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Nonlinear Static Analysis of Cable Roof Structures with Unified Kinematic Description

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Abstract A finite element analysis technology applicable to the prediction of the static nonlinear response of cable roof structure is presented. The unified kinematic description is employed to formulate the present cable element and different strain definitions such as Green-Lagrange strain, Biot strain and Hencky strain can be adopted. The Newton–Raphson method is used to trace the nonlinear load-displacement path. In the iteration process, the compressive stress of a cable element is not allowed. For the verification of the present cable element, four numerical examples are tackled. Finally, numerical results obtained by using the present cable element are provided as new benchmark test results for cable structures under static loads.

Keywords: Cable Roof Structure, Unified Kinematic Description, Nonlinear Static Analysis, Green-Lagrange Strain, Tangent Stiffness

1. INTRODUCTION

Cable roof structure is one of the important structural systems which can provide a large column-free architectural space. It can basically resist tensile force only and therefore usually needs a pre-stressing force to preserve its form and structural stability. For this reason, it sometimes produces complex structural behaviours against external load.

For the analysis of cable structure, the elastic catenary cable was first presented by O'Brien and Francis (1960). In their study, an iterative method was mainly used to solve a two-dimensional cable structure subjected to static concentrated loads. Later, the elastic catenary element was presented by Peyrot and Goulois (1978, 1979). They used O'Brien's expression to obtain the flexibility matrix. Then tangent stiffness matrix was produced by taking the inverse of the flexibility matrix and used in the direct stiffness method.

The finite element (FE) analysis technique has been also used for cable analysis. Three types of element have been mainly adopted: 1) straight element, 2) curved element, and 3) curved element with rotational degrees of freedom. Baron and Venkatesan (1971) developed the stiffness matrix of a three-dimensional two-node straight truss element including the effect of stress stiffening. They used the direct stiffness method and a nonlinear iterative scheme to solve cable structures subjected to static concentrated forces. Similarly, other two-node truss elements were developed by Webster (1975) and Broughton et al. (1994). Argyris and Scharpf (1972) developed a FE computer procedure based on the displacement method for the analysis of large pre-stressed networks. The effect of the geometric nonlinear contribution was first reviewed in their study. Gambhir and Batchelor (1977) developed a curved element with rotational degree of freedom for shallow cable nets. The large displacement formulation was used to evaluate the static and dynamic response of three-dimensional cable nets. They described that a cubic displacement field is sufficient for the prediction of the first frequency of shallow nets. However, for globally deep networks, the accuracy can be increased by employing a quintic order of displacement field for the normal component of displacement. Desai et al. (1988) formulated the stiffness matrix of a three-node cable element using a parabolic assumed function. A comparison between two-node element and three-node element was presented. They showed that the three-node cable element is more accurate in a static analysis. Mitsugi (1994) formulated a stiffness matrix for the hyper-cable element which is for a cable connected to intermediate pulleys along its length. Kwan (1998) reviewed the existing technique for static analysis of cable structures and explained nonlinear behaviour of cable networks using linked structure. He also provided the computation time for a cable structure and compared with Lewis's result (1989). Recently, Thai and Kim (2011) developed a spatial two-node catenary cable element for the nonlinear analysis of cable structures subjected to static and dynamic loadings. The tangent stiffness matrix and internal force vector of the element are derived explicitly based on the exact analytical expressions of elastic catenary. For static analysis, the Newton–Raphson method is adopted for solving the

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nonlinear equation of motion. Salehi Ahmad Abad et al. (2013) proposed two elements for three-dimensional FE analysis of cable structures. The first one is the continuous catenary cable (CCC) element which is the extension of the classic catenary cable element. The second element, discrete catenary cable (DCC) element, is introduced by transforming the continuous equations of the CCC element into discrete formulation, giving the capability of dividing the cable into several straight elements with axial behavior.

In this study, the cable element is proposed on the basis of the unified kinematic description. The linear shape function is introduced to derive the tangent stiffness matrix of two-node cable element. Strain definitions such as Green-Lagrange strain, Biot strain and Hencky strain can be incorporated to produce the tangent stiffness matrices in the present FE formulation. The Newton–Raphson method is adopted for tracing nonlinear load-displacement path. In particular, the compressive stress of a cable element is not allowed by forcing its values to zero. For the verification of the implementation of the present cable element, four examples, which have been studied by many authors, are thoroughly analyzed. The numerical results obtained by using the present cable element on the basis of unified kinematic description are provided as new benchmark test results for cable structures under static loads.

2. UNIFIED KINEMATIC DESCRIPTION

2.1 Geometry mapping

The deformation of a body can be described by using the mapping between the initial and deformed configurations. The point in the initial configuration is denoted as **X** and the point in the deformed configuration is denoted as **x**.

