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Power Allocation and Capacity Analysis of Secondary User in Heterogeneous Spectrum Sharing Systems

Sungmook Lim
Dept. of Electronic Engineering
Korea National University of Transportation, 50 Daehak-ro, Chungju-si, Chungbuk, 380-702 Korea

Kyunbyoung Ko
Dept. of IT Convergence & Dept. of Control and Instrumentation Engineering
Korea National University of Transportation, 50 Daehak-ro, Chungju-si, Chungbuk, 380-702 Korea

ABSTRACT

In this paper, we considered heterogeneous spectrum sharing system where the number of subcarriers of the primary user (PU) was twice as much as that of the secondary user (SU). In this case, due to non-orthogonality and inter-carrier interference (ICI) from SU to PU, it is difficult to satisfy the interference constraint of PU. In order to mitigate ICI and satisfy the interference constraint, we proposed a new transmission scheme of the SU with power allocation scheme. The proposed scheme will only generate subcarrier-by-subcarrier interference. Therefore, it can easily satisfy the interference constraint of the PR and enhance the capacity of the SU. In addition, we derived the ergodic capacity of the SU. Based on numerical results, we confirmed that the proposed schemes could guarantee SU with a reliable capacity while satisfying the interference constraint of the PU. In addition, the derived capacity well matched the numerical results.

***Key words**: Spectrum Sharing System, Heterogeneous System, OFDM, Power Allocation, Capacity.*

1. INTRODUCTION

Recently, spectrum sharing systems have paid much attention as an attractive solution to overcome spectrum scarcity [1], [2]. In spectrum sharing systems, secondary users (SUs) can share the spectrum licensed to primary users (PUs) under the constraint that SUs do not cause severe interference to PUs [3], [4]. The primary receiver (PR) has the interference power constraint and the secondary transmitter (ST) controls its power in order to satisfy the PR's constraint [4], [5].

Conventional research on spectrum sharing systems has mainly focused on the power allocation schemes and the capacity maximization of the SU under the condition of the interference constraint of the PU. Ghasemi and Sousa proposed the power allocation scheme of the SU under average and peak power constraints in various fading environments as well as analyzed the capacity of the SU [4]. In this work, it is assumed that the ST has perfect knowledge of the channel state information (CSI) of the ST-PR link.

In [6] and [7], the authors consider the situation where the ST can only obtain the partial or outdated CSI of the ST-PR link because of the channel variation and feedback delay. They proposed power allocation schemes of the SU by introducing *interference outage concept* and analyzed the capacity.

All of these works have commonly considered the homogeneous spectrum sharing systems. In other words, both the PU and SU utilize the same bandwidth. When the results of conventional research are applied to orthogonal frequency division multiplexing (OFDM)-based spectrum sharing system, the PU and SU utilize the same bandwidth, tone spacing (*i.e.*, the number of subcarriers) and the length of the cyclic prefix (CP).

However, it is not guaranteed that the system configuration of the PU is the same as that of the SU. In homogeneous spectrum sharing systems based on OFDM, the tone spacing and the CP length of the PU could be different from those of the SU. In this case, conventional power allocation schemes cannot be applied, because inter-carrier interference (ICI) between the PU and the SU occurs due to breaking the orthogonality. Therefore, a novel power allocation scheme suitable for heterogeneous spectrum sharing systems is necessary. In [8], in homogeneous spectrum sharing systems where the tone spacing of the SU is less than that of the PU, a power allocation scheme of the SU was proposed and the capacity was analyzed.

