Neutrality between a Vertically Integrated Cable Provider and an Over-the-Top Video Provider

Wei Dai, Ji Won Baek, and Scott Jordan

Abstract: We are concerned with whether a vertically integrated broadband and content provider can unreasonably advantage itself over competing content providers, either by selling quality-of-service (QoS) to content providers at unreasonably high prices, or by refusing to provide access to QoS to competing content. We address this question by modeling the competition between one such vertically integrated provider and one over-the-top (OTT) content provider. The broadband provider decides whether to deploy QoS, and if so it also determines the QoS price if sold to either the OTT content provider or directly to users. We analytically determine when the broadband provider will sell QoS and when the OTT content provider or users will purchase QoS. We characterize the optimal QoS and video service prices. The Internet service provider (ISP)'s market share increases with the difference in the value of the two video services and decreases with the difference in the corresponding costs. Numerical results illustrate the effect of QoS price on content price, the variation of prices and profit with QoS price, and the variation of prices and market shares with the benefit of QoS. The ISP may sell QoS to users at a lower price than when QoS is sold to the OTT provider.

Index Terms: Net neutrality, pricing, quality-of-service (QoS).

1. INTRODUCTION

HISTORICALLY many broadband providers have offered both packet-switched broadband Internet service and circuit-switched video service to end users. However, most broadband providers are now moving their circuit-switched video service to video-over-IP [1], and are multiplexing this video-over-IP traffic with their broadband Internet traffic. These vertically integrated video services are thus competing with over-the-top (OTT) video services, both for customers and for network capacity.

Broadband providers are deploying quality-of-service (QoS) technologies to improve the performance of video-over-IP, either by reserving network resources for their video traffic (e.g., using IntServ) or by prioritizing their video traffic over other traffic (e.g., using DiffServ) [2] 1. Broadband providers may have an incentive to sell QoS to OTT video service providers or directly to end users. Alternatively, vertically integrated broadband and video service providers may have an incentive to refuse to provide access to QoS to competing video service providers, if that decision will result in a higher total profit for its broadband and video services.

One of the questions driving the net neutrality debate has been whether a vertically integrated broadband and content provider can unreasonably advantage itself over competing content providers either by selling QoS to content providers at unreasonably high prices or by refusing to provide access to QoS to competing content.

There is a small academic literature on the impact of QoS pricing on the competition between a vertically integrated broadband provider’s content service and OTT content services. One set of papers explores competition from an economics viewpoint. Kocsis and de Bijl [3] apply economic concepts to explore whether bargaining power allows broadband providers to extract surplus from OTT service providers through QoS. However, no mathematical model is proposed. Weisman and Kulick [4] similarly apply economic concepts to explore price discrimination by a broadband provider. They claim that the weight of the economic evidence suggests that both differential pricing and price discrimination by broadband providers toward content providers increases both static and dynamic efficiency, and are thus likely welfare enhancing. However, again no mathematical model is proposed.

Another set of papers focuses on OTT service providers. Baldry et al. [5] argue that the OTT providers are taking advantage of flat rate end user broadband Internet access to provide services that compete with the broadband provider’s content services. They believe that addressing this issue will require regulation, and they propose some regulatory approaches. However, no mathematical analysis is given to validate and solve the competition issues. Nooren et al. [6] use a systematic value chain analysis to investigate how net neutrality interacts with video distribution. However, the focus is not on competition between OTT providers and a broadband provider’s integrated services, and no mathematical model is proposed to evaluate the video distribution value chain.

Another set of papers analyzes vertical integration or traffic discrimination from the Internet policy perspective. Jordan [7] provides an analysis of potential discrimination using QoS. He proposes a prohibition on unreasonable discrimination, which would allow charging users for QoS but place limits on charging content providers for QoS. Grunwald [8] analyzes the possibility that Internet service providers (ISPs) may use QoS or traffic prioritization to discriminate certain services against others, and lead to unfair competition. Waterman and Choi [9] argue that vertically integrated broadband providers have plausible incentives to favor their affiliated content and to restrict entry of OTT content providers. As with the literature discussed above, none

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1 In the latter case, we presume that video packets are marked as high priority by the content provider and that the broadband provider respects this marking if either the content provider or the end user has purchased QoS.
of these papers propose a mathematical model of the competition.

As a complement to these approaches, here we postulate that a simple mathematical model may lend further insight into whether a vertically integrated broadband and content provider can unreasonably advantage itself over competing content providers either by selling QoS to content providers at unreasonably high prices or by refusing to provide access to QoS to competing content. We are interested in when a broadband provider will find it profitable to deploy QoS, and when it does, whether it will choose to sell QoS. If a broadband provider charges content providers for QoS, we are interested in when a content provider will choose to purchase QoS. If a broadband provider charges end users directly for QoS, we are interested in which users will choose to purchase QoS. We are particularly interested in a comparison of these two approaches to charging for QoS.

The mathematical model presented in Section II considers competition between one vertically integrated broadband and video service provider (henceforth referred to as the ISP) and one OTT video service provider. The two video service providers offer services that differ both by the amount of content they provide and the type of content (e.g., the number of movies versus television programs). End users are similarly differentiated by their preference for the type of content. End users decide which service to subscribe to (if any), so as to maximize their surplus.

The ISP also decides whether to deploy QoS, which incurs an incremental network cost per user. QoS, if used, increases user utility proportionally. We consider two types of markets for the sale of QoS: A market in which the broadband provider may charge the OTT content provider for QoS, and an alternative market in which the broadband provider may only charge end users directly for QoS. The ISP sets its video service price and the QoS price (if it deploys QoS) to maximize its profit, defined as the sum of its video service revenue and its QoS revenue (if any) minus the corresponding incremental costs. The OTT content provider sets its video service price to maximize its profit, defined as its video service revenue minus the corresponding incremental cost and the QoS cost (if any).

We both derive analytical results and provide numerical results. For the analytical results, we must make a set of assumptions to simplify the model and the range of parameters considered. In Section III, we analyze the duopoly competition game under different QoS prices. The price and market share of the ISP and of the OTT content provider at the Nash equilibrium are derived. In Section IV, we present numerical results based on current Internet statistics. In addition to verifying the analytical results, we illustrate when the broadband provider does not sell QoS; the effect of QoS price on content price; the variation of each content provider’s price and profit with QoS price; and the variation of prices and market shares with the benefit of QoS.

II. MARKET MODELS FOR QoS

In this section, we formulate a mathematical model that considers competition between one vertically integrated broadband and video service provider (denoted ISP/CP1) and one OTT video service provider (denoted CP2). The two video service providers offer services that differ both by the amount of content they provide and the type of content (e.g., the number of movies versus television programs). A common economic model for horizontal product differentiation is the Hotelling model [10], which we use here. Content provider $j$ is characterized by a pair of parameters $(H_j, V_j)$, where $H_j \in [0, 1]$ characterizes the type of content and $V_j$ is the maximum amount that any user would pay for the service. $V_j$ can also characterize the amount of content, since a user is willing to pay more for the video service with more content. User $i$ is similarly characterized by her preference, denoted $\theta_i \in [0, 1]$, for the type of content. The distance $|\theta_i - H_j|$ between the preference of user $i$ and the type of content provided by CP $j$ is the basis for the valuation of user $i$ for the video service provided by CP $j$. The utility of user $i$, when subscribing to CP $j$ without QoS, is given by:

$$ U_{i,j}^b = \max (V_j - t_j|\theta_i - H_j|, 0) $$

where $t_j > 0$ is the marginal penalty on user utility caused by the deviation of CP $j$’s content type from user $i$’s preference. If user $i$ is served with QoS, the network performance experienced by the user is improved. It is common to assume that user utility is an increasing function of network performance, with a scaling factor indicating users’ willingness to pay [11], [12], which is $U_{i,j}^q$ in our model. Denote $r$ by the increased percentage from the performance function after adopting QoS. We can derive that user utility is also increased by $r$. Thus, the utility of user $i$, when subscribing to CP $j$ with QoS, is given by:

$$ U_{i,j}^q = \max ((1 + r)(V_j - t_j|\theta_i - H_j|), 0). $$

A. ISP Chooses not to Deploy QoS

We first consider the competition between ISP/CP1 and CP2 when the ISP chooses not to deploy QoS. Denote the price charged by CP $j$ by $P_j$. The surplus of user $i$, when subscribing to CP $j$ without QoS, is then defined as utility minus the payment to CP $j$:

$$ S_{i,j} = U_{i,j}^b - P_j = \max (V_j - t_j|\theta_i - H_j|, 0) - P_j $$

Denote the content provider choice of user $i$ by $T_i \in \{0, 1, 2\}$, where $T_i = 0$ indicates user $i$ chooses not to subscribe to any CP. End users decide which service to subscribe to (if any), so as to maximize their surplus:

$$ T_i = \arg \max_j S_{i,j}. \quad (1) $$

Denote the set of users who subscribe to CP $j$ by $\Pi_j = \{i | T_i = j\}$. Thus the demands for both content providers are functions of the set of prices: $N_j(P_1, P_2) = |\Pi_j|$.  

