

Robust Nonlinear Control of a Mobile Robot

Ghania Zidani*, Said Drid†, Larbi Chrifi-Alaoui**, Djemaï Arar*** and Pascal Bussy**

Abstract – A robust control intended for a nonholonomic mobile robot is considered to guarantee good tracking a desired trajectory. The main drawbacks of the mobile robot model are the existence of nonholonomic constraints, uncertain system parameters and un-modeled dynamics. in order to overcome these drawbacks, we propose a robust control based on Lyapunov theory associated with sliding-mode control, this solution shows good robustness with respect to parameter variations, measurement errors, noise and guarantees position and velocity tracking. The global asymptotic stability of the overall system is proven theoretically. The simulation results largely confirm the effectiveness of the proposed control.

Keywords: Wheeled mobile robot (WMR), Kinematic control, Dynamic control, Nonlinear methods, Theorem Lyapunov, Nonholonomic mobile robot

1. Introduction

One of the basic issues in the field of mobile robotics is the running path. The trajectory tracking is to guide the robot through intermediate points to arrive at the final destination. This guide is done under a time constraint, ie, the robot must reach the goal within a predefined time. In the literature, the problem is treated as the continuation of a robot reference (virtual processor) which moves to the desired trajectory with a certain pace. The real robot must follow this virtual robot accurately and try to minimize the error in distance, varying its linear and angular velocities [1-9].

There are lots of works on its tracking control. Their aims are mainly kinematic models; one method for dynamic models has been suggested [1]. In this case generally use linear and angular velocities of the robot (Fierro & Lewis, 1997; Fukao et al., 2000) or torques (Rajagopalan & Barakat , 1997; Topalov et al., 1998) as an input control vector [2]. The most authors determine the problem of mobile robot stability using nonlinear backstepping algorithm (Tanner & Kyriakopoulos, 2003) with steady parameters (Fierro & Lewis, 1997), or with the known functions (Oriollo et al., 2002) [1-6]. Other goals at the control architectures, the hybrid of the kinematic control, and the dynamic controller, the neural network controller, is proposed as some trajectory tracking methods [3].

In this paper, first, a kinematic controller is introduced

to the WMR. Second, the dynamic controller, PI then Lyapunov theory associated to a sliding mode control, is proposed to make the actual velocity of the mobile robot to reach the wheel velocity control desired.

2. Kinematic Model

Fig. 1 shows the typical model of a nonholonomic wheeled mobile robot. This last is operated by two independent wheels and with a passive wheel ensuring its stability. The posture of the WMR can be represented as

$$q = [x \ y \ \theta] \quad (1)$$

where the (x, y) is the center of mass (COM) position of the WMR in the world $X-Y$ coordinate, and θ is the included angle between the X -axis and X' -axis

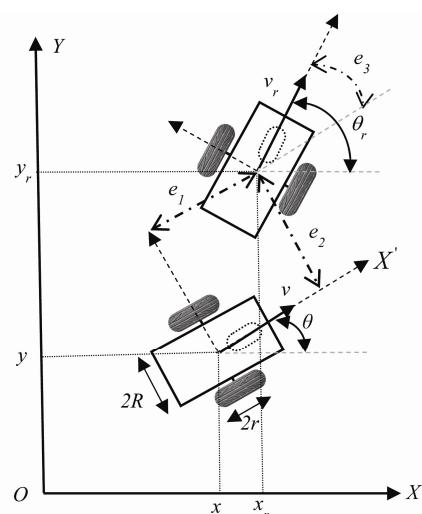


Fig. 1. Error posture of a nonholonomic WMR

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representing the WMR [5, 7].

