the effect of the two CSI imperfections will be illustrated in the numerical results section.

4.2 Power Allocation

Optimal power allocation is pivotal to reduce power consumption and energy costs of the multi-antenna multi-relaying network. Indeed, this is an attractive design choice for optimizing the network performance without expending additional resources.

Based on the asymptotic analysis at high SNRs, especially for the case of CSI is outdated, in respect that the diversity gain is reduced to one, it is feasible to present an easy-to-compute solution to the optimum power allocation scheme of λ to minimize the average SER. Applying the former array gain analysis in (44), we can formulate the optimization problem as

$$\min_{\lambda} \kappa \quad \textit{subject to} \quad 0 < \lambda < 1 \tag{47}$$

As a result, the optimal power allocation is the value of λ_* that satisfies

$$\frac{\partial \kappa}{\partial \lambda} = 0 \tag{48}$$

For the case of outdated CSI, substituting (46) and (35) into (48), we can obtain a closed-form solution of λ_*, given by

$$\lambda_* = \frac{\sqrt{N_t\left(1-\rho_1^2\right)^{N_t-1}\sigma_1^2}}{\sqrt{N_t\left(1-\rho_1^2\right)^{N_t-1}\sigma_1^2} + \sqrt{\sum_{m=0}^{N_r-1}\binom{N_r-1}{m}\frac{(-1)^m N_r \sigma_2^2}{\left(m(1-\rho_2^2)+1\right)}}} \tag{49}$$

While for the case of no time delays in TB vector feedback and relay selection, applying the similar steps with the outdated CSI case, substituting (45) and (35) into (48), the general solution for the optimum power allocation is the value satisfying

$$\frac{\lambda_*^{N_t+1}}{\left(1-\lambda_*\right)^{N_r+1}} = \frac{\left(\left(1+1/\delta\right)\eta\right)^{N_t-N_r}\sigma_2^{2N_r}}{N_r N_t!N_t^{N_t}\sigma_1^{2N_t}} \tag{50}$$

5. Numerical Results

This section presents the numerical and the Monte-Carlo simulation results study of the detrimental effect of delay and channel estimation errors on the system performance. Both the theoretical expressions of outage probability and average SER, including the exact, lower bound, and asymptotic analysis at high SNRs, and the Monte Carlo simulation results are provided to demonstrate the validity and usefulness of our analytical expressions. Rayleigh fading channels are employed by all the communication links in our system. Without loss of generality, we set $P=1$, $\sigma_1^2=\sigma_2^2=1$, $\lambda=1/2$ and $N_t=N_r=4$.

In **Fig. 2** and **Fig. 3**, the outage probability and average SER for BPSK of the AF system are presented for various delays $f_d T_d$ and channel estimation qualities δ, respectively. Furthermore, the curves of the perfect CSI ($f_d T_d=0,\ and\ \delta\to\infty$) are also plotted for comparison. As it can be clearly seen from both figures, analytical and simulated outage probability and average SER curves match excellently, which confirm the accuracy of our mathematical analysis and the tightness of the derived lower bound as well as the asymptotic (high-SNR) analysis. As expected, the outage and symbol error performance are aggravated significantly due to the outdated and erroneous CSI.

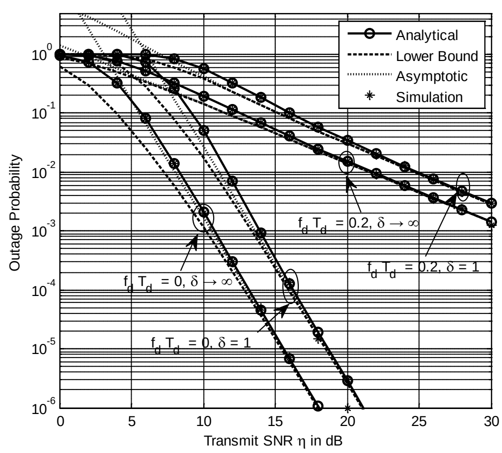

Fig. 2. Outage probability v.s. transmit SNR.