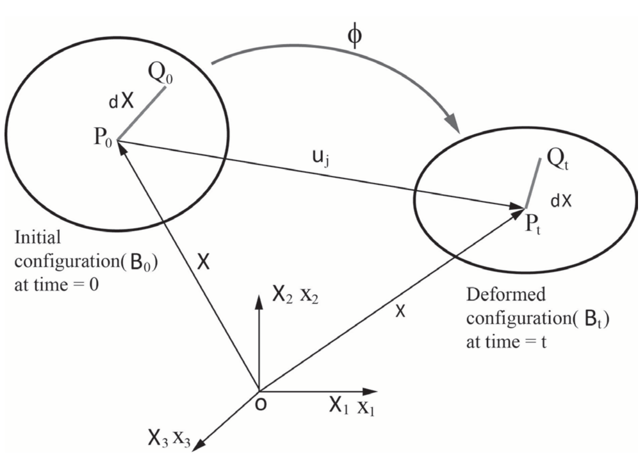

Figure 1. Motion of a particle

If there is a mapping from the initial configuration to the deformed configuration, the following relationship can be achieved as

$$\phi(\mathbf{X}, t) = \mathbf{x} \tag{1}$$

and the displacement mapping can be written as

$$u(\mathbf{X}, t) = \phi(\mathbf{X}, t) - \mathbf{X} \tag{2}$$

The deformation gradient can be defined as

$$\mathbf{F} = \frac{\partial \phi}{\partial \mathbf{X}} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}\,. \tag{3}$$

2.2 Strain definitions

Three different definitions of strain (e) such as Green-Lagrange ($\boldsymbol{\varepsilon}^G$), Biot ($\boldsymbol{\varepsilon}^B$) and Hencky ($\boldsymbol{\varepsilon}^H$) strains can be introduced to represent nonlinear structural behaviours as follows

$$\boldsymbol{\varepsilon}^G = \frac{1}{2}\left(\mathbf{F}^T\mathbf{F} - \mathbf{I}\right) \tag{4}$$

$$\boldsymbol{\varepsilon}^B = \mathbf{F} - \mathbf{I} \tag{5}$$

$$\boldsymbol{\varepsilon}^H = log(\mathbf{F})\ . \tag{6}$$

In one-dimensional problem, the deformation gradient of (3) can be written as

$$\mathrm{F} = \mathrm{L}/\mathrm{L}_0 \tag{7}$$

where L and L_0 are the length of deformed bar and the length of the initial bar respectively as illustrated in Figure 2.

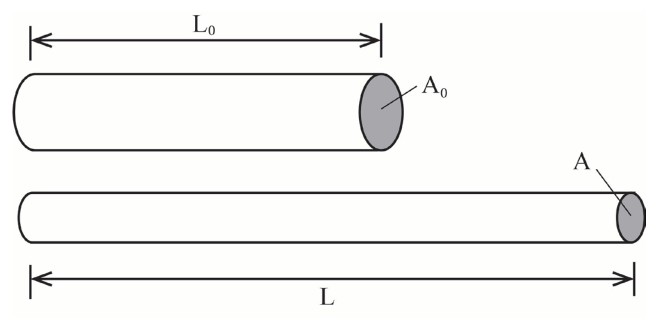

Figure 2. One-dimensional strain

Therefore, three strain definitions for one-dimensional cable can be written as

$$\varepsilon^G = \frac{1}{2}\left(\frac{\mathrm{L}}{\mathrm{L}_0}\frac{\mathrm{L}}{\mathrm{L}_0} - 1\right) \tag{8}$$

$$\varepsilon^B = \frac{\mathrm{L}}{\mathrm{L}_0} - 1 \tag{9}$$

$$\varepsilon^H = log\left(\frac{\mathrm{L}}{\mathrm{L}_0}\right) \tag{10}$$

The strain magnitude with respect to the value of deformation gradient is illustrated in Figure 3.

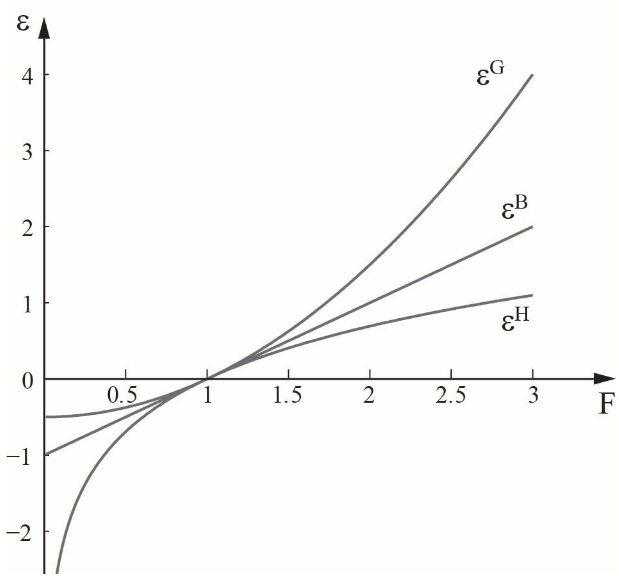

Figure 3. Strain variations with respect to deformation gradient

2.3 Stress definition

The stress of the cable in the current configuration can be written as

$$s = s_0 + Ee \tag{11}$$

where e is the axial strain, s is the current stress which is conjugate of e and s_0 is the stress in the initial configuration.

Stress definition of (11) can be rewritten in force term

$$N = sA_0 = N_0 + EA_0 e \ . \tag{12}$$

2.4 Strain energy

Strain energy of the cable in current configuration can be written as

$$U = \int_{\mathbf{B}_0} u \,\mathrm{dV} = uV_0$$
$$= \left(s_0 e + \frac{1}{2} E e^2\right) A_0 L_0 = L_0 \left(N_0 e + \frac{1}{2} E A_0 e^2\right) \tag{13}$$

where u is the energy density.