Know the derivative of the posture control $V = (v \ w)^T$ is easy. A simple geometric consideration gives

$$\begin{cases} \dot{x} = v \cos \theta \\ \dot{y} = v \sin \theta \\ \dot{\theta} = \omega \end{cases} \quad (2)$$

What is written in matrix form

$$\dot{q} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ w(t) \end{bmatrix} = J(\theta)V \quad (3)$$

whith

$$J(q) = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \quad (4)$$

The mobile robot is nonholonomic, this signify the wheels roll without slipping, ie [3]

$$\dot{y} \cos \theta - \dot{x} \sin \theta = 0 \quad (5)$$

3. Dynamic Model

The dynamic equation of the WMR with n-generalized coordinates $q \in \mathbb{R}^{n \times 1}$, and inputs $r = n - m$, can be expressed as [1, 3, 7]

$$M(q)\ddot{q} + V_m(q, \dot{q})\dot{q} + F(q) + \tau_p = B(q)\tau - A^T(q)\lambda \quad (6)$$

where $M(q) \in \mathbb{R}^{n \times n}$ is a positive symmetric definite inertia matrix, $V_m(q, \dot{q}) \in \mathbb{R}^{n \times n}$ is the centripetal and coriolis matrix, $F(q) \in \mathbb{R}^{n \times 1}$ denotes the gravitational vector, $\tau_p \in \mathbb{R}^{n \times 1}$ is bounded unknown disturbance, $B(q) \in \mathbb{R}^{n \times (n-m)}$ denotes the input transformation matrix, $\tau \in \mathbb{R}^{(n-m) \times 1}$ is the control input vector, $A \in \mathbb{R}^{m \times n}$ is a matrix associated with the constraints.

The parameters in (6) are given as

$$\begin{aligned} M(q) &= \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}, \quad A^T(q) = \begin{bmatrix} -\sin \theta \\ \cos \theta \\ 0 \end{bmatrix}, \quad F(q) = 0, \\ B(q) &= \frac{1}{r} \begin{bmatrix} \cos \theta & \cos \theta \\ \sin \theta & \sin \theta \\ L & -L \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \text{ and } V_m(q, \dot{q}) = 0 \end{aligned}$$

I and m are the moment of inertia and the mass of the WMR, respectively, the motors torques τ_r and τ_l act on the right and left wheels respectively [1, 7]. r and R are the radius of the wheel and the distances between the two driving wheels, respectively.

Substituting (2) and its derivative in Eq. (5) pre-multiplied by $J^T(q)$, and without considering uncertainties and disturbances the Eq. (5) can be written as the following [1, 3, 7, 8]

$$\bar{M}(q)\dot{V} = \bar{B}(q)\tau \quad (7)$$

The dynamic model of a WMR unicycle type is simplified and given by

$$\dot{V}(t) = E.\tau(t) \quad (8)$$

where

$$\begin{aligned} E &= \bar{M}^{-1}(q)\bar{B}(q) = \frac{1}{m.r.I} \begin{bmatrix} I & I \\ Rm & -Rm \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \\ \bar{M}(q) &= \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \text{ and } \bar{B}(q) = \frac{1}{r} \begin{bmatrix} 1 & 1 \\ L & -L \end{bmatrix} \end{aligned}$$

4. Tracking Controller Design

The problem can be interpreted as consisting of the slave robot to robot reference, whose trajectory is given by $t \rightarrow [x_r(t), y_r(t)]$. It is then desired to control the zero error vector $[x_r(t) - x(t), y_r(t) - y(t)]$, where $[x_r(t) y_r(t) \theta_r(t)]^T$ denotes the coordinate vector generalized robot reference and $[x(t) y(t) \theta(t)]^T$ the vector of generalized coordinates of the real robot [1-6].

For the tracking control problem, a time-varying reference mobile robot model is given as [7]

$$\dot{q}_r = J(\theta_r)V_r \quad (9)$$

where \dot{V}_r and $\dot{q}_r(t)$ denote the reference velocity and posture of the WMR, $J(\theta_r)$ is the Jacobean defined in the Eq. (4).