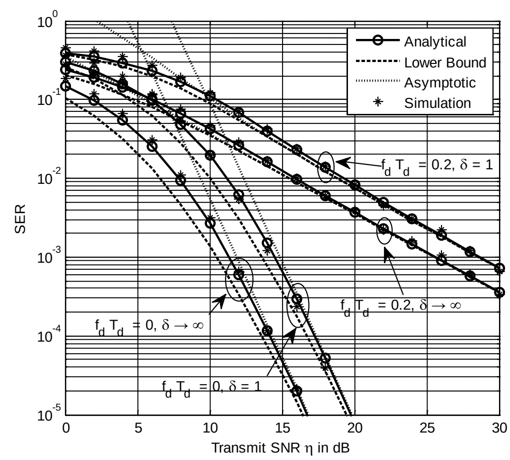

Fig. 3. Average SER v.s. .transmit SNR.

Fig. 4 draws the ergodic capacity of the AF system versus SNR for various delays $f_d T_d$ and channel estimation qualities δ. As can be clearly seen from both figures, analytical and simulated capacity curves match very well, and both feedback delay and CEE will degrade the system capacity. Besides, it is interesting to find that, as far as lower SNR region is concerned, the ergodic capacity is more sensitive to the quality of channel estimation, while at high SNRs, feedback delay will play the dominate role in leading the deleterious effects.

To further evaluate which effect is more influential and under which condition, we present another set of simulations on the performance loss due to CEE and feedback delay, separately. **Fig. 5** shows the resulting percentage outage performance loss due to the feedback delay and CEE versus the transmit SNR. The outage loss is defined as

$$L_{delay} = \frac{P^{out}_{delay\&CEE} - P^{out}_{CEE}}{P^{out}_{delay\&CEE}} \tag{51}$$

and

$$L_{CEE} = \frac{P^{out}_{delay\&CEE} - P^{out}_{delay}}{P^{out}_{delay\&CEE}} \tag{52}$$

where $P^{out}_{delay\&CEE}$, P^{out}_{delay}, and P^{out}_{CEE} denote the outage probability as in **Fig. 2** of the delay and error case, no CEE case, and no delay case, respectively. It can be seen from **Fig. 5** that the outage loss due to the feedback delays increases with the transmit SNR, while the CEE one first increases to a peak value but then drops. The effect of the CEE is more influential at low-to-medium SNR, and then it will hand over the dominate role to the feedback delay. It is more straightforward to obtain these findings on the influence of the delay and CEE which are in line with the above performance figures.

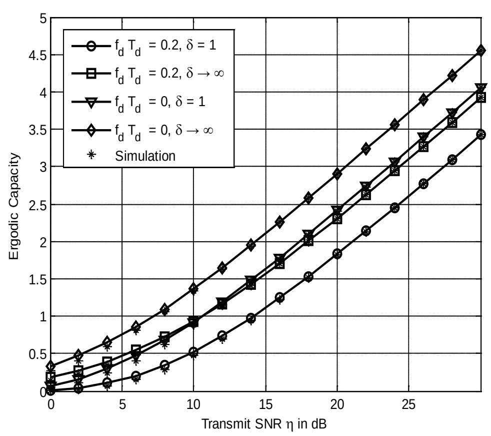

Fig. 4. Ergodic capacity v.s. transmit SNR.

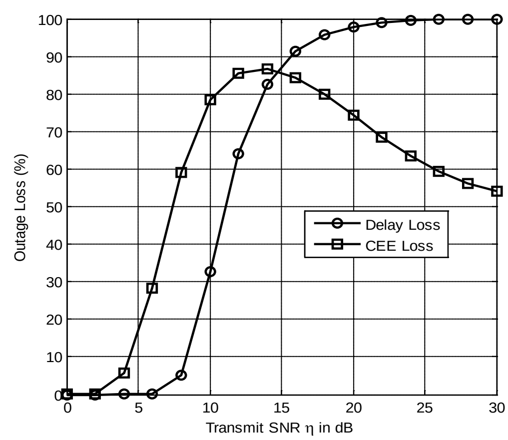

Fig. 5. Percentage outage performance loss.