2.5 Internal force

Internal force can be obtained by the differentiation of the strain energy with respect to the nodal displacement $\mathbf{u}$.

$$\mathbf{q} = \frac{\partial U}{\partial \mathbf{u}} = \frac{\partial U}{\partial e}\frac{\partial e}{\partial \mathbf{u}} \ . \tag{14}$$

2.6 Tangent stiffness

Tangent stiffness of the cable can be obtained by the following expression.

$$\mathbf{K} = \frac{\partial \mathbf{q}}{\partial \mathbf{u}} = \frac{\partial^2 U}{\partial \mathbf{u}^2} \ . \tag{15}$$

3. CABLE FINITE ELEMENT

3.1 Element Kinematics

In this study, two-node cable element is formulated and implemented to analyze the cable roof structures. The geometry of cable element is illustrated in Figure 4.

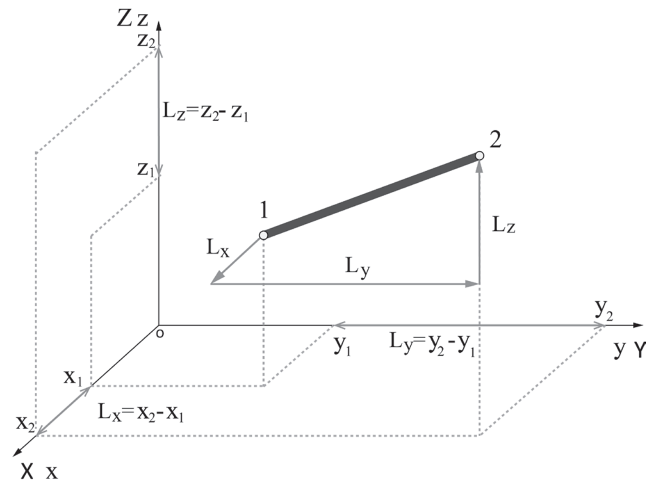

Figure 4. Cable element in three-dimensional space

The position vector of nodal points in the current configuration can be written as

$$\mathbf{x} = \{\mathbf{x}_1\ \mathbf{x}_2\}^T = \{x_1\ y_1\ z_1\ x_2\ y_2\ x_2\}^T \tag{16}$$

where $\mathbf{x}_1$ and $\mathbf{x}_2$ are the position vectors of node 1 and node 2 respectively.

The length of bar in current configuration is as follows

$$L = \sqrt{L_x^2 + L_y^2 + L_z^2} \tag{17}$$

where the projected length into the global axes are

$$\mathrm{L}_x = x_2 - x_1;\ \mathrm{L}_y = y_2 - y_1;\ \mathrm{L}_z = z_2 - z_1. \tag{18}$$

3.2 Displacement

The displacement is obtained by subtracting the position vectors $\mathbf{x}$ and $\mathbf{X}$.

$$\mathbf{u} = \mathbf{x} - \mathbf{X} = \{u_1\ v_1\ w_1\ u_2\ v_2\ w_2\}^T \tag{19}$$

where $u_a\ v_a\ w_a$ are the displacement at the node a in x-, y-, z-direction respectively.

3.3 Internal force

Internal force of the cable can be obtained by using (14)

$$\mathbf{q} = \frac{\partial U}{\partial \mathbf{u}} = \frac{\partial U}{\partial e}\frac{\partial e}{\partial \mathbf{u}} = \frac{\partial U}{\partial e}\frac{\partial e}{\partial L}\frac{\partial L}{\partial \mathbf{u}} = L_0 N \frac{\partial e}{\partial L}\boldsymbol{\Theta} \tag{20}$$

where the terms $\frac{\partial U}{\partial e}$ and $\frac{\partial L}{\partial \mathbf{u}}$ are calculated as

$$\frac{\partial U}{\partial e} = L_0(N_0 + EA_0 e) = L_0 N \tag{21}$$

$$\frac{\partial L}{\partial \mathbf{u}} = \begin{Bmatrix} \partial L/\partial u_1 \\ \partial L/\partial v_1 \\ \partial L/\partial w_1 \\ \partial L/\partial u_2 \\ \partial L/\partial v_2 \\ \partial L/\partial w_2 \end{Bmatrix} = \begin{Bmatrix} -L_X/L \\ -L_Y/L \\ -L_Z/L \\ L_X/L \\ L_Y/L \\ L_Z/L \end{Bmatrix} = \begin{Bmatrix} -\hat{\boldsymbol{\Theta}} \\ \hat{\boldsymbol{\Theta}} \end{Bmatrix} = \boldsymbol{\Theta}\,. \tag{22}$$

3.4 Tangent stiffness matrix

Substituting (20) into (15), the tangent stiffness of the cable in discretized domain can be calculated as

$$\mathbf{K} = \frac{\partial \mathbf{q}}{\partial \mathbf{u}} = \frac{\partial}{\partial \mathbf{u}}\left(L_0 N \frac{\partial e}{\partial L} \boldsymbol{\Theta}\right). \tag{23}$$