We define the error between the desired positions and orientations, and actual by

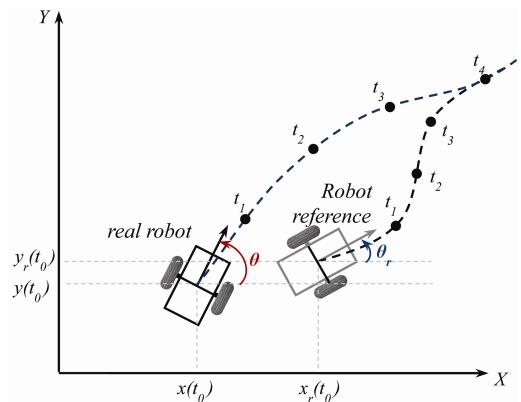
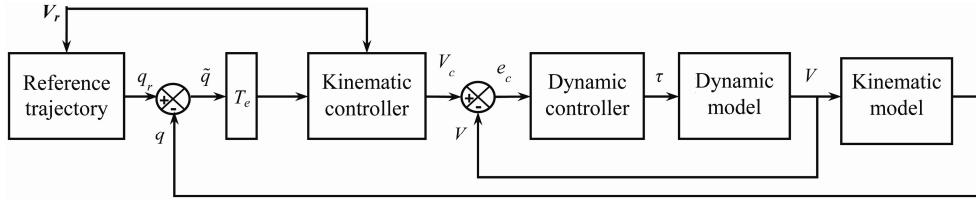


Fig. 2. Characterization of the trajectory tracking.


Fig. 3. Architecture robot controller

$$\tilde{q} = q_r - q = \begin{bmatrix} (x_r - x) \\ (y_r - y) \\ (\theta_r - \theta) \end{bmatrix} = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} \quad (10)$$

We define q_e as following

$$q_e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \quad (11)$$

T_e is called the matrix $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The vector of error variations can be expressed as :

$$\dot{q}_e = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix} = v \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + w \begin{bmatrix} e_2 \\ -e_1 \\ -1 \end{bmatrix} + \begin{bmatrix} v_r \cos e_3 \\ v_r \sin e_3 \\ w_r \end{bmatrix} \quad (12)$$

5. Design of Hierarchical Controller

Fig. 3 illustrates the architecture of the controller designed to control a WMR.

6. Kinematic Controller

Proposition : Let the Lyapunov function candidate

$$L_0 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1-\cos\theta}{k_2} \geq 0, k_2 > 0 \quad (13)$$

where ; the derivative L_0 is given by

$$\dot{L}_0 = \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 \frac{\sin e_3}{k_2} \quad (14)$$

Eq. (11) gives

$$\begin{cases} e_1 = (x_r - x) \cos\theta + (y_r - y) \sin\theta \\ e_2 = -(x_r - x) \sin\theta + (y_r - y) \cos\theta \\ e_3 = \theta_r - \theta \end{cases}$$

Then

$$\begin{cases} \dot{e}_1 = -v + we_2 + v_r \cos e_3 \\ \dot{e}_2 = -we_1 + v_r \cos e_3 \\ \dot{e}_3 = w_r - w \end{cases}$$

By substitution in the Eq. (14)

$$\begin{aligned} \dot{L}_0 = & (-v + we_2 + v_r \cos e_3)e_1 + (-we_1 + v_r \sin e_3)e_2 \\ & + (w_r - w)\frac{\sin e_3}{k_2} \end{aligned}$$

The control law for the system to be stable is $\dot{L}_0 \leq 0$, so we choose

$$\begin{cases} v_c = v_r \cos e_3 + k_1 e_1 \\ w_c = w_r + k_2 v_r e_2 + k_3 \sin e_3 \end{cases}$$