3We ignore the user payment to the ISP for internet access, since it affects neither the competition for video service nor the sale of QoS.

4For convenience, we denote $V_0 = 0$ and $P_1 = 0$. 

2If an ISP deploys QoS and attempts to sell it to the OTT content provider, then the ISP is presumed to use it for its own video service.
Denote the incremental cost to serve each user who subscribes to CP $j$ without QoS by $C_j$.\(^5\) Denote the profit earned by CP $j$ by $\pi_j = N_j(P_j - C_j)$\(^6\). Both content providers maximize their profits by controlling their prices: $\max \pi_j$.

The competition between CP1 and CP2 through setting $P_1$ and $P_2$ is a continuous game, because $P_1$ and $P_2$ can be any positive values. Thus, we focus on the local Nash equilibriums\([13]\) in these continuous games which are determined by the following derivative-based conditions:

**Definition 1:** A price set $(P_1^*, P_2^*) \in \mathbb{R}^+ \times \mathbb{R}^+$ is a local Nash equilibrium if there exist open sets $P_1 \subset \mathbb{R}^+$, $P_2 \subset \mathbb{R}^+$ such that $P_1^* \in P_1$, $P_2^* \in P_2$,

$$\begin{align*}
\pi_1(P_1^*, P_2^*) &\geq \pi_1(P_1, P_2^*), \forall P_1 \in P_1 \setminus \{P_1^*\}, \\
\pi_2(P_1^*, P_2^*) &\geq \pi_2(P_1^*, P_2), \forall P_2 \in P_2 \setminus \{P_2^*\}.
\end{align*}$$

If $P_1 = \mathbb{R}^+$ and $P_2 = \mathbb{R}^+$, then $(P_1^*, P_2^*)$ is a global Nash equilibrium.

A local Nash equilibrium $(P_1^*, P_2^*)$ satisfies $\partial \pi_1/\partial P_1|_{(P_1,P_2)} = 0$ and $\partial \pi_2/\partial P_2 < 0$. The local Nash equilibrium derived in the following model analysis may also be a global Nash equilibrium. We will illustrate these equilibriums in more detail in the next section.

**B. QoS Sold to Content Provider**

We next consider a market in which the ISP deploys QoS and uses it for its own integrated video service CP1. The ISP may decide to offer QoS to CP2 at a price $P_q$ per user, or it may decide not to offer QoS to CP2 and to use QoS solely for CP1 (denoted by $P_q = \infty$). If the ISP offers QoS to CP2, then CP2 decides whether or not to purchase QoS for all of its users; $Q = 0$ denotes that CP2 does not purchase QoS and $Q = 1$ denotes CP2 purchases QoS. We will consider the ISPs and CP2’s choices in Sections III and IV.

User $i$’s surplus when subscribing to CP $j$ is thus given by:

$$S_{i,j} = U_{i,j}^r - P_j,$$

As before, end users decide which service to subscribe to (if any), so as to maximize their surplus according to (1). Denote the incremental network cost per user to deploy QoS by $d$. In the short term (i.e., given $P_1$ and $Q$), ISP/CP1 and CP2 compete by setting $P_1$ and $P_2$ to maximize their profits $\pi_1 = N_1(P_1 - C_1 - d) + QN_2(P_2 - d)$ and $\pi_2 = N_2(P_2 - C_2 - QP_2)$, respectively. The first term in $\pi_1$ is the profit earned from CP1’s users; the second term in $\pi_1$ is the profit, if any, earned from selling QoS to CP2.

The profit of ISP/CP1 and the profit of CP2 at the local Nash equilibrium $(P_1^*, P_2^*)$ by $\pi_1^*$ and $\pi_2^*$, respectively. $\pi_1^*$ and $\pi_2^*$ are functions of $P_2^*$ and $Q$. Thus, in the long term (considered in Sections III and IV), the ISP sets the QoS price $P_q$ to maximize profit, and CP2 decides whether or not to buy QoS: $\max \pi_1^*, \max \pi_2^*$.

This sequential game can be formulated in four stages as follows. Stage 1), ISP/CP1 sets QoS price $P_q$; Stage 2), CP2 decides whether or not to buy QoS (i.e., $Q$); Stage 3), ISP/CP1 and CP2 set content prices $P_1$ and $P_2$; Stage 4 users decide which content provider to subscribe to (i.e., $T_i$). Thus, Stage 1) and Stage 2) formulate a typical Stackelberg game, where ISP/CP1 is the leader and CP2 is the follower. The Hotelling competition in Stage 3 and Stage 4) formulate a sub-game of the whole sequential game.

**C. QoS Sold to Users**

We finally consider a market in which the ISP deploys QoS and offers it to end users at price $P_q$. Subscribers to either content provider can experience the benefit from QoS only if they directly buy QoS in addition to the video service. The surplus of user $i$, when subscribing to CP $j$, is thus the maxima of the surplus when buying QoS and the surplus when not buying QoS:

$$S_{i,j} = \max(U_{i,j}^b - P_j, U_{i,j}^r - P_j - P_q).$$

In the short term (given $P_q$), as before end users decide which service to subscribe to (if any) and whether or not to buy QoS, so as to maximize their surplus according to (1). Denote the number of users who buy QoS and subscribe to CP $j$ by $N_{q,j}$. ISP/CP1 and CP2 compete by setting $P_1$ and $P_2$ to maximize profits $\pi_1 = N_1(P_1 - C_1) + (N_1^q + N_2^q)(P_2 - d)$ and $\pi_2 = N_2(P_2 - C_2)$, respectively. In the long term, ISP maximizes the profit of ISP/CP1 by controlling $P_q$: $\max \pi_1^*$. This sequential game can thus be formulated in three stages as follows. Stage 1), ISP/CP1 sets QoS price $P_q$; Stage 2), ISP/CP1 and CP2 set content prices $P_1$ and $P_2$; Stage 3) users decide which content provider to subscribe to (i.e., $T_i$) and whether or not to buy QoS. Similarly, the Hotelling competition in Stage 2) and Stage 3) formulate a sub-game of the whole sequential game.

**III. MARKET SHARES UNDER DIFFERENT QOS PRICES**

In this section, we analyze the duopoly competition game under a fixed QoS price $P_q$, i.e., competition through the video service prices $P_1$ and $P_2$. The price, market share and profit of the ISP and of the OTT content provider at the Nash equilibrium are derived. For these analytical results, we must make a set of assumptions to simplify the model and the range of parameters considered. Some of these assumptions will be relaxed in the numerical results section below.

**Assumption A:** $H_1 = 0$, $H_2 = 1$, $t_1 = t_2 \equiv t$.

This places CP1 and CP2’s content types at the extreme points, and user utility decreases with the same marginal penalty under both content providers, when the content type deviates from a user’s preference.

**Assumption B:** User preferences $\theta$ are uniformly distributed between 0 and 1, i.e., $f(\theta) = 1$, for $0 \leq \theta \leq 1$.

It is common in economic models to assume a uniform distribution of user preferences. Since the range $0 \leq \theta \leq 1$ is arbitrary, the distribution does not by itself matter.

**Assumption C:** $V_1 + V_2 > 3d + C_1 + C_2$ and $|V_1 - V_2| - (C_1 - C_2) < 3d$. 
When QoS is not deployed, the condition $V_1 + V_2 > 3t + C_1 + C_2$ assures that the maximum user willingness to pay (i.e., $V_j$) is high enough such that each user will subscribe to either CP1 or CP2 (i.e., full market coverage of users). The condition $|V_1 - V_2| - (C_1 - C_2) < 3t$ or $|V_1 - V_1| - (V_2 - C_2) < 3t$ assures that no content provider can monopolize the market. CP $j$, with higher maximum user willingness to pay (i.e., larger $V_j$) and lower cost (i.e., smaller $C_j$), has an advantage over its competitor. Thus, the term $V_1 - C_1$ can be used to indicate the competitive power of CP $j$. This condition requires that the advantage of CP $j$ over its competitor should not exceed a certain threshold that is $3t$. Otherwise, one CP’s competitive power is so large that it is able to monopolize the market.

**Assumption D:** $(r^2 + 2r + 3)V_1 + (2r + 3)V_2 > (2r^2 + 9r + 9)t + (3 + r)(C_1 + d) + (3 + 2r)C_2$ and $-(2 + r)t < \frac{1}{(1 + r)V_1 - V_2} - \frac{(C_1 + d - C_2)}{(3 + 2r)t}$.

When the ISP uses QoS solely for CP1’s subscribers, this assumption assures that 1) every user will subscribe either to CP1 or CP2; and 2) no content provider can monopolize the market.

**Assumption E:** $(1 + r)(V_1 + V_2) > 3(1 + r)t + (C_1 + C_2)$ and $|(1 + r)| - (V_1 - V_2) - (C_1 - C_2) < 3(1 + r)t$.