Using (23), the tangent stiffness matrix can be obtained in the following form

$$\mathbf{K} = \mathbf{K}_m + \mathbf{K}_g \tag{24}$$

where the material stiffness matrix ($\mathbf{K}_m$) and geometrical stiffness matrix ($\mathbf{K}_g$) are

$$\mathbf{K}_m = EA_0 L_0 \left(\frac{\partial e}{\partial L}\right)^2 \boldsymbol{\Theta}\boldsymbol{\Theta}^T \tag{25}$$

$$\mathbf{K}_g = NL\left[\frac{1}{L}\frac{\partial e}{\partial L}\check{\mathbf{I}} + \left(\frac{\partial^2 e}{\partial L^2} - \frac{1}{L}\frac{\partial e}{\partial L}\right)\boldsymbol{\Theta}\boldsymbol{\Theta}^T\right] \tag{26}$$

in which $\check{\mathbf{I}}$ is

$$\check{\mathbf{I}} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}. \tag{27}$$

Since the unified kinematic description is used here, the tangent stiffness matrix of (25) and (26) will be led into the equations summarized in Table 1.

Table 1. Tangent stiffness terms associated different strains

Strains (e)	$\frac{\partial e}{\partial L}$	$\frac{\partial^2 e}{\partial L^2}$	$\mathbf{K}_m$	$\mathbf{K}_g$
$\boldsymbol{\varepsilon}^G$	$\frac{L}{L_0^2}$	$\frac{1}{L_0^2}$	$\frac{EA_0L^2}{L_0^3}\boldsymbol{\Theta}\boldsymbol{\Theta}^T$	$\frac{N}{L_0}\check{\mathbf{I}}$
$\boldsymbol{\varepsilon}^B$	$\frac{1}{L_0}$	0	$\frac{EA_0}{L}\boldsymbol{\Theta}\boldsymbol{\Theta}^T$	$\frac{N}{L}\left[\check{\mathbf{I}} - \boldsymbol{\Theta}\boldsymbol{\Theta}^T\right]$
$\boldsymbol{\varepsilon}^H$	$\frac{1}{L}$	$-\frac{1}{L^2}$	$\frac{EA_0L_0}{L^2}\boldsymbol{\Theta}\boldsymbol{\Theta}^T$	$\frac{NL_0}{L^2}\left[\check{\mathbf{I}} - 2\boldsymbol{\Theta}\boldsymbol{\Theta}^T\right]$

4. NUMERICAL EXAMPLES

In order to verify the present cable element, a single cable under central point load is first tested. Furthermore, three examples are analyzed and the results are compared to reference solutions (Jayaraman and Knudson, 1981; Michalos and Birnstiel, 1960; O'Brien and Francis, 1964; Saafan, 1970; West and Kar, 1973). Note that all the present solution is obtained with the tolerance $Tol = 10^{-10}$ for the displacement residual in Newton-Rapson method.

4.1 Single cable with central point load

In this example, we consider a single horizontal cable which is pre-stressed by $N_0 = 10$. The cable is kinematically indeterminate structure. The cable is fixed at both end supports and it is subjected to a central point load. In the analysis, the elastic modulus $E = 1000$ and section area $A = 1$ are used. For the FE analysis, three nodes and two cable elements are used to discretize the cable structure. All units are assumed to be consistent.

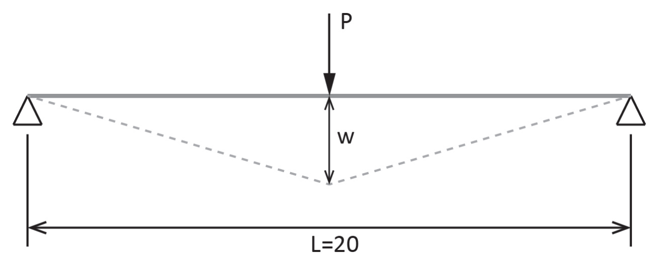

Figure 5. Single horizontal cable subjected to point load

The relationship between the central point load P and the deflection w (Kollr, 2003) can be written as

$$P = 8EA\frac{w^3}{L^3} + 4N_0\frac{w}{L} \tag{28}$$

where P is the applied central load, w is the deflection, N_0 is the initial stress. The present result is compared with analytical solution of (28) and illustrated in Figure 6. In this example, it is turned out to be that the present numerical result has an excellent agreement with the analytical solution.