In this paper, we consider the heterogeneous spectrum sharing system where the tone spacing of the PU is less than

* *Corresponding author, Email: kbko@ut.ac.kr*
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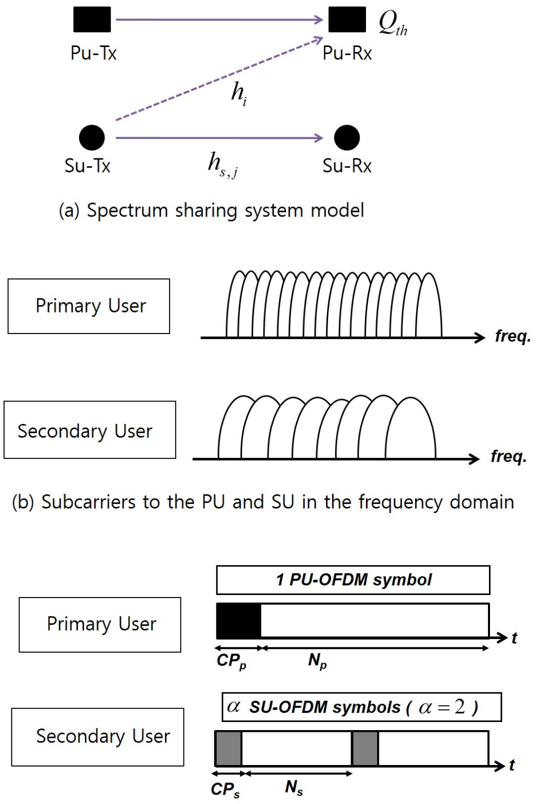
that of the SU (*i.e.*, the number of subcarriers of the PU is more than that of the SU). In this case, due to non-orthogonality between the PU and the SU, ICI from all subcarriers of the SU to all subcarriers to the PU occurs, resulting in violating the interference constraint of the PU. We propose the transmission



(a) Spectrum sharing system model

(b) Subcarriers to the PU and SU in the frequency domain

(c) OFDM symbol of the PU and SU in the time domain

Fig. 1. Heterogeneous Spectrum Sharing System Model

scheme of the SU to mitigate ICI from the SU to the PU. In addition, we propose a power allocation scheme to satisfy the interference constraint of the PU. Because the proposed transmission scheme incurs only subcarrier-by-subcarrier ICI, it enables a simple power allocation of the SU. Furthermore, we derive the ergodic capacity of the SU. From numerical results, we confirm that the proposed schemes can guarantee the reliable capacity of the SU, while satisfying the interference constraint of the PU. In addition, the derived capacity well matches the numerical result.

2. SYSTEM MODEL OF HETEROGENEOUS SPECTRUM SHARING SYSTEMS

As shown in Fig. 1(a), we consider the heterogeneous spectrum sharing system consisting of a pair of the Primary transmitter (PT) and PR and a pair of ST and the secondary receiver (SR). The PT is assumed to be located far from the SR, so interference from the PT to the SR is ignored. Both the PU and the SU use OFDM, and the numbers of subcarriers of the PU and the SU are N_p and N_s, respectively. N_p is more than N_s ($N_p = \alpha N_s$ and α is integer larger than 1. In this paper, without loss of generality, α is assumed to be 2, but it can be easily expanded to the different value of α .

When $\alpha = 2$, symbols of the PU and SU are mapped into subcarriers in the frequency domain, as seen in Fig. 1(b). In the PT, N_p -point inverse fast Fourier transform (IFFT) is performed and CP_p is inserted to a PU-OFDM symbol where $CP_p = N_p / 4$. On the other hand, in the ST, N_s -point IFFT is performed, and then CP_s is inserted to a SU-OFDM symbol where $CP_s = N_s / 4$. It is noted that the ST transmits two SU-OFDM symbols during the transmission period of a PU-OFDM symbol because $(N_p + CP_p) = 2 \times (N_s + CP_s)$, as depicted in Fig 1(c).

The transmitted PU-OFDM symbol after the CP insertion in time domain, $x_p[n]$ can be expressed as follows:

$$x_p[n] = \sum_{k=0}^{N_p-1} X_{p,k} e^{j2\pi kn/N_p}, \ -CP_p \le n \le N_p - 1 \quad (1)$$

where $X_{p,k}$ is a binary phase shift keying modulation (BPSK) symbol (-1 or 1) allocated to the $k-th$ subcarrier of the PU. On the other hand, the ST sequentially transmits two SU-OFDM symbols during a PU-OFDM symbol transmission, and the transmitted SU-OFDM symbols after the CP insertion in time domain, $x_s[n]$ can be expressed as follows:

$$\begin{aligned} x_s[n] &= \sum_{j=1}^{2} \sum_{k=0}^{N_s-1} \sqrt{P_{s,k,j}} X_{s,k,j} e^{j2\pi k(n+(j-1)N_s)/N_s} \\ &= \sum_{k=0}^{N_s-1} (\sqrt{P_{s,k,1}} X_{s,k,1} + \sqrt{P_{s,k,2}} X_{s,k,2}) e^{j2\pi kn/N_s} \quad (2) \\ & , -CP_s \le n \le N_s - 1 \end{aligned}$$