When QoS is provided by the ISP for free, this assumption assures that 1) every user will subscribe to either CP1 or CP2; and 2) no content provider can monopolize the market.

**Assumption F:** $V_1 \geq V_2$, $C_1 \geq C_2$.

This assumption restricts the range of the parameters considered in the analysis to the case in which the ISP provides a video service that is of higher value to its customers than the OTT’s service is to its customers ($V_1 \geq V_2$), and in which its cost per subscriber is correspondingly higher.

**Assumption G:** $rV_1 \geq d$, $rV_2 \geq d$.

This assumption assures that if QoS is offered at cost directly to users, there are some subscribers of each content provider that would purchase it.

### A. ISP Chooses not to Deploy QoS

With these assumptions, we return to consideration of competition between ISP/CP1 and CP2 when the ISP chooses not to deploy QoS. In this market, we can show that there is a partition of users between the two content providers, and that CP1’s market share increases with the difference between the maximum value of the two services ($V_1 - V_2$) and decreases with the difference between their two costs ($C_1 - C_2$). Denote the user preference threshold $\theta^{(b)} = 0.5 + [(V_1 - V_2) - (C_1 - C_2)]/6t$.

**Theorem 1:** Suppose assumptions A–C hold, and QoS is not deployed by the ISP. There exists a global Nash equilibrium such that users with preferences $0 \leq \theta < \theta^{(b)}$ subscribe to CP1 and users with preferences $\theta^{(b)} \leq \theta \leq 1$ subscribe to CP2. The price and profit of each CP are:

\[
\begin{align*}
P_1^{(b)} &= (3t + (V_1 - V_2) + 2C_1 + C_2)/3, \\
P_2^{(b)} &= (3t - (V_1 - V_2) + C_1 + 2C_2)/3, \\
p_1^{(b)} &= (3t + (V_1 - V_2) - (C_1 - C_2))^2/18t, \\
p_2^{(b)} &= (3t - (V_1 - V_2) + (C_1 - C_2))^2/18t.
\end{align*}
\]

The threshold $\theta^{(b)}$ that determines the market shares of ISP/CP1 and CP2 is determined by $(V_1 - V_2)/t$ and $(C_1 - C_2)/t$.

In addition to the aforementioned dependence on the difference between the maximum value of the two services and the difference between their two costs, we also observe that the market share depend on $t$, the marginal penalty on user utility caused by the deviation from user $i$’s preference. Smaller values of $t$ result in video subscribers who are more homogeneous in their preferences. When the difference between CP1’s and CP’s maximum value exceeds the corresponding cost difference (i.e., $V_1 - V_2 > C_1 - C_2$), CP1’s market share is greater than 50% and increasing with the homogeneity of user preferences. Similarly, when $V_1 - V_2 < C_1 - C_2$, CP2’s market share is greater than 50% and increasing with the homogeneity of user preferences.

### B. QoS Sold to Content Provider

We next return to the market in which the ISP deploys QoS and uses it for its own integrated video service CP1. We first consider the situation when CP2 does not purchase QoS from the ISP, either because the ISP decides not to sell QoS to CP2 or because the QoS price is high enough that CP2 chooses not to purchase it. We can show that there is again a partition of users between the two content providers, and that CP1’s market share increases with the difference between the maximum value of the two services, which is now $(1 + r)V_1 - V_2$, and decreases with the difference between their two costs, which is now $C_1 + d - C_2$.

Denote the user preference threshold:

\[
\theta^{(c)} = \frac{3 + r + \frac{1}{6 + 3r}[(1 + r)V_1 - V_2] - \frac{(C_1 + d - C_2)}{(6 + 3r)t}}{6 + 3r}.
\]

**Theorem 2:** Suppose assumptions A, B, D, and G hold, and CP2 does not purchase QoS from the ISP (i.e., $Q = 0$). There exists a global Nash equilibrium such that users with preferences $0 \leq \theta < \theta^{(c)}$ subscribe to CP1 and users with preferences $\theta^{(c)} \leq \theta \leq 1$ subscribe to CP2. The price and profit of each CP are:

\[
\begin{align*}
P_1^{(c)^{(0)}} &= (3 + r + [(1 + r)V_1 - V_2])/2 + (C_1 + d + C_2), \\
P_2^{(c)^{(0)}} &= (3 + r)(1 - [(1 + r)V_1 - V_2] + (C_1 + d + 2C_2)/3, \\
p_1^{(c)^{(0)}} &= (3 + r)(1 - [(1 + r)V_1 - V_2] - (C_1 + d - C_2) - 2(1 + r)t), \\
p_2^{(c)^{(0)}} &= (3 + r)(1 - [(1 + r)V_1 - V_2] - (C_1 + d + C_2) - 2(1 + r)t).
\end{align*}
\]

We then consider the situation when CP2 does purchase QoS from the ISP at a price $P_2^q$ per user. We can show that there is again a partition of users. However, in some cases not all users subscribe to a video service. The cases depend on the QoS price. Denote the following QoS price thresholds, which satisfy $P_2^{(c)^{(1)}} \leq P_2^{(c)^{(2)}} \leq P_2^{(c)^{(3)}} < P_2^{(c)^{(4)}}$:

\[
\begin{align*}
P_2^{(c)^{(1)}} &= (1 + r)V_1 - 3(1 + r)t - (C_1 + C_2)/2, \\
P_2^{(c)^{(2)}} &= (1 + r)(V_1 + 4V_2) - 6(1 + r)t - 2C_1 - 4C_2)/6, \\
P_2^{(c)^{(3)}} &= \min (1 + r)V_2 - C_2, \\
P_2^{(c)^{(4)}} &= (1 + r)V_1 + V_2 - (C_1 + d + C_2) - 2(1 + r)t).
\end{align*}
\]

Denote the following user preference thresholds:

\[
\begin{align*}
\theta^{(c)^{(1)}} &= 0.5 + [(1 + r)V_1 - V_2] - (C_1 - C_2))/6(1 + r)t, \\
\theta^{(c)^{(2)}} &= 1 - ((1 + r)V_2 - (C_2 + P_2^q))/2(1 + r)t, \\
\theta^{(c)^{(3)}} &= \min ((1 + r)V_1 - (C_1 + d))/2(1 + r)t, 1).
\end{align*}
\]
**Theorem 3:** Suppose assumptions A, B, and E–G hold, the ISP may sell QoS to content providers, and CP2 purchases QoS from the ISP (i.e., \( Q = 1 \)). There exists a global Nash equilibrium as follows:

Case 1: When \( P_q < P_q^{(c_1)} \), users with preferences \( 0 \leq \theta < \theta^{(c_1)} \) subscribe to CP1 and users with preferences \( \theta^{(c_1)} < \theta \leq 1 \) subscribe to CP2. The price and profit of each CP are:

\[
\begin{align*}
    P_1^{(c_1)} &= \frac{3}{4}((1+r)\theta + 2)+(1+r)(V_1-V_2)+2C_1+C_2 - P_q, \\
    P_2^{(c_1)} &= \frac{3}{4}((1+r)\theta + 2)+(1+r)(V_1-V_2)+2C_1+C_2 - P_q, \\
    \pi_1^{(c_1)} &= \frac{3}{8}(1+r)((1+r)(V_1-V_2)-(C_1-C_2))^2 + (P_q - d), \\
    \pi_2^{(c_1)} &= \frac{3}{8}(1+r)((1+r)(V_1-V_2)-(C_1-C_2))^2 + (P_q - d).
\end{align*}
\]

Case 2: When \( P_q^{(c_1)} \leq P_q < P_q^{(c_2)} \), users with preferences \( 0 \leq \theta < \theta^{(c_1)} \) subscribe to CP1 and users with preferences \( \theta^{(c_1)} < \theta \leq 1 \) subscribe to CP2. The price and profit of each CP are:

\[
\begin{align*}
    P_1^{(c_2)} &= \frac{3}{4}((1+r)\theta + 2)+\frac{3}{4}((1+r)(V_1-V_2)-(C_1-C_2)), \\
    P_2^{(c_2)} &= \frac{3}{4}((1+r)\theta + 2)+\frac{3}{4}((1+r)(V_1-V_2)-(C_1-C_2)), \\
    \pi_1^{(c_2)} &= \theta^{(c_1)}((P_1^{(c_2)} - P_q) - C_1) + (P_q - d), \\
    \pi_2^{(c_2)} &= (1 - \theta^{(c_1)})((P_2^{(c_2)} - P_q) - C_2).
\end{align*}
\]