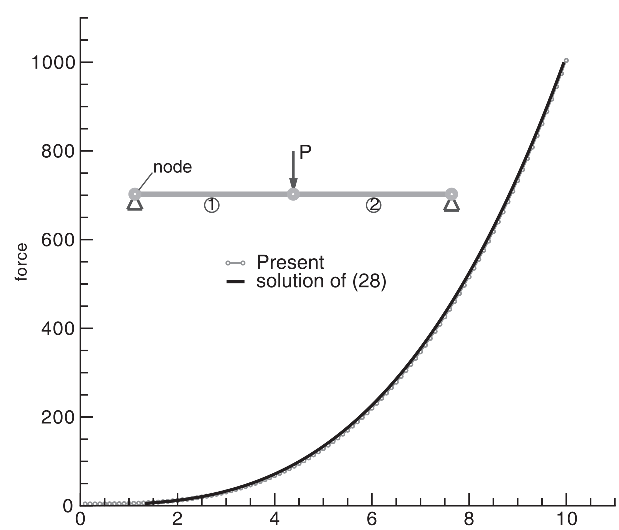

Figure 6. Force-displacement path at loading point

4.2 Poskitt cable truss

A two-dimensional structure shown in Figure 7 is considered. It consists of an upper cable and a lower cable and 14 vertical hangers. Hangers are equally spaced and all displacement in the out-of direction is prevented. A point load $P = \lambda P_0 = \lambda \times 115.4$ N in the vertical direction is applied at the lower cable of the fifth hanger from the left support. A mesh of 44 cable elements with 32 nodes is used. The nodal coordinates are summarized in Table 2. Material properties are assumed : elastic modulus $E = 12800\ \mathrm{kN}/cm^2$ and the section area of the upper and lower cables $A = 0.01866\ cm^2$. The section area of hangers is $A = 0.0080654\ cm^2$. Both ends of cable structure are fixed. Pre-tensioning stress is applied to the lower and upper cables about 0.6896 kN. In this example, 22 load steps (λ) are used to produce the load-displacement path as illustrated in Figure 8. Noted that there is no load-displacement path information of this problem in other references, we therefore provide it here.

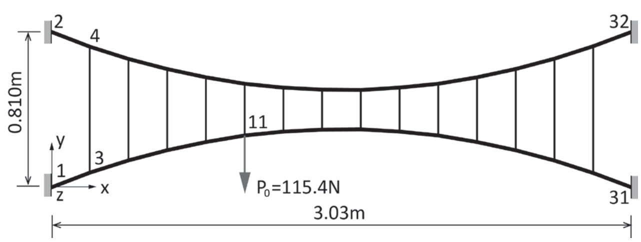

Figure 7. The geometry of Poskitt truss

The vertical displacement of lower cable at the load $P_0 = 115.4$ N is illustrated in Figure 9. The horizontal displacement of lower cable and upper cable are provided in Figure 10.

Table 2. The nodal coordinate data of Poskitt truss (m)

node	Nodal coordinates		node	Nodal coordinates	
	x	y		x	y
1	0	0	17	1.6184	0.3028
2	0	0.8103	18	1.6184	0.5073
3	0.2023	0.0771	19	1.8207	0.2907
4	0.2023	0.733	20	1.8207	0.5199
5	0.4046	0.1412	21	2.0230	0.2705
6	0.4046	0.6690	22	2.0230	0.5398
7	0.6069	0.1941	23	2.2253	0.2367
8	0.6069	0.6162	24	2.2253	0.5733
9	0.8092	0.2367	25	2.4276	0.1938
10	0.8092	0.5733	26	2.4276	0.6165
11	1.0115	0.2705	27	2.6299	0.1401
12	1.0115	0.5398	28	2.6299	0.6701
13	1.2138	0.2908	29	2.8322	0.0752
14	1.2138	0.5194	30	2.8322	0.7351
15	1.4161	0.3010	31	3.0345	0
16	1.4161	0.5093	32	3.0345	0.8103

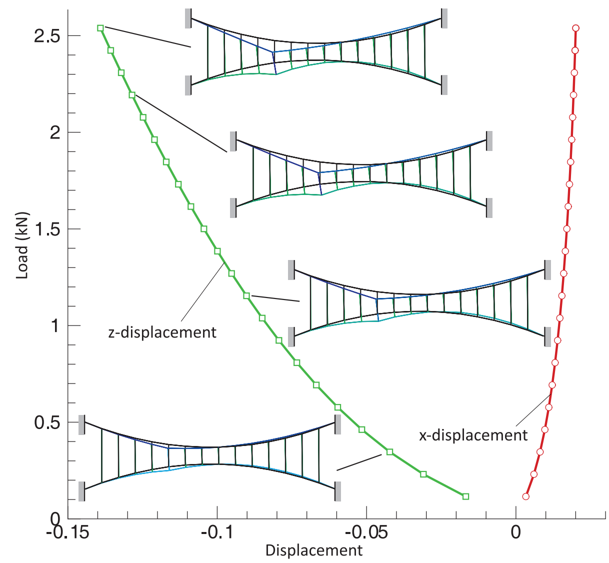

Figure 8. Poskitt truss : load-displacement path of the node 11

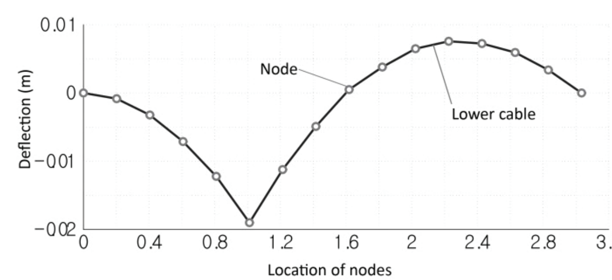

Figure 9. Poskitt truss: deflection of lower cable

Since the present results are almost identical to the results in References (Poskitt, 1967; Meek, 1991; Lewis, 2003), the present result only is therefore provided in Figures 9 and 10.