$X_{s,k,j}$ and $P_{s,k,j}$ are a BPSK symbol (-1 or 1) and its transmission power allocated to the $k-th$ subcarrier of the $j-th$ SU-OFDM symbol of the SU, respectively. The PR has the interference constraint, Q_{th} , at every subcarrier, so the ST should control its transmission power, $P_{s,k,j}$, in order to satisfy the interference constraint of the PR.

In the SR, the received signal of the $j-th$ SU-OFDM symbol, $y_{s,j}[n]$, is

$$y_{s,j}[n] = x_{s,j}[n] * h_{s,j}[l] + w_{s,j}[n], \ l = 1, 2, \cdots L \quad (3)$$

where $x_{s,j}[n]$ is the transmitted $j-th$ SU-OFDM symbol, $w_{s,j}[n]$ is the additive white Gaussian noise (AWGN) with zero mean and σ_s^2 variance, and $h_{s,j}[l]$ is the $l-th$ coefficient of L-tap Rayleigh fading channels. L is less than CP_s, so there is no inter-symbol interference (ISI). After removing the CP, N_s-point FFT is performed and $\hat{X}_{s,k,j}$ is detected at the $k-th$ subcarrier of the $j-th$ SU-OFDM symbol of the SU. Because there is no ISI, $\hat{X}_{s,k,j}$ can be equivalently expressed as

$$\hat{X}_{s,k,j} = P_{s,k,j}X_{s,k,j}H_{s,k,j} + W_{s,k,j} \tag{4}$$

where $H_{s,k,j}$ and $W_{s,k,j}$ are the channel coefficient and the noise coefficient of the $k-th$ subcarrier of the $j-th$ SU-OFDM symbol of the SU in frequency domain.

On the other hand, the two SU-OFDM symbols transmitted by the ST cause the interference at the PR. The interference signal during receiving a PU-OFDM symbol at the PR, $I[n]$, can be expressed

$$I[n] = x_s[n] * h_i[l],\ l = 1, 2, \cdots L_i \tag{5}$$

where $h_i[l]$ is the $l-th$ coefficient of L_i-tap Rayleigh fading channels between the ST and the PR. L_i is less than CP_p. In the PR, after removing the CP, N_p-point FFT is performed and then, $X_{s,k,j}$ interferes with all subcarriers of the PU because of non-orthogonality between the PU and the SU. Therefore, it is very difficult to satisfy the interference constraint of the PU by controlling $P_{s,k,j}$ and even if it is, the capacity of the SU is expected to be very low.

3. THE PROPOSED TRANSMISSION SCHEME AND POWER ALLOCATION OF SU

3.1 The Proposed Transmission Scheme

When the PU and the SU utilize the different number of subcarriers, severe ICI from the ST to the PR occurs [9]. As mentioned above, because the length of the PU-OFDM symbol is twice as much as that of the SU-OFDM symbol, two SU-OFDM symbols concurrently interferes with one PU-OFDM symbol. It causes non-orthogonality between the signals of the PU and SU, so a symbol loaded to the $k-th$ subcarrier of the SU affects all subcarriers of the PU as interference. It is difficult to satisfy the interference constraint of the PU as well as it causes the capacity degradation of the PU and the SU.