Case 3: When \( P_q^{(c_2)} \leq P_q < P_q^{(c_3)} \), users with preferences \( 0 \leq \theta < \theta^{(c_2)} \) subscribe to CP1 and users with preferences \( \theta^{(c_2)} < \theta \leq 1 \) subscribe to CP2. The price and profit of each CP are:

\[
\begin{align*}
    P_1^{(c_3)} &= \frac{3}{4}((1+r)\theta + 2)+(2V_1-V_2)-(C_1-C_2), \\
    P_2^{(c_3)} &= \frac{3}{4}((1+r)\theta + 2)+(2V_1-V_2)-(C_1-C_2), \\
    \pi_1^{(c_3)} &= \theta^{(c_2)}((P_1^{(c_3)} - P_q) - C_1) + (P_q - d), \\
    \pi_2^{(c_3)} &= \frac{3}{4}((1+r)\theta + 2)-(C_1-C_2).
\end{align*}
\]

Case 4: When \( P_q^{(c_3)} \leq P_q < P_q^{(c_4)} \), users with preferences \( 0 \leq \theta < \theta^{(c_3)} \) subscribe to CP1 and users with preferences \( \theta^{(c_3)} < \theta \leq 1 \) subscribe to CP2. The price and profit of each CP are:

\[
\begin{align*}
    P_1^{(c_4)} &= \frac{3}{4}((1+r)\theta + 2)+(1+r)(V_1-V_2)-C_2-P_q, \\
    P_2^{(c_4)} &= \frac{3}{4}((1+r)\theta + 2)+(1+r)(V_1-V_2)-C_2-P_q, \\
    \pi_1^{(c_4)} &= \theta^{(c_3)}((P_1^{(c_3)} - P_q) - C_1) + (P_q - d), \\
    \pi_2^{(c_4)} &= \frac{3}{4}((1+r)\theta + 2) - (C_1-C_2).
\end{align*}
\]

Theorem 3 gives the price, market share, and profit of ISP/CP1 and of CP2 under different QoS prices. When the QoS price is small (case 1), both CP1 and CP2 pass the entire QoS charge to the end users, and thus their market shares do not depend on the QoS price. Note, however, that in addition to passing on the QoS charge, CP1 extracts an additional portion of its subscribers’ surplus due to QoS. When the QoS price is a bit higher (case 2), neither content provider passes the entire QoS charge to the end users, because doing so would reduce their market share and profit. As \( P_q \) increases, although the market shares remain constant, CP2’s profit decreases due to the lack of pass-through of the entire QoS charge, and ISP/CP1’s profit increases due to the sale of QoS. When the QoS price is moderate (case 3), CP2 can no longer afford to keep its price constant, and it will resume increasing its price with further increases in the QoS price. In response, CP1 can afford to decrease its price, resulting in an increasing CP1 market share and increasing ISP/CP1 profit. When the QoS price is large (case 4), there is now a set of users with moderate preferences that no longer subscribe to either content provider, i.e., \( \theta^{(c_3)} < \theta^{(c_2)} \). CP2’s profit continues to decrease with increases in the QoS price, but CP1’s profit is a unimodal function of the QoS price.

We now turn to CP2’s decision of whether or not to buy QoS (i.e., \( Q \)), which depends on the QoS improvement of user utility (i.e., \( r \)), as well as the price of QoS per user (i.e., \( P_q \)). We start by determining the maximum price at which CP2 will purchase QoS. This can be easily done by comparing CP2’s profit when it purchases QoS (given in Theorem 3) to its profit when it doesn’t (given in Theorem 2). If the QoS price \( P_q < P_q^{(c_3)} \) (Theorem 3, case 1), then CP2’s profit is independent of \( P_q \). Furthermore, it can be shown that its profit when it purchases QoS \((\pi_1^{(c_3)})\) is greater than when it does not \((\pi_2^{(c_3)})\). It follows that CP2 should purchase QoS if it is sufficiently cheap. If the QoS price is \( P_q \geq P_q^{(c_3)} \) (Theorem 3 cases 2–4), then CP2’s profits decrease monotonically with the QoS price \( P_q \). It follows that there exists an upper bound for which CP2 will purchase QoS. The upper bound can be derived from this comparison:

\[
\begin{align*}
    P_q^{(c_3)} &= \min \left\{ \frac{P_q^{(c_2)} - C_2}{1 - \theta^{(c_3)}}, \frac{P_q^{(c_3)} - C_2}{1 - \theta^{(c_3)}} < P_q^{(c_3)} \right\} \\
    &= \min \left\{ \frac{(1+r)V_2 - C_2 - \frac{\sqrt{\pi_2^{(c_3)}}}{1+r}}{1+r}, \left(1 + \frac{\sqrt{\pi_2^{(c_3)}}}{1+r}\right) \right\}.
\end{align*}
\]

**Lemma 1:** Suppose assumptions A, B, and E–G hold, the ISP may sell QoS to content providers, in a short time scale CP1 and CP2 compete through price, and in a long time scale CP2 decides whether to purchase QoS. Then CP2 will purchase QoS if and only if \( P_q \leq P_q^{(c)} \).

**C. QoS Sold to Users**

Finally, we return to the market in which the ISP deploys QoS and offers it to end users at price \( P_q \). We can show that if the QoS price is low enough, then subscribers with preferences close to the content mix of one of the two content providers, i.e., those who place a high value on the service, will choose to purchase QoS. Denote the following user preference thresholds:

\[
\theta \triangleq \min \left\{ \theta \left( (rV_1 - P_q)/rt, 1 \right), 0 \right\}, \\
\bar{\theta} \triangleq \min \left\{ \max \left( 1 - (rV_2 - P_q)/rt, 0 \right), 1 \right\}.
\]

**Lemma 2:** Suppose assumption A holds.

For \( P_q \leq rV_2 \), subscribers of CP1 with \( \theta \leq \bar{\theta} \) and subscribers of CP2 with \( \bar{\theta} \leq \theta \leq 1 \) purchase QoS.

For \( rV_2 < P_q \leq rV_1 \), subscribers of CP1 with \( \theta \leq \bar{\theta} \) purchase QoS and no subscribers of CP2 who purchase QoS exist.

For \( P_q > rV_1 \), no subscribers of CP1 or of CP2 who purchase QoS exist.
Given these decisions, we can show that if there exists a Nash equilibrium, then there is again a partition of users between the two content providers:

**Lemma 3:** Suppose assumptions A–E hold, and the ISP may sell QoS to users. If there exists a local Nash equilibrium, then there exists a threshold \( \hat{\theta} \) such that users with preferences \( 0 \leq \theta < \hat{\theta} \) subscribe to CP1 and users with preferences \( \hat{\theta} \leq \theta \leq 1 \) subscribe to CP2.

However, the existence of a local Nash equilibrium is not guaranteed. Denote the following QoS price thresholds:

\[
\begin{align*}
P_q^{(u1)} &= \max\left(\frac{r(V_1+V_2)}{6} - \frac{rt}{2} - \frac{r(C_1-C_2)}{6(1+r)}, 0\right), \\
P_q^{(u2)} &= \frac{r(1+r)V_1 + 5(3r)V_2 - (3+3)3r - [(C_1+d)-C_2]}{6+3r}, \\
P_q^{(u3)} &= \frac{r(3+3r)V_1V_2 - (3+3r)(C_1+C_2)}{6+3rl}, \\
P_q^{(u4)} &= \frac{r(5V_1+V_2)(C_1-C_2)-3rt}{6}.
\end{align*}
\]

Denote the following user preference thresholds:

\[
\begin{align*}
\theta^{(u1)} &= \frac{1}{6} (1+r)(V_1-V_2)+3(1+r)(C_1-C_2), \\
\theta^{(u2)} &= \frac{1}{6} (1+r)(V_1-V_2)+3(1+r)(C_1-C_2), \\
\theta^{(u3)} &= \frac{1}{6} (3+3r)(V_1V_2)-(C_1+C_2), \\
\theta^{(u4)} &= \frac{1}{6} (3+3r)(V_1V_2)-(C_1+C_2).
\end{align*}
\]

**Theorem 4:** Suppose assumptions A–G hold and the ISP may sell QoS to users. When \( P_q^{(u1)} \leq P_q \leq P_q^{(u2)} \) or \( P_q^{(u3)} \leq P_q \leq P_q^{(u4)} \), there is no local Nash equilibrium. When \( 0 \leq P_q < P_q^{(u1)} \) or \( P_q^{(u2)} < P_q < P_q^{(u3)} \), or \( P_q^{(u4)} < P_q \), there is a local Nash equilibrium as follows:

Case 1: When \( 0 \leq P_q < P_q^{(u1)} \), users with preferences \( 0 \leq \theta < \theta^{(u1)} \) subscribe to CP1 and users with preferences \( \theta^{(u1)} \leq \theta \leq 1 \) subscribe to CP2. All users purchase QoS. The price and profit of each CP are:

\[
\begin{align*}
\Pi_1^{(u1)} &= (3+1+r)t + (1+r)(V_1-V_2) + 2C_1 + C_2)/3, \\
\Pi_2^{(u1)} &= (3+1+r)t - (1+r)(V_1-V_2) + C_1 + 2C_2)/3, \\
\pi_1^{(u1)} &= \frac{1}{18(1+r)t}(3+1+r)(V_1-V_2)(C_1-C_2)^2 + (P_q - d), \\
\pi_2^{(u1)} &= \frac{1}{18(1+r)t}(3+1+r)(V_1-V_2)(C_1-C_2)^2 .
\end{align*}
\]

