CORRIGENDUM TO “REFLEXIVE PROPERTY ON IDEMPOTENTS” [BULL. KOREAN MATH. SOC. 50 (2013), NO. 6, 1957–1972]

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In [2], Theorem 4.2 is incorrect, and so we here provide a correct theorem and proof.

**Theorem 4.2.** If $R$ is a non-Abelian RIP ring of minimal order, then $R$ is of order 16 and is isomorphic to $\text{Mat}_2(\mathbb{Z}_2)$ or the ring
\[
S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \text{Mat}_2 \left( \frac{\mathbb{Z}_2[x]}{x^2\mathbb{Z}_2[x]} \right) \mid a, d \in \mathbb{Z}_2 \text{ and } b, c \in (x + x^2\mathbb{Z}_2[x])^2 \mathbb{Z}_2[x] \right\}.
\]

**Proof.** Let $R$ be a non-Abelian RIP ring of minimal order. Then it is true that $R$ cannot be local since local rings are Abelian. By the Wedderburn-Artin theorem, $R/J(R) \cong \bigoplus_{i=1}^n \text{Mat}_{k_i}(D_i)$ for some $k_i$’s and fields $D_i$’s. Here assume that $k_i = 1$ for all $i$. Then we have three cases of $|J(R)| = 2$, $|J(R)| = 4$, and $|J(R)| = 8$.

If $|J(R)| = 8$, then $R/J(R) \cong \mathbb{Z}_2$ and so $R$ is local, a contradiction. Thus $|J(R)| = 2$ or $|J(R)| = 4$.

Let $|J(R)| = 4$. Then $R/J(R) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2$. Since $J(R)$ is nilpotent, there exist orthogonal nonzero idempotents $e_1, e_2$ with $e_1 + e_2 = 1$ (i.e., $e_2 = 1 - e_1$) by [3, Proposition 3.7.2], and moreover we have $R = \{x + y \mid x \in \text{Id}(R), y \in J(R)\}$, where $\text{Id}(R) = \{0, 1, e_1, e_2\}$. Then $R$ is an RIP ring and moreover $R \cong S$ by the proof of [1, Proposition 2.7(1) and Theorem 2.11].

Let $|J(R)| = 2$. Then $R/J(R) \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$. Since $J(R)$ is nilpotent, there exist orthogonal nonzero idempotents $e_1, e_2, e_3$ with $e_1 + e_2 + e_3 = 1$ by [3, Proposition 3.7.2], and moreover we have $R = \{x + y \mid x \in \text{Id}(R), y \in J(R)\}$, where $\text{Id}(R) = \{0, 1, e_1, e_2, e_3, 1 - e_1, 1 - e_2, 1 - e_3\}$. For every $r = x + y \in R$ with $x \in \text{Id}(R)$ and $y \in J(R)$, we have $e_ire_j = e_ire_j$ for $i \neq j$ since $e_ixe_j = 0$. If $e_iye_j \neq 0$ then $J(R) = \{0, e_iye_j\}$. Thus $e_jJ(R)e_i = 0$, and so $e_jRe_i = 0$

Received June 16, 2016.
since $e_j\text{Id}(R)e_j = 0$. But since $R$ is RIP, we get $e_iRe_j = 0$ and this yields $e_iye_j = 0$, a contradiction. Therefore we can conclude that $e_iRe_j = 0$ for all $i, j$ with $i \neq j$.

Now suppose that $eRf = 0$ for $e, f \in \text{Id}(R)$. Then $e$ and $f$ are orthogonal each other, say $e = e_1$ and $f = e_2$. But since $R$ is RIP, $e_2Re_1 = 0$ and so we get $e_2ye_1 = 0$ since $e_2xe_1 = 0$. This entails

$r = (e_1 + e_2 + e_3)r(e_1 + e_2 + e_3) = e_1re_1 + e_2re_2 + e_3re_3$.

Since $R$ is non-Abelian, $e_k$ is non-central for some $k \in \{1, 2, 3\}$. If $e_1$ is non-central then there exists $s \in R$ such that $e_1s - se_1 \neq 0$. Note $e_1s - se_1 \in J(R)$. Then we have

$$e_1s - se_1 = (e_1 + e_2 + e_3)(e_1s - se_1)(e_1 + e_2 + e_3) = e_1(e_1s - se_1)e_1 + e_2(e_1s - se_1)e_2 + e_3(e_1s - se_1)e_3 = 0,$$

a contradiction. Each case of ($e_2$ is non-central) and ($e_3$ is non-central) also induces a contradiction through a similar computation. The computations for other cases of $e$ and $f$ are also similar, inducing contradictions.

Summarizing, we have two cases of $J(R) = 0$ and $|J(R)| = 4$, and $R$ is isomorphic to either $\text{Mat}_2(\mathbb{Z}_2)$ (when $|J(R)| = 0$) or the ring $S$ (when $|J(R)| = 4$).

In [2], Corollary 4.3 is incorrect, and so we here provide a correct expression.

**Corollary 4.3.** Let $R$ be a ring with $J(R) = 0$. Then $R$ is a non-Abelian RIP ring of minimal order if and only if $R$ is a non-Abelian semi prime ring of minimal order if and only if $R$ is a non-Abelian reflexive ring of minimal order if and only if $R$ is a non-Abelian right idempotent reflexive ring of minimal order if and only if $R$ is a non-Abelian left idempotent reflexive ring of minimal order.

**References**


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