In order to mitigate ICI due to non-orthogonality and satisfy the interference constraint of the PU in a simple way, we propose a novel transmission scheme of the SU. When $\alpha = 2$, the PT generates the first PU-OFDM symbol, $x_{s,1}[n]$, and then the second PU-OFDM symbol, $x_{s,2}[n]$, is duplicated with $x_{s,1}[n]$ after cyclic shift. As shown in Fig. 2, when the samples of $x_{s,1}[n]$ is of the form $x_{s,1}[n] = \begin{bmatrix} A & B & C & D \end{bmatrix}$ in time domain, $x_{s,2}[n]$ is designed to be of the form $x_{s,1}[n] = \begin{bmatrix} B & C & D & A \end{bmatrix}$ by left cyclic shift with the length

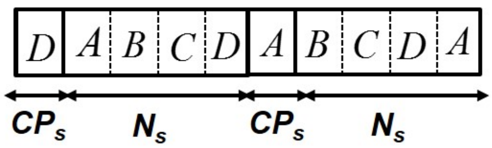


Fig. 2. Two SU-OFDM Symbols Generated by the Proposed Transmission Scheme

of CP_s.

After inserting CP_s into $x_{s,1}[n]$ and $x_{s,2}[n]$, respectively, the transmitted SU-OFDM symbols, $x_s[n]$, is of the form as

$$x_s[n] = \begin{bmatrix} D & A & B & C & D & A & B & C & D & A \end{bmatrix} \tag{6}$$

3.2 The Proposed Power Allocation

In (5) and (6), $x_s[n]$ interferes with the PR. After removing

CP_p which is twice as much as CP_s, the remaining interference part is of the form as

$$I[n] = \begin{bmatrix} B & C & D & A & B & C & D & A \end{bmatrix} \tag{7}$$

In (7), $I[n]$ has the repeated pattern in time domain, so the interference occurs to the even number of subcarriers in frequency domain after N_p-point FFT due to the characteristic of FFT. In addition, the interference of the $m-th$ subcarrier of the PR occurs from only $X_{s,k,j}$ where $j \in \{1,2\}$ and $k = m/2$ and k is integer.

Therefore, the interference model at the $m-th$ subcarrier of the PR in frequency domain can be equivalently expressed

$$Y_m = \left(\sqrt{P_{s,k,1}}X_{s,k,1} + \sqrt{P_{s,k,2}}X_{s,k,2}\right)H_{i,m} \tag{8}$$

where $k = m/2$ and k is integer, and $H_{i,m}$ is the channel coefficient between the $m-th$ subcarrier of the PU and the $k-th$ subcarrier of the SU in the frequency domain. $H_{i,m}$

can be obtained from FFT of $h_i[l]$. It is noted that ICI only occurs in the manner of subcarrier-by-subcarrier and there is no ICI due to adjacent subcarriers in the proposed transmission scheme of the ST.

In this case, interference power can be simply expressed

$$P_{i,m} = \left(P_{s,k,1}+P_{s,k,2}\right)G_{i,m} \tag{9}$$

where $G_{i,m} =| H_{i,m} |^2$. $G_{i,m}$ is the exponential distribution with unit mean, so its probability density function (pdf) is

$$f_{G_i}(g_i) = e^{-g_i} \tag{10}$$

When the interference constraint at the $m-th$ subcarrier of the PR is denoted as Q_{th}, the proposed power allocation of the $k-th$ subcarrier of the ST is as follows:

$$P_{s,k} = P_{s,k,1} = P_{s,k,2} = Q_{th} / 2G_{i,m} \tag{11}$$

The proposed power allocation scheme can satisfy the interference constraint of each subcarrier of the PR.

3.3 Capacity Analysis of the SU

In the proposed transmission scheme, the ST transmits a SU-OFDM symbol twice in a duplicated form. Therefore, two identical symbols at the $k-th$ subcarrier in the SR are detected by using maximal ratio combining (MRC).