Case 2: When \( P_q^{(u2)} < P_q < P_q^{(u3)} \), users with preferences \( 0 \leq \theta < \theta^{(u2)} \) subscribe to CP1 and users with preferences \( \theta^{(u2)} \leq \theta \leq 1 \) subscribe to CP2. All CP1 subscribers purchase QoS; only CP2 subscribers with preferences \( \hat{\theta} \leq \theta \leq 1 \) purchase QoS, where \( \theta^{(u2)} < \hat{\theta} \). The price and profit of each CP are:

\[
\begin{align*}
\Pi_1^{(u2)} &= \frac{(3+1+r)t + (1+r)(V_1-V_2) + 2(C_1+d)+C_2}{3} - P_q, \\
\Pi_2^{(u2)} &= \frac{(3+1+r)t - (1+r)(V_1-V_2) + C_1 + 2C_2}{3}, \\
\pi_1^{(u2)} &= \frac{1}{18(1+r)t}(3+1+r)(V_1-V_2)(C_1+C_2)^2 + (1-\hat{\theta})(P_q - d), \\
\pi_2^{(u2)} &= \frac{1}{18(1+r)t}(3+1+r)(V_1-V_2)(C_1+C_2)^2 .
\end{align*}
\]

Case 3: When \( P_q^{(u4)} < P_q \), users with preferences \( 0 \leq \theta \leq \theta^{(u3)} \) subscribe to CP1 and users with preferences \( \theta^{(u3)} \leq \theta \leq 1 \) subscribe to CP2. Only CP1 subscribers with preferences \( 0 \leq \theta \leq \theta \leq 1 \) and CP2 subscribers with preferences \( \theta \leq \hat{\theta} \leq 1 \) purchase QoS, where \( \theta < \theta^{(u4)} \leq \hat{\theta} \). The price and profit of each CP are:

\[
\begin{align*}
\Pi_1^{(u3)} &= (3+1+r)(V_1-V_2) + 2C_1 + C_2)/3, \\
\Pi_2^{(u3)} &= (3+1+r)(V_1-V_2) + C_1 + 2C_2)/3, \\
\pi_1^{(u3)} &= \frac{1}{18(1+r)t}(3+1+r)(V_1-V_2)(C_1+C_2)^2 + (P_q - d), \\
\pi_2^{(u3)} &= \frac{1}{18(1+r)t}(3+1+r)(V_1-V_2)(C_1+C_2)^2 .
\end{align*}
\]

Theorem 4 gives the price, market share, and profit of ISP/CP1 and of CP2 under different QoS prices, when QoS is sold directly to users. It also shows how many users purchase QoS. When the QoS price is small (case 1), all users buy QoS. Thus, we see similar behaviors as when QoS is sold to the content providers (Theorem 3, case 1). When the QoS price is moderate (case 2), while all CP1 subscribers continue to purchase QoS, some CP2 subscribers (those with lower valuations on CP2’s video service) choose to no longer purchase QoS. The marginal users on which the content providers compete are thus indifferent between CP1 with QoS and CP2 without QoS. We see similar behaviors as when QoS is used solely for CP1 (Theorem 2), except that ISP/CP1 earns an additional profit from the sale of QoS to CP2’s users. When the QoS price is high (case 3), there are some subscribers to both CP1 and CP2 who do not value the service enough to purchase QoS at this price. The marginal users on which the content providers compete are thus indifferent between CP1 without QoS and CP2 without QoS (Theorem 1). We see similar behavior as when ISP chooses not to deploy QoS, except that ISP/CP1 earns an additional profit from the sale of QoS to CP1’s users and CP2’s users.

**IV. NUMERICAL RESULTS**

In this section, we numerically evaluate the competition between a vertically integrated cable broadband provider (ISP) that also offers a multichannel video programming distributor (MVPD) service (CP1) and an OTT provider (CP2) that offers a competing video service. The average prices charged by MVPD providers are approximately \( P_1 = \$140/month \) and by OTT providers approximately \( P_2 = \$10/month \) [14]. The average profit margins over the past five years for DirecTV (a MVPD provider) and Netflix (an OTT provider) are 9.41% and 4.22%, respectively [15, 16], which gives \( C_1 = P_1(1 - 0.0941) \approx 858/month \) and \( C_2 = P_2(1 - 0.0422) \approx 89.5/month \). We set QoS cost per user for the MVPD provider \( d_1 = \gamma C_1 \) and for the OTT provider \( d_2 = \gamma C_2 \), where \( \gamma = 0.3 \). Among 118 million United States households, 60.6 million subscribe only to MVPD service, 6.4 million subscribe only to OTT service, and 40.3 million subscribe to both [14, 17], i.e., 51.4% of households have \( (S_{1,1} > 0, S_{1,2} \leq 0) \), 5.4% have \( (S_{1,1} \leq 0, S_{1,2} > 0) \) and 34.2% have \( (S_{1,1} > 0, S_{1,2} > 0) \). We set \( H_1, H_2, t_1, t_2, V_1 \), and \( V_2 \) to fit these user subscription choices at the competition equilibrium without QoS; this gives \( V_1 = 92.4, V_2 = 16.5, t_1 = 55, t_2 = 34 \) ($ per month), \( H_1 = 0.44 \), and \( H_2 = 0.71 \). The increased proportion of user utility is \( r = 0.3 \). Each user can choose either to...
subscribe to a single content provider or can choose not to subscribe to either, so as to maximize their surplus $S_{i,j}$. When QoS is sold to the OTT provider, the OTT provider can choose to buy QoS for all its users at price $P_q$ (in $/ per user), or can choose not to buy QoS. When QoS is sold to the users, subscribers to the MVPD service have the option to buy QoS at price $P_q d_1 / d_2$, and subscribers to the OTT service provider have the option to buy QoS at price $P_q$.

Fig. 1 shows the variation with the QoS price of the MVPD price, the OTT price and the total user payment for video service and QoS (if purchased). When QoS is sold to the OTT provider at a relatively low price ($P_q < $3.5), the OTT price increases slightly with the QoS price, passing on a part of the QoS charge to the end users. In contrast, the cable provider does not have much incentive to change its MVPD price, which is already optimized to maximize the joint profit of its broadband and video businesses. When QoS is sold to the OTT provider at a moderate to high price ($P_q \geq $4), the OTT provider chooses not to buy QoS from the ISP, and some users drop it due to its lower service quality. The MVPD continues to deploy and use QoS, and it decreases its MVPD price so as to gain a larger market share. The cable provider thus earns 71% market share at a slightly lower MVPD price, while the OTT provider is only left with 10% market share.

When QoS is sold to end users at a relatively low price ($P_q < $2.5), all video service subscribers buy QoS. The cable provider decreases its MVPD price proportionally with the QoS price, so that the total price $P_1 + P_q d_1 / d_2$ to its MVPD subscribers remains constant. In response, the OTT decreases its price slightly, but not enough to hold the total price $P_2 + P_q$ to its subscribers constant. As a result, a few users switch from the OTT to the MVPD. When QoS is sold to end users at a moderate price ($2.5 \leq P_q < $3.5), the cable provider seeks to further dominate the market by reducing its MVPD price, so that the total price paid by its MVPD subscribers who purchase QoS falls. Although the OTT provider also reduces its price in response, it still loses market share from 14% to 9% while the MVPD market share increases from 54% to 78%. When QoS is sold to end users at a moderate to high price ($3.5 \leq P_q < $4), some video subscribers stop buying QoS. In this environment, the cable provider has less advantage from selling QoS, and it ends the price war, which leads to both a higher MVPD price and a higher OTT price. If QoS is sold to end users at a high price ($P_q \geq $4), no users buy QoS, and the MVPD price and OTT price are the same as that of the case when the ISP does not deploy QoS.