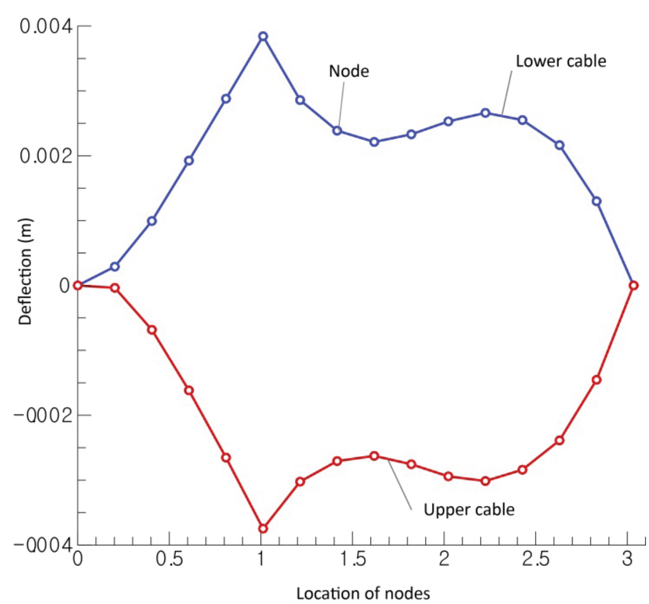

Figure 10. Poskitt truss: horizontal displacement of lower cable and upper cable.

4.3 Pre-stressed cable net under vertical load

A pre-stressed cable net is considered. It is first studied by Saafan (1973) and subsequently by West and Kar (1970) and Jayaraman and Knudson (1981). Later, Tibert (1999) provided a comprehensive review on this subject. The geometry of the cable net is illustrated in Figures 11 and 12. The initial data for the analysis are given in Table 3. The cable net is discretized with twelve elements and twelve nodes. Four point loads $P = \lambda P_0 = \lambda \times 8.0$ kip are applied at the nodes 4, 5, 8 and 9 in the vertical direction. Since the geometry of cable net and loadings are all symmetry, the vertical displacements at all nodes having the point load will be therefore the same values and lateral displacements in x- and y-direction should be the same.

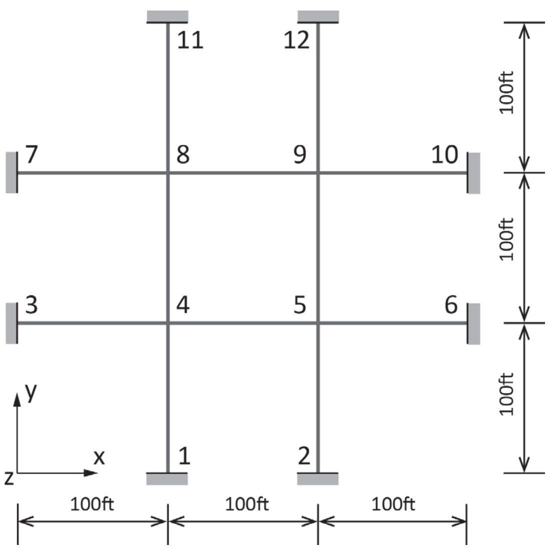

Figure 11. Cable net: x-y plane view

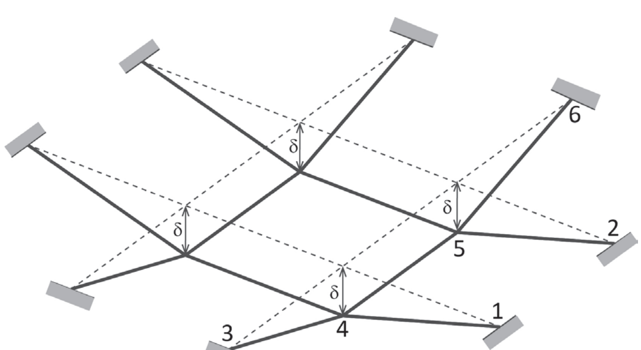

Figure 12. Cable net: perspective view ()

Table 3. Data for analysis of cable net

Descriptions	Data
Cross section area (A)	0.227in²
Elastic modulus (E)	12000kips/in²
Self-weight	0.0001 kips/ft
Horizontal pre-stressing force	5.459 kips
Inclined pre-stressing force	5.325 kips
Load (P_0)	8.0 kips

Twenty load steps are used in this example to produce the overall behaviour of cable net structure. The load-displacement history of cable net is newly provided in Figure 13. To investigate the performance of this cable element, the iteration number is monitored. All load steps have 4 or 5 iterations to get the converged solution. Note that less iteration will be required with larger tolerance value.

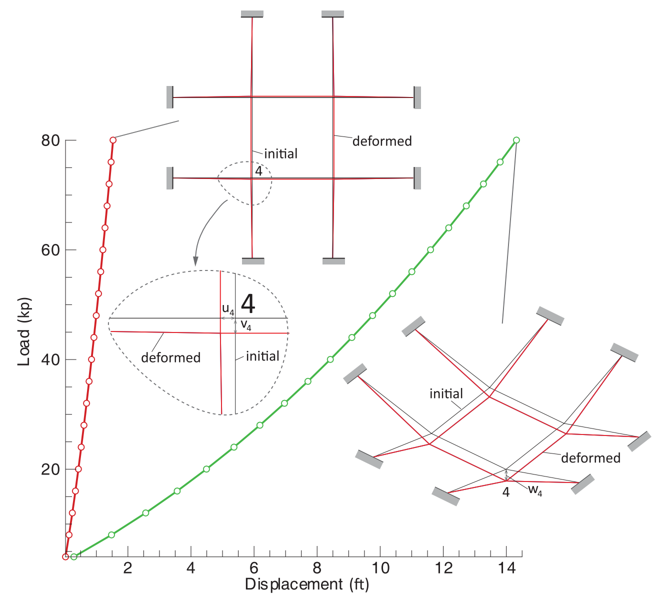

Figure 13. Displacement at node 4

The present result with a particular load (P_0 = 8.0 kip) is summarized with other reference solutions (Saafan, 1970; West & Kar, 1973; Jayaraman & Knudson, 1981; Tibert, 1999) in Table 4. It is found to be that the present result has a good agreement with other solutions.