The average capacity per subcarrier of the SR can be expressed as:

$$C = \iiint_{g_{i,m},g_{s,k,1},g_{s,k,2}} \log\left(1+\frac{(g_{s,k,1}+g_{s,k,2})Q_{th}}{2g_{i,m}\sigma^2}\right) \times f(g_{i,m})f(g_{s,k,1})f(g_{s,k,2})dg_{i,m}dg_{s,k,1}dg_{s,k,2} \tag{12}$$

where $g_{s,k,1}$ and $g_{s,k,2}$ are $g_{s,k,1} =| H_{s,k,1} |^2$ and $g_{s,k,2} =| H_{s,k,2} |^2$, and $H_{s,k,1}$ and $H_{s,k,2}$ are the channel coefficient of the $k-th$ subcarrier of the SU in the frequency domain, respectively. $H_{s,k,1}$ and $H_{s,k,2}$ can be obtained from FFT of $h_{s,1}[l]$ and $h_{s,2}[l]$, respectively. Because $H_{s,k,1}$ and $H_{s,k,2}$ are Rayleigh fading channel coefficient, $g_{s,k,1}$ and $g_{s,k,2}$ are the exponential distributions with unit mean, so their probability density functions (pdf) are

$$f(g_{s,k,1}) = e^{-g_{s,k,1}} \tag{13}$$

$$f(g_{s,k,2}) = e^{-g_{s,k,2}} \tag{14}$$

In order to obtain the solution of (12) in a closed form, we need to get the joint pdf of $g_{i,m}$, $g_{s,k,1}$ and $g_{s,k,2}$. Let $g_1 = g_{s,k,1} + g_{s,k,2}$, then g_1 is the sum of two independent and identical random variables followed by exponential distribution with unit mean. Therefore, g_1 has a gamma distribution with $\beta = 2$ and its pdf is [10]

$$f(g_1) = g_1^{\beta-1}\frac{e^{-g_{s,k,1}}}{\Gamma(\beta)} = g_1 e^{-g_1} \tag{15}$$

Then, (12) can be simplified

$$\begin{aligned} C &= \iint_{g_{i,m},g_1} \log\left(1+\frac{g_1 Q_{th}}{2g_{i,m}\sigma^2}\right)f(g_{i,m})f(g_1)dg_{i,m}dg_1 \\ &= \int_0^\infty \log\left(1+\frac{g_2 Q_{th}}{2\sigma^2}\right)f(g_2)dg_2 \end{aligned} \tag{16}$$

where $g_2 = g_1 / g_{i,m}$. Because g_1 and $g_{i,m}$ are independent, so the pdf of g_2 can be obtai

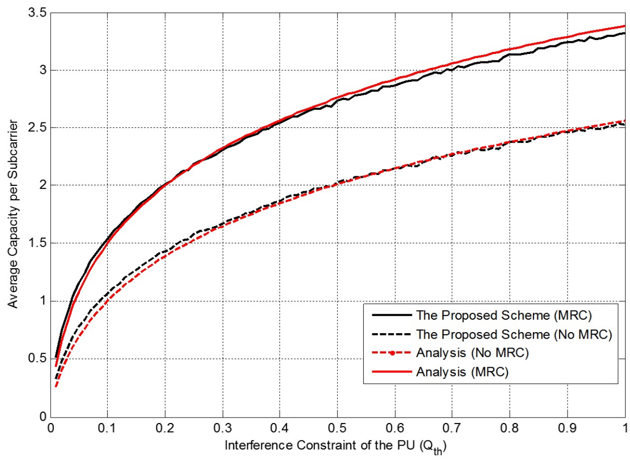


Fig. 3. The ergodic capacity of the SU as the interference constraint of the PU varies

$$\begin{aligned} f(g_2) &= \int_0^\infty g_{i,m} f_{g_1}(g_{i,m}g_1) f_{g_{i,m}}(g_{i,m})dg_{i,m} \\ &= \int_0^\infty g_{i,m}g_{i,m}g_2 e^{-g_{i,m}g_2} e^{-g_{i,m}} dg_{i,m} \\ &= \frac{2g_2}{(g_2+1)^3} \end{aligned} \tag{17}$$

From (16) and (17), the ergodic capacity of the SU can be obtained

$$C = \int_0^\infty \log\left(1+\frac{g_2 Q_{th}}{2\sigma^2}\right)\frac{2g_2}{\left(g_2+1\right)^3}\,dg_1$$
$$= \frac{Q_{th}\left(-\sigma^2+2\sigma^2 Q_{th}-Q_{th}{}^2\right)\left(\log\sigma^2-\log Q_{th}\right)}{\left(\sigma^2-Q_{th}\right)^2} \quad (18)$$

4. SIMULATION RESULTS

In this section, we show the capacity per subcarrier of the SU as the signal to noise ratio (SNR) and the interference constraint (Q_{th}) at the PR and compare the analyzed ergodic capacity and the numerical result.