Fig. 2 shows the variation with the QoS price of the profits of the cable provider and the OTT provider. When the QoS price is relatively small ($P_q \leq $2.5), regardless of whom it sells QoS to, the cable provider can always earn more profit by increasing the QoS price. In contrast, the OTT provider earns less profit because it is losing market share and may slightly reduce its OTT price. When QoS is sold to the OTT provider, the cable provider earns the maximum profit by pricing QoS high enough ($P_q \geq $4) so that the OTT provider does not buy it, which is also the sub-game perfect equilibrium in the sequential game defined in Section II-B. However, this maximum profit is very close to profit earned when $P_q = $3.5 (the maximum QoS price that the OTT provider can accept). Thus, the cable provider may either use QoS solely for its MVPD service or sell QoS to the OTT provider at a high price. In contrast, the OTT provider always earns the minimum profit when $P_q$ is large. When QoS is sold to users, the cable provider starts a price war when $2.5 \leq P_q < $3.5, which leads to lower profits for both providers. However, when $P_q \geq $3.5, some light users do not buy QoS. The profit of the cable provider decreases due to less advantage from selling QoS. When $P_q \geq $5, the QoS price is too expensive for all users, if sold directly to them. The QoS price is also too expensive for the OTT provider in this case, where QoS is thus only provided for MVPD users. Both providers earn less profit. When QoS is sold to users, the cable provider’s profit is maximized when $P_q = $2.5, which is also
the sub-game perfect equilibrium in the sequential game defined in Section II-C. The QoS price $P_q$ in the sub-game perfect equilibrium when QoS is sold to users ($P_q = 7.5$) is smaller than that of the equilibrium ($P_q = 8$) when QoS is sold to the OTT provider.

In the next set of figures, we examine the effect of changes in the user utility due to QoS, i.e., in the parameter $r$. In some situations considered below, we will consider a regulated QoS price, in which case the price is set so that the return rate equals 20%, i.e., $P_q = 1.2d_2$.

Fig. 3 shows the variation with the QoS improvement $r$ of the QoS price that maximizes the profit of the cable provider under different pricing strategies. It is straightforward that QoS price increases with $r$, since both users and the OTT provider are willing to pay more for QoS when it has a greater impact on user utility. The QoS price that maximizes cable provider profit when QoS is sold to the OTT provider is larger than that in when QoS is sold to users. The regulated QoS price does not change with $r$, since it is based on the cost of providing QoS. When the QoS price is unregulated, it increases substantially above the regulated price.

Fig. 4 shows the variation with the QoS improvement of the MVPD price, the OTT price and the total user payment for video service and QoS (if purchased). Most MVPD subscribers are served with QoS regardless of whether the QoS price is regulated. The OTT price is fairly insensitive to QoS improvement, since the OTT provider does not purchase QoS at higher prices. When the QoS price is unregulated, the cable provider either increases the QoS price (if QoS is sold to end users) or increases its MVPD price (otherwise). When the QoS price is regulated, the cable provider responds by increasing its MVPD service price based on the increased willingness to pay.

Fig. 5 shows the variation with the QoS improvement of MVPD market share and OTT market share, where “MVPD w/QoS” indicates the percentage of all users who subscribe to the MVPD service and buy QoS, where “MVPD total” indicates the percentage of all users who subscribe to the MVPD service, and where similar definition also apply to the OTT service. When $r = 0.1$, although QoS is not deployed, the cable provider earns a high market share since $V_1$ is much larger than $V_2$. When $r > 0.1$ and QoS is sold to the OTT provider, the cable provider gains a larger advantage and its market share increases with $r$. When $r > 0.1$ and QoS is sold to users, although OTT market share initially increases with $r$ because the OTT provider is in a slightly better position, OTT market share later decreases with $r$, as the cable provider gains more advantages from selling QoS at higher prices.

Fig. 6 shows the variation with the QoS improvement of the profits of the cable provider and the OTT provider. When the QoS price is unregulated, although the OTT provider initially earns more profit when $r$ increases from 0.1 to 0.2 due to the increased value of QoS, the OTT provider earns less profit as $r$.

\footnote{No QoS prices are shown when $r = 0.1$, because it is not economically beneficial for the cable provider to adopt QoS in this case.}
increases further when the cable provider gains competitive advantage when by selling QoS at a higher price. When the QoS price is regulated, a similar pattern holds except that the profit of the cable provider increases at a slightly lower rate. The cable provider can always benefit from QoS technology improvement, while the OTT provider may be hurt by the cable provider’s increased competitive advantage.

Fig. 7 shows the variation with the user type of user surplus under different QoS pricing strategies. When QoS is not adopted, users with preferences $0.04 \leq \theta < 0.78$ subscribe to the MVPD, and users with preferences $0.78 < \theta \leq 0.88$ subscribe to OTT. Heavy MVPD subscribers, whose user type $\theta$ is close to MVPD content type $H_1$, experience higher surplus than light MVPD users, since they are charged the same price. Similarly, heavy OTT users experience higher surplus than light OTT users.

When QoS is sold to the OTT provider, only MVPD subscribers are served with QoS; see the case when $P_q \geq 4$ in Fig. 2. The OTT provider chooses not to buy QoS from the ISP, i.e., $Q = 0$. Users with preferences $0.07 \leq \theta < 0.77$ subscribe to the MVPD, and users with preferences $0.77 < \theta \leq 0.87$ subscribe to OTT. Although heavy MVPD subscribers experience higher surplus than that in the case when QoS is not deployed, light MVPD users who benefit less from QoS experience lower surplus due to higher MVPD prices. Fewer users subscribe to the MVPD service as a result. All OTT subscribers experience slight lower surplus compared with the case without QoS, since OTT price increases slightly along with the increased MVPD price. As a result, some marginal users switch from OTT service to MVPD service.

When QoS is sold to users, users with preferences $0.15 \leq \theta < 0.68$ subscribe to the MVPD, and users with preferences $0.68 < \theta \leq 0.82$ subscribe to OTT; see the case when $P_q = \$2.5$ in Fig. 2. Although all users purchase QoS from the ISP, they experience lower surplus than that in the case when QoS is not deployed, since the cable provider has one more degree of freedom (the QoS price $P_q$) to better extract profits from all users. However, the OTT provider can earn a larger market share in this case than that of the case when QoS is sold to OTT provider, and thus earn more profit.
equilibrium for prices can be thus obtained in the following three cases:

Case 1: If \( S_{i,j}^{b}(\theta^{(b)}) = (V_1 + V_2 - P_1^{(b)} - P_2^{(b)} - t)/2 < 0 \), i.e., the marginal users with preference \( \theta_i = \theta^{(b)} \) do not subscribe to either content provider, then CP1 and CP2 are maximizing their profits separately on different sets of users, instead of competing with each other for the marginal users. Thus, under assumption B, user \( i \) with preference \( 0 \leq \theta_i \leq (V_1 - P_1^{(b)})/2 \) will subscribe to CP1 and user \( i \) with preference \( (t - V_2 + P_2^{(b)})/2 \leq \theta_i \leq 1 \) will subscribe to CP2, which gives the demands for both content providers \( N_j = (V_j - P_j^{(b)})/t \). The profit of CP \( j \) is thus \( \pi_j = (V_j - P_j^{(b)})((P_j^{(b)} - C_j))/t \). The prices in Nash equilibrium satisfy \( \partial \pi_j / \partial P_j |_{P_j = P_j^{(b)}} = 0 \), which gives \( P_1^{(b)} + P_2^{(b)} = (V_1 + V_2 + C_1 + C_2)/2 \). Considering \( (V_1 + V_2 - P_1^{(b)} - P_2^{(b)} - t)/2 < 0 \), we have \( P_1^{(b)} + P_2^{(b)} > V_1 + V_2 - t \), which gives \( V_1 + V_2 < 2t + C_1 + C_2 \). This contradicts assumption C \( (V_1 + V_2 > 3t + C_1 + C_2) \), and therefore there is no Nash equilibrium in case 1.

Case 2: If \( S_{i,j}^{b}(\theta^{(b)}) = (V_1 + V_2 - P_1^{(b)} - P_2^{(b)} - t)/2 > 0 \), then user \( i \) with preference \( 0 \leq \theta_i < \theta^{(b)} \) will subscribe to CP1, user \( i \) with preference \( \theta^{(b)} < \theta_i \leq 1 \) will subscribe to CP2, and user \( i \) with preference \( \theta_i = \theta^{(b)} \) will subscribe to either CP1 or CP2. Under assumption B, the demands for CP1 and CP2 are \( N_1 = \theta^{(b)} \) and \( N_2(P_1, P_2) = 1 - \theta^{(b)} \), respectively. The profit of CP \( j \) is \( \pi_j = (t + V_j - V_3 - j - P_j^{(b)} + P_3^{(b)})(P_j^{(b)} - C_j)/t \). The prices in the Nash equilibrium satisfy \( \partial \pi_j / \partial P_j |_{P_j = P_j^{(b)}} = 0 \), which gives \( \theta^{(b)} = 0.5 + [(V_1 - V_2) - (C_1 - C_2)]/6t \) and

So, \( P_1^{(b)} + P_2^{(b)} = 2t + C_1 + C_2 \), which gives \( V_1 + V_2 > 3t + C_1 + C_2 \) consistent with assumption C. The corresponding profits of CP1 and CP2 are

The condition \( (V_1 - V_2 - (C_1 - C_2))/8t \) in assumption C ensures \( N_j > 0 \), so that both CP1 and CP2 earn positive profit and market share in this local Nash equilibrium.