Fig. 6 plots the value of κ in the array gain versus delay coefficients (including TB feedback delay coefficient ρ_{d_1} of the first hop and relay selection delay coefficient ρ_{d_2} of the second hop) under perfect channel estimation. On the whole, we see that as ρ_d decreases, the value of κ at first increases significantly, but then approaches to a limit, which means that the array gain degrades as the delay increases performance. Besides, it can be observed that the section plane of the surface on ρ_{d_1} is steeper than that of ρ_{d_2}. So we may conclude that the system performance is more sensitive to the TB vector feedback delay compared with the relay section delay. **Fig. 6** also compares the array gain of the system with and without optimum power allocation. It can be clearly seen that optimum power allocation offers superior performance over uniform power allocation since the surface of κ with optimum power allocation (the gray surface) is always covered by the equal power allocation case (the gridded surface).

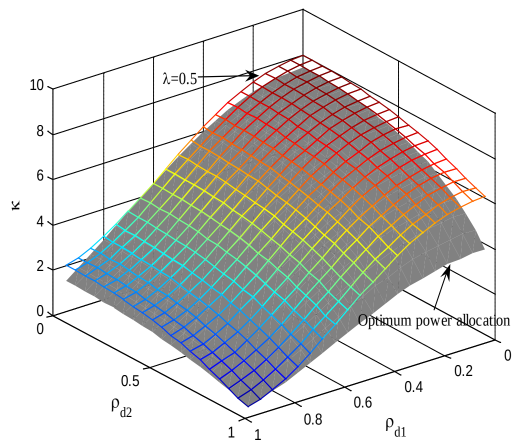

Fig. 6. The value of κ versus ρ_{d1} and ρ_{d2}.

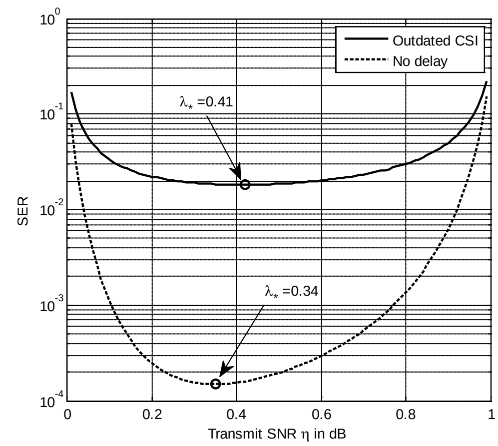

Fig. 7. Average SERs v.s. power allocation factor.

Fig. 7 further compares the average SER versus power allocation factor λ and shows the optimal power allocation λ_* with and without feedback delays at $\eta = 15dB$ and $\delta = 1$. It can be clear seen that for the delayed and no-delay CSI cases, the SER are minimized at $\lambda_* = 0.41$ and $\lambda_* = 0.34$, respectively, which precisely agrees with the analysis in (49) and (50). We corroborate that optimal power allocation offers superior performance over uniform power allocation, especially for the no-delay case.

6. Conclusion

We investigate the effect of imperfect channel estimation and outdated CSI on the performance of the multiple-antennas and multiple-relays assisted downlink networks with relay selection and transmit beamforming. Both analytical and simulated results indicate that delay effect results in the coding gain loss and the diversity order loss, and CEE will merely cause the coding gain loss. The array gain performance results shows that the system is more sensitive to TB delay compared with relay selection delay and also verify the superiority of optimum power allocation. These results will be helpful to predict practical relaying system performance with channel estimation errors and feedback delays.

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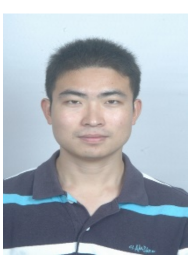

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