For simulation, we consider $N_p = 32$, $N_s = 16$ (*i.e.*, $\alpha = 2$), $CP_p = N_p / 4 = 8$ and $CP_s = N_s / 4 = 4$. The received SNR is assumed to be 10dB. The channel between the ST and SR is assumed to be 2-tap Rayleigh fading channel and it does not vary during one SU-OFDM symbol transmission. On the other hand, the channel between the ST and the PR is assumed to be 4-tap Rayleigh fading channel and it does not vary during one PU-OFDM symbol transmission.

In Fig. 3, we show the ergodic capacity of the SU as Q_{th} varies from 0.01 to 1. In the proposed transmission scheme, interference from the ST to the PR occurs in the subcarrier-by-subcarrier manner, so the proposed power allocation can satisfy the interference constraint at the PR. Furthermore, the capacity of the SU increases as Q_{th} increases. It is because high Q_{th} means the PR is robust to interference, so the ST can allocate higher power.

In addition, the derived capacity well matches the numerical result. In the proposed transmission scheme, the ST transmits the identical SU-OFDM symbol twice, so there is the capacity loss. Nevertheless, the proposed transmission scheme and the power allocation scheme can satisfy the interference constraint of the PR. Furthermore, the PR performs the MRC, so the capacity can be enhanced by increasing SNR.

5. CONCLUSIONS

In this paper, we consider the heterogeneous spectrum sharing system where the number of subcarriers of the PU is twice as much as that of the SU. In this case, due to of non-orthogonality, ICI from the SU to the PU occurs, which makes it difficult to satisfy the interference constraint of the PU. In order to mitigate ICI and satisfy the interference constraint, we propose a new transmission scheme of the SU. In the proposed transmission scheme, after a PU-OFDM symbol is generated, the identical symbol with cyclic shift is sequentially transmitted. The proposed scheme generates only subcarrier-by-subcarrier interference, so it enables the simple power allocation to satisfy the interference constraint. Based on the proposed transmission scheme, we also propose the simple power allocation to satisfy the interference constraint of the PU. In addition, we derive the ergodic capacity of the SU in a closed form. Through the proposed schemes, the SU can share the licensed bandwidth of the PU and it can enhance the capacity, while satisfying the interference constraint.

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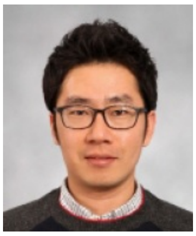

Sungmook Lim

He received the B.S. and Ph.D. degrees in Electrical and Electronic Engineering at Yonsei University, Seoul, Korea in 2005 and 2012, respectively. From September 2012 to March 2014, he was a Postdoctoral Fellow at Yonsei University where his research interests were in 5G wireless communications. Since March 2014, he joined the Department of Electronics Engineering at Korea National University of Transportation as an Assistant Professor. His current research interests include the field of 5G wireless communications focusing on multicarrier and multi-antenna systems, cooperative relaying, and ITS.

Kyunbyoung Ko

He (corresponding author) received the B.S., M.S., and Ph.D. degrees in Electrical and Electronic Engineering at Yonsei University, Seoul, Korea in 1997, 1999, and 2004, respectively. From March 2004 to February 2007, he was a senior engineer in Samsung Electronics, Suwon, Korea where he developed Mobile WiMAX systems for broadband wireless services. Since March 2007, he joined the Department of Control and Instrumentation Engineering at Korea National University of Transportation as an Associate Professor. His current research interests include the field of wireless communications focusing on multicarrier and multi-antenna systems, cooperative relaying, and ITS.