We can show that this local Nash equilibrium is also a global Nash equilibrium by verifying it in the following profit function on a full region of the charged price \( P_j^{(b)} \):

\[
\pi_j = \begin{cases} 
\frac{P_j^{(b)} - P_j^{(b)}}{(t + V_j - V_3 - j - P_j^{(b)} + P_3^{(b)})(P_j^{(b)} - C_j)}, & \text{if } V_j - V_3 - j - t < 0, \\
(P_j^{(b)} - P_j^{(b)})/t, & \text{if } V_j - V_3 - j - t \leq 0.
\end{cases}
\]

where \( P_j^{(b)} - P_j^{(b)} \leq V_j - V_3 - j - t \) corresponds to the special case when \( P_j^{(b)} \) is so low that all users subscribe to CP \( j \); \( P_j^{(b)} - P_j^{(b)} \geq V_j - V_3 - j + t \) corresponds to the special case when \( P_j^{(b)} \) is so high that no users subscribe to CP \( j \).

APPENDIX

A. Proof of Theorem 1

Under assumption A, marginal users (i.e., users with positive surplus who are indifferent between the two content providers) have preference \( \theta^{(b)} \) and satisfy:

\[ S_{i,1} = S_{i,2} \Rightarrow V_1 - t\theta^{(b)} - P_1 = V_2 - t(1 - \theta^{(b)}) - P_2 \Rightarrow \theta^{(b)} = (V_1 - V_2 - P_1^{(b)} + P_2^{(b)} + t)/2t. \]

User \( i \) with preference \( \theta_i \leq \theta^{(b)} \) prefers CP1 to CP2, and user \( i \) with preference \( \theta_i > \theta^{(b)} \) prefers CP2 to CP1. The Nash
Case 3: If $S^{(b)}_{2,j}(\theta^{(b)}) = (V_1 + V_2 - P^{(b)}_1 - P^{(b)}_2 - t)/2 = 0$ or $P^{(b)}_1 + P^{(b)}_2 = V_1 + V_2 - t$, then user $i$ with preference $0 \leq \theta_i < \theta^{(b)}$ will subscribe to CP1, user $i$ with preference $\theta^{(b)} < \theta_i \leq 1$ will subscribe to CP2, and user $i$ with preference $\theta_i = \theta^{(b)}$ will either subscribe to either content provider or to neither. Under assumption B, the demands for CP1 and CP2 are $N_1 = \theta^{(b)}$ and $N_2 = 1 - \theta^{(b)}$, respectively. The profit of CP $j$ is $\pi_j = (t + V_j - V_{3-j} - P^{(b)}_j + P^{(b)}_{3-j})(P^{(b)}_j - C_j)/t$. The prices in the Nash equilibrium satisfy $\partial \pi_1 / \partial P_1|_{P_1 = P^{(b)}_1} - \partial \pi_2 / \partial P_2|_{P_2 = P^{(b)}_2} \leq 0$ (as in case 1) and $\partial \pi_1 / \partial P_1|_{P_1 = P^{(b)}_1} - \partial \pi_2 / \partial P_2|_{P_2 = P^{(b)}_2} \geq 0$ (as in case 2), which gives $(C_1 + C_2 + V_1 + V_2)/2 \leq P^{(b)}_1 + P^{(b)}_2 \leq 2t + C_1 + C_2$. Considering $P^{(b)}_1 + P^{(b)}_2 = V_1 + V_2 - t$, we have $2t + C_1 + C_2 \leq V_1 + V_2 \leq 3t + C_1 + C_2$. This contradicts assumption C ($V_1 + V_2 > 3t + C_1 + C_2$), and therefore there is no Nash equilibrium in case 3.

The results in Theorem 1 can be obtained from Nash equilibrium in case 2.

B. Proof of Theorem 2

This theorem can be easily proven by replacing $V_1$, $t_1$, and $C_1$ in the proof of Theorem 1 with $(1 + r)V_1$, $(1 + r)t_1$, and $(1 + r)C_1$, respectively.

C. Proof of Theorem 3

Under assumption A, marginal users who are indifferent between CP1 and CP2 have preference $\theta^{(1)}_{\text{max}}$ and satisfy:

$S^{(c)}_{1,1} = S^{(c)}_{1,2} \Rightarrow (1 + r)(V_1 - t\theta^{(1)}_{\text{max}}) - P_1 = (1 + r)(V_2 - t(1 - \theta^{(1)}_{\text{max}})) - P_2 \Rightarrow \theta^{(1)}_{\text{max}} = \max \left( \min \left( (1 + r)V_1 - P_1 / (1 + r)t, 0 \right), 1 \right)$.

Marginal users who are indifferent between CP1 and none have preference $\theta^{(1)}_{\text{max}}$ and satisfy:

$S^{(c)}_{1,2} = 0 \Rightarrow (1 + r)(V_2 - t(1 - \theta^{(1)}_{\text{max}})) - P_2 = 0 \Rightarrow \theta^{(2)}_{\text{max}} = \max \left( \min \left( (1 + r)V_2 - P_2 / (1 + r)t, 0 \right), 1 \right)$.

Thus, the complete profit functions in the full region of $(P_1, P_2)$ can be derived as follows. If $P_1 - P_2 \leq (1 + r)(V_1 - V_2 - t)$, which means no users will subscribe to CP2, the profits of ISP/CP1 and CP2 are:

$\pi^{(0)}_1 = \theta^{(1)}_{\text{max}}(P_1 - C_1 - d) \quad \pi^{(0)}_2 = 0$.

If $(1 + r)(V_1 - V_2 - t) < P_1 - P_2 < (1 + r)(V_1 - V_2 + t)$ and $S^{(c)}_{1,2}(\theta^{(1)}_{\text{max}}) > 0$, which means users with preferences $0 \leq \theta_i < \theta^{(1)}_{\text{max}}$ will subscribe to CP1 and users with preference $\theta^{(1)}_{\text{max}} < \theta_i \leq 1$ will subscribe to CP2, the profits of ISP/CP1 and CP2 are:

$\pi^{(1)}_1 = \theta^{(1)}_{\text{max}}(P_1 - C_1 - d) + (1 - \theta^{(1)}_{\text{max}})(P_2 - d)$,

$\pi^{(1)}_2 = (1 - \theta^{(1)}_{\text{max}})(P_2 - C_2 - P_2)$.

If $(1 + r)(V_1 - V_2 + t) < P_1 - P_2 < (1 + r)(V_1 - V_2 + t)$ and $S^{(c)}_{1,1}(\theta^{(1)}_{\text{max}}) \leq 0$, which means users with preference $0 \leq \theta_i < \theta^{(1)}_{\text{max}}$ will subscribe to CP1, users with preference $\theta^{(2)}_{\text{max}} < \theta_i \leq 1$ will subscribe to CP2 and users with preference $\theta^{(2)}_{\text{max}} \leq \theta_i \leq \theta^{(2)}_{\text{max}}$ will subscribe to neither CP1, the profits of ISP/CP1 and CP2 are:

$\pi^{(2)}_1 = \theta^{(1)}_{\text{max}}(P_1 - C_1 - d) + (1 - \theta^{(2)}_{\text{max}})(P_2 - d)$,

$\pi^{(2)}_2 = (1 - \theta^{(2)}_{\text{max}})(P_2 - C_2 - P_2)$.

If $P_1 - P_2 \geq (1 + r)(V_1 - V_2 + t)$, which means no users will subscribe to CP1, the profits of ISP/CP1 and CP2 are:

$\pi^{(3)}_1 = (1 - \theta^{(2)}_{\text{max}})(P_1 - d)$,

$\pi^{(3)}_2 = (1 - \theta^{(2)}_{\text{max}})(P_2 - C_2 - P_2)$.

Under assumptions A, B, and E, when $P_q < P^{(c)}_1$, we can show user $i$ with preference $0 \leq \theta_i < \theta^{(1)}_{\text{max}}$ will subscribe to CP1, user $i$ with preference $\theta^{(1)}_{\text{max}} \leq \theta_i \leq 1$ will subscribe to CP2, and user $i$ with preference $\theta_i = \theta^{(1)}_{\text{max}}$ will subscribe to either content provider or to neither (similar to Theorem 1, case 2). The corresponding prices $(P^{(c)}_1, P^{(c)}_2)$ and profits $(\pi^{(c)}_1, \pi^{(c)}_2)$ in the Nash equilibrium can thus be derived from $\partial \pi^{(1)}_j / \partial P_j |_{P_j = P^{(c)}_j} = 0$. We can further verify that this local Nash equilibrium is also a global Nash equilibrium according to the above profit function in the full region of $(P_1, P_2)$.