Table 4. Displacement with P_0=8.0 kip at the node 4

References	Displacement at loading point		
	u_4	v_4	w_4
Straight bar (Ref1)	-0.1324	-0.1324	-1.477
Straight bar (Ref2)	-0.1325	-0.1324	-1.468
Elastic catenary (Ref3)	-0.1300	-0.1319	-1.463
Straight bar (Ref3)	-0.1322	-0.1322	-1.477
Straight bar (Ref4)	-0.1322	-0.1322	-1.477
Elastic parabola (Ref4)	-0.1338	-0.1338	-1.483
Elastic catenary (Ref4)	-0.1328	-0.1328	-1.474
Associate catenary (Ref4)	-0.1338	-0.1338	-1.484
Present	-0.1323	-0.1323	-1.469

Ref1: Saafan (1970). Ref2:West & Kar (1973).
Ref3:Jayaraman & Knudson (1981). Ref4: Tibert(1999).

Table 5. Displacement at specific free nodes (a)

Node (a)	z-Coord.	Kwan(1998)			Thai and Kim(2011)			Present		
		u_a	v_a	w_a	u_a	v_a	w_a	u_a	w_a	w_a
1	1368	0	0	0	-	-	-	0	0	0
2	2432	0	0	0	-	-	-	0	0	0
3	3192	0	0	0	-	-	-	0	0	0
4	3648	0	0	0	-	-	-	0	0	0
5	3800	0	0	0	-	-	-	0	0	0
11	1032	15.55	-4.46	81.70	15.55	-4.46	81.66	15.5538	4.4601	-81.6998
12	1835	11.50	-5.55	61.22	11.05	-5.54	61.18	11.5041	5.5490	-61.2277
13	2408	7.38	-4.20	33.31	7.38	-4.19	33.28	7.3862	4.1965	-33.3164
14	2752	5.34	-3.11	17.88	5.34	-3.11	17.87	5.3404	3.1112	-17.8842
15	2867	4.11	-2.80	11.16	4.10	-2.80	11.15	4.1077	2.7988	-11.1562
22	792	14.43	-3.53	97.14	14.42	-3.53	97.10	14.4287	3.5279	-97.1436
23	1408	11.27	-4.47	72.90	11.26	-4.46	72.84	11.2683	4.4666	-72.8983
24	1848	7.25	-2.97	31.98	7.25	-2.97	31.94	7.2528	2.9701	-31.9848
25	2118	5.67	-2.12	10.54	5.67	-2.11	10.52	5.6744	2.1160	-10.5343
26	2200	4.77	-0.60	-11.34	4.77	-0.60	-11.34	4.7754	0.5976	11.3450
33	648	11.71	-1.71	92.44	11.70	-1.71	92.40	11.7089	1.7118	-92.4431
34	1152	9.55	-2.11	66.94	9.54	-2.11	66.89	9.5465	2.1087	-66.9433
35	1512	6.30	-1.15	20.21	6.30	-1.15	20.17	6.3025	1.1547	-20.2033
36	1728	4.92	-0.23	-14.05	4.91	-0.23	-14.06	4.9182	0.2264	14.0525
37	1800	4.65	0.52	-35.79	4.65	0.52	-35.77	4.6515	-0.5251	35.7881
44	600	10.63	0	88.73	10.62	0	88.68	10.6289	0.0000	-88.7260
45	1067	8.80	0	62.83	8.79	0	62.77	8.7994	0.0000	-62.8246
46	1400	5.83	0	13.99	5.83	0	13.95	5.8330	0.0000	-13.9885
47	1600	4.64	0	-22.52	4.63	0	-22.52	4.6375	0.0000	22.5202
48	1667	4.55	0	-45.89	4.54	0	-45.87	4.5487	0.0000	45.8875
52	600	0.92	0	5.86	0.92	0	5.86	-0.9209	0.0000	-5.8603
72	1840	3.85	-0.78	-30.12	3.85	-0.78	-30.10	3.8509	0.7793	30.1165
81	2867	4.11	2.80	11.16	4.10	2.80	11.16	4.1077	-2.7988	-11.1562
85	1032	5.40	1.87	32.17	5.40	1.87	32.15	-5.4008	-1.8722	-32.1685
16	2752	-	-	-	-	-	-	3.3051	0.5044	6.7076
17	2408	-	-	-	-	-	-	1.8681	0.8644	2.8741
18	1835	-	-	-	-	-	-	-1.1042	1.7925	-12.7494
19	1032	-	-	-	-	-	-	-5.4008	1.8722	-32.1685
27	2118	-	-	-	-	-	-	4.4444	-0.6910	30.0767
28	1848	-	-	-	-	-	-	3.8509	-0.7793	30.1165
29	1408	-	-	-	-	-	-	1.0266	0.5615	5.3189
30	792	-	-	-	-	-	-	-3.3993	1.1658	-25.2982
38	1728	-	-	-	-	-	-	4.9523	-1.4445	63.1802
39	1512	-	-	-	-	-	-	4.6272	-1.1142	56.0464
40	1152	-	-	-	-	-	-	2.3739	-0.1607	25.9804
41	648	-	-	-	-	-	-	-1.5657	0.4242	-11.8888
49	1600	-	-	-	-	-	-	5.0161	0.0000	74.4089
50	1400	-	-	-	-	-	-	4.8781	0.0000	66.7895
51	1067	-	-	-	-	-	-	2.8118	0.0000	34.5377
52	600	-	-	-	-	-	-	-0.9209	0.0000	-5.8603