Therefore the Eq. (14) can be rewritten as

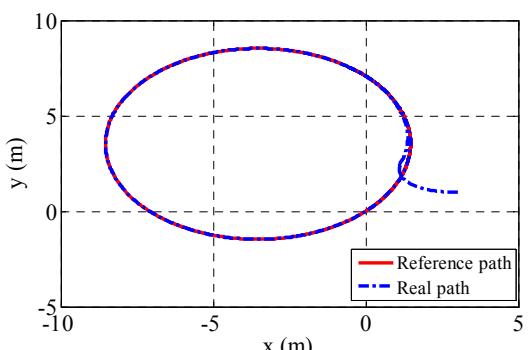
$$\dot{L}_0 = (-k_1 e_1)e_1 + (-k_3 \sin e_3)\frac{\sin e_3}{k_2} = -k_1 e_1^2 - \frac{k_3}{k_2} (\sin e_3)^2$$

For $\dot{L}_0 \leq 0$ must k_1, k_2 and $k_3 \in R_+$

We have opted for the following values $k_1 = 10$, $k_2 = 5$ and $k_3 = 4$.

7. Dynamic Controller

We used in the present work two control technicals, first one is simple, it is a PI controller, the second is based on the nonlinear method.


Fig. 4. The robot trajectory in X-Y plane

7.1 PI control in dynamic level

To ensure that the movement of the WMR can follow the desired velocity controller generated by the kinematic, two dynamic controllers are introduced in this section. Both controllers are PI controllers responsible for providing left and right torques capable of powering the left and right wheels.

The parameters of both controllers are $k_p = k_i = 2$.

7.2 Simulation results

To show the effectiveness of the proposed controller, simulations were performed in Matlab-Simulink. The examples chosen is that the tracking of a circular path.

The parameters of the robot used in the simulation are: $m = 4 \text{ kg}$, $I = 2.5 \text{ kg m}^2$, $R = 0.15 \text{ m}$ and $r = 0.03 \text{ m}$.

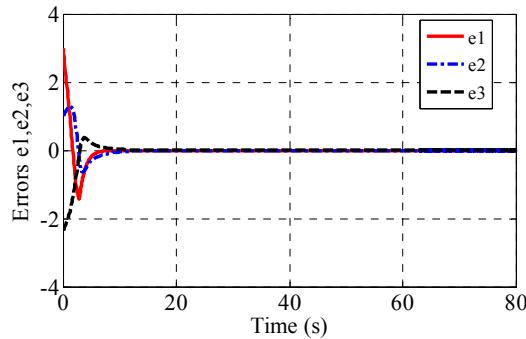


Fig. 5. Tracking errors trajectory e_1 , e_2 and e_3

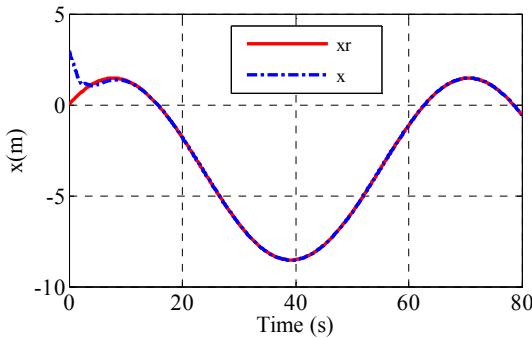


Fig. 6. The tracking errors in X-coordinate

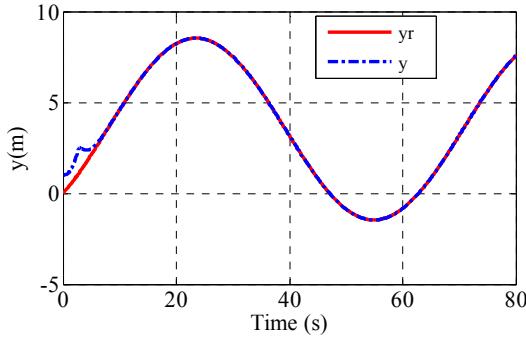


Fig. 7. The tracking errors in Y-coordinate

The actual initial posture of the mobile robot is $q(0) = [3 \ 1 \ 180^\circ]^T$. The initial posture of the robot is defined by reference: $q_r(0) = [0 \ 0 \ 0]^T$.

PI controller gave us satisfactory results. Figs. 5, 6 and 7 show significant oscillations errors