Under assumptions A, B, and E, when $P^{(c)}_1 \leq P_q < P^{(c)}_2$, we can show user $i$ with preference $0 \leq \theta_i < \theta^{(1)}_{\text{max}}$ will subscribe to CP1, user $i$ with preference $\theta^{(1)}_{\text{max}} \leq \theta_i \leq 1$ will subscribe to CP2, and user $i$ with preference $\theta_i = \theta^{(1)}_{\text{max}}$ will subscribe to either content provider or to neither (similar to Theorem 1, case 3). The corresponding prices $(P^{(c)}_1, P^{(c)}_2)$ and profits $(\pi^{(c)}_1, \pi^{(c)}_2)$ is a local Nash equilibrium, since it satisfies $\partial \pi^{(2)}_j / \partial P_j |_{P_j = P^{(c)}_j} \leq 0$, $\partial \pi^{(1)}_j / \partial P_{3-j}|_{P_{3-j} = P^{(c)}_{3-j}} \leq 0$. We can verify that this local Nash equilibrium is also a global Nash equilibrium according to the above profit function in the full region of $(P_1, P_2)$.

Under assumptions A, B, and E, when $P^{(c)}_2 < P_q < P^{(c)}_3$, we can show user $i$ with preference $0 \leq \theta_i < \theta^{(1)}_{\text{max}}$ will subscribe to CP1, user $i$ with preference $\theta^{(1)}_{\text{max}} \leq \theta_i \leq 1$ will subscribe to CP2, and user $i$ with preference $\theta_i = \theta^{(1)}_{\text{max}}$ will subscribe to either content provider or to neither (similar to Theorem 1, case 3). The corresponding prices $(P^{(c)}_1, P^{(c)}_2)$ and profits $(\pi^{(c)}_1, \pi^{(c)}_2)$ is a local Nash equilibrium, since it satisfies $\partial \pi^{(2)}_1 / \partial P_1 |_{P_1 = P^{(c)}_1} \leq 0$, $\partial \pi^{(1)}_1 / \partial P_{3-j}|_{P_{3-j} = P^{(c)}_{3-j}} \leq 0$ and $\partial \pi^{(2)}_2 / \partial P_{3-j}|_{P_{3-j} = P^{(c)}_{3-j}} \leq 0$. We can
verify that this local Nash equilibrium is also a global Nash equilibrium to the above profit function in the full region of \((P_1, P_2)\).

Under assumptions A, B, and E, when \(P_{q_{c0}}^{(c0)} \leq P_q \leq P_{c1}^{(c1)}\), we can show that marginal users with preference \(\theta_1 = \theta_{\text{marg}}\) do not subscribe to either content provider (similar to Theorem 1, case 1). The corresponding prices \((P_1^{(c0)}, P_2^{(c0)})\) and profits \((\pi_1^{(c0)}, \pi_2^{(c0)})\) in the Nash equilibrium can be derived from \(\partial\pi_1^{(c0)}/\partial P_1|_{P_1^{(c0)}} = 0\) and \(\partial\pi_2^{(c0)}/\partial P_2|_{P_2^{(c0)}} = 0\). We can verify that this local Nash equilibrium is also a global Nash equilibrium according to the above profit function in the full region of \((P_1, P_2)\).

**D. Proof of Lemma 1**

According to Theorem 3, the profit of CP2 is a decreasing function of \(P_q\) when \(P_q \geq P_{c1}^{(c1)}\). If CP2 earns less profit when it buys QoS at price \(P_q = P_{c1}^{(c1)}\) than in the case when CP2 does not buy QoS, i.e.,

\[
\pi_2^{(c0)} > \pi_2^{(c2)}(P_q^{(c2)}) = (1 - \theta^{(c1)})(C_2 - P_{2}^{(c2)} - P_{1}^{(c2)} - C_2) \Rightarrow P_2^{(c2)} - C_2 - \frac{\pi_2^{(c0)}}{1 - \theta^{(c1)}} < P_q^{(c2)}.
\]

The maximum QoS price CP2 will accept is \(P_q^{(c)} = \frac{P_2^{(c2)} - C_2 - \frac{\pi_2^{(c0)}}{1 - \theta^{(c1)}}}{P_2^{(c2)} - C_2} \). If CP2 earns more profit when it buys QoS at price \(P_q = P_{c1}^{(c1)}\) than that in the case when CP2 does not buy QoS, i.e., \(P_2^{(c2)} - C_2 - \frac{\pi_2^{(c0)}}{1 - \theta^{(c1)}} \leq P_q^{(c2)}\), the maximum QoS price CP2 will accept can be obtained from

\[
\pi_2^{(c0)} = \pi_2^{(c2)}(P_q^{(c)}) = \frac{(1 + r)V_2 - C_2 - P_q^{(c)}}{1 + r}.
\]

**E. Proof of Lemma 2**

Under assumption A, \(U_{q_{11}}^{(b)} \geq U_{q_{11}}^{(b)} \Rightarrow r(V_1 - \theta_1) \geq P_q\). Thus, \(P_q > \lambda \Rightarrow \) gives \(P_q > r(V_1 - \theta_1)\). Let \(U_{q_{11}}^{(b)} \geq U_{q_{11}}^{(b)}\), which means no subscribers to CP1 will purchase QoS. When \(P_q \leq r(V_1 - \theta_1)\), \(r(V_1 - \theta_1) \geq P_q \Rightarrow 0 \leq \theta_1 \leq \theta_{\text{marg}}\), which means only subscribers to CP1 with preferences \(0 \leq \theta_1 \leq \theta_{\text{marg}}\) purchase QoS. Similar results can be obtained for subscribers to CP2.

**F. Proof of Lemma 3**

Under assumption A, if there exists a local Nash equilibrium, it can be easily shown that only two scenarios can occur. In scenario 1, there exists a threshold \(\theta\) such that users with preferences \(0 \leq \theta < \theta_{\text{marg}}\) subscribe to CP1 and users with preferences \(\theta < \theta_{\text{marg}}\) subscribe to CP2. In scenario 2, there exists two thresholds \(\theta_{1}\) and \(\theta_{2}\) (\(\theta_{1} < \theta_{2}\)) such that users with preference \(0 \leq \theta < \theta_{1}\) subscribe to CP1 and users with preference \(\theta < \theta_{2}\) subscribe to CP2. We will prove that scenario 2 will not occur.

If scenario 2 occurs and neither marginal users with preference \(\theta_1 = \theta_{2}\) nor marginal users with preference \(\theta_1 = \theta_{2}\) purchase QoS, then based on the uniform distribution in assumption B, demands for both content providers are \(N_j = (V_j - P_{j}^{(b)})/t\). The profits of CP1 and CP2 are

\[
\pi_1 = (V_1 - P_1)(P_1 - C_1)/t + (P_q - d)(\theta_1 + 1 - \theta_2),
\pi_2 = (V_2 - P_2)(P_2 - C_2)/t.
\]

The prices in the Nash equilibrium satisfy \(\partial\pi_2/P_2|_{P_2^{(u)}} = 0\), which gives \(P_{1}^{(u)} + P_{2}^{(u)} = (V_1 + V_2 + C_1 + C_2)/2\). Considering marginal subscribers in CP2 will not subscribe to CP1, i.e., \(U_{q_{11}}^{(b)} < 0\) for \(\theta_1 = \theta_{2}\), we have \(V_1 + V_2 - P_{1}^{(u)} - P_{2}^{(u)} - t < 0\), which gives \(V_1 + V_2 < 2t + C_1 + C_2\). This contradicts assumption C (\(V_1 + V_2 > 3t + C_1 + C_2\)), and therefore this case will not occur.

If scenario 2 occurs and marginal users with preference \(\theta_1 = \theta_{2}\) buy QoS, but marginal users with preference \(\theta_1 = \theta_{2}\) do not buy QoS, we can similarly prove that this case will not occur since it contradicts assumption D. If scenario 2 occurs and both marginal users with preference \(\theta_1 = \theta_{2}\) and marginal users with preference \(\theta_1 = \theta_{2}\) buy QoS, we can similarly prove that this case will not occur since it contradicts assumption E. If scenario 2 occurs and marginal users with preference \(\theta_1 = \theta_{2}\) buy QoS, but marginal users with preference \(\theta_1 = \theta_{2}\) do not buy QoS, we can again prove that this case will not occur since it contradicts assumption F.

Based on the above analysis, scenario 2 will not occur.

**G. Proof of Theorem 4**

Under assumptions A, B, E, and G, when \(0 \leq P_q < P_{q_{11}}^{(u)}\) and \(P_2 = P_{q_{11}}^{(u)}\) and \(P_1 = P_{q_{11}}^{(u)}\), it follows from Lemma 2.3 that \(\theta < \theta_{1} = \theta_{2} \leq \theta_{\text{marg}}\) and \(\partial\pi_2/P_2|_{P_2 = P_{q_{11}}^{(u)}} = 0\) for \(j = 1, 2\). Thus, \(P_1 = P_{q_{11}}^{(u)}\) for \(j = 1, 2\) is a local Nash equilibrium based on Definition 1. The profits of CP1 and CP2 \((\pi_1^{(u)}, \pi_2^{(u)})\) under the local Nash equilibrium can also be directly derived.

Under assumptions A–D, F, and G, the proofs for cases 2 and 3 follow using a similar analysis.

**REFERENCES**


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