4.4 Three-dimensional saddle net

Three-dimensional saddle net with the dimension of $50\ m \times 40\ m$ is considered. It consists of 142 cable elements which are equally spaced with $5\ \mathrm{m} \times 5\ \mathrm{m}$ grid. The equal pre-tension force $\mathrm{N_0} = 60\ \mathrm{kN}$ is applied to all cables. The external loads with the magnitude of $\mathrm{P} = 1\ \mathrm{kN}$ are applied to the half structure in the positive x- and negative z-directions as illustrated in Figure 14. The elastic modulus and section area are $\mathrm{E} = 147\mathrm{kN/mm^2}$ and $\mathrm{A} = 306\ \mathrm{mm^2}$ respectively. Note that the geometry of saddle net has doubly symmetry in x-, y-direction.

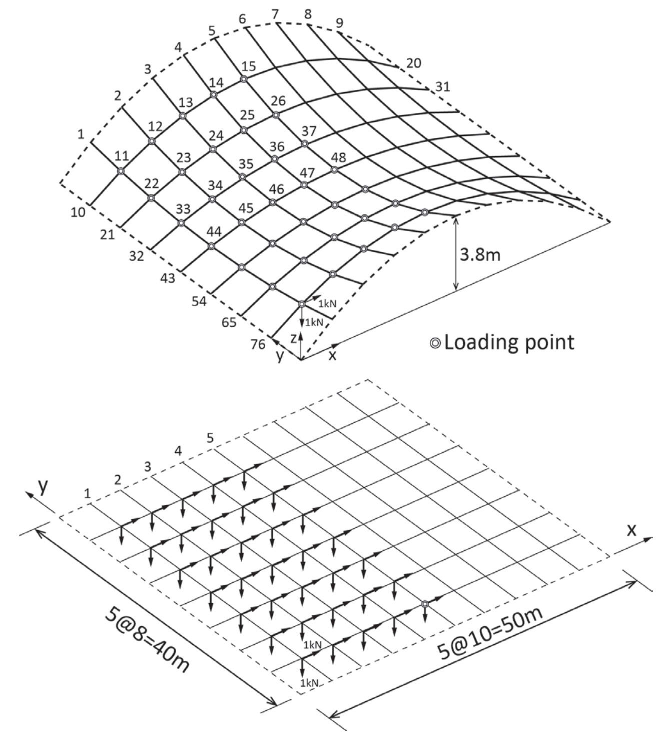

Figure 14. The geometry of saddle net :
(top) perspective view and (bottom) x-y plane view

This example was tested by several researchers (Lewis, 2003; Kwan, 1998; Thai and Kim, 2011). Lewis focused on the performance evaluation of different solution algorithms to find the form of the saddle net against external load. Lewis described that the tangent stiffness matrix approach was unable to find a converged solution for this problem. She also provided that the dynamic relaxation method can produce the converged solution which is required 260 iterations using viscous damping and 329 iterations using kinetic damping respectively. Kwan described a simple analytical approach and provided a reference solution. He used the Gauss-Newton algorithm to calculate the similar solution to the one produced by Lewis (1989). However, the detailed result on the performance evaluation was not provided. Thai and Kim also produced numerical solution by using catenary cable element. The present result is provided with other solutions in Table 5. The 5 iterations are required to produce the present solution. The present solution is in excellent agreement with other solutions (Lewis, 2003; Kwan, 1998; Thai and Kim, 2011). Full displacement information of the saddle net at free nodes is also provided for future reference solution.

5. CONCULSIONS

A cable roof structures are analyzed by using a two-node cable element formulated by using unified kinematic description. The tangent stiffness matrix based on Green-Lagrange strain definition is consistently used in numerical tests and the present numerical results have a very good agreement with other reference solutions. From numerical tests, it is observed that the present element with Newton-Raphson method shows a good performance since it can produce a converged solution with a few iterations without any unstable situation. Comprehensive load-displacement paths are newly provided for all numerical examples. In particular, full displacement information of saddle net is described here for future benchmark test. Throughout this study, it is turned out to be that the unified kinematic description can provide a very simple process for the derivation of all terms required in the geometrically nonlinear cable element and it ultimately lead us to formulate an efficient FE cable element for the geometrically nonlinear analysis of two- and three- dimensional cable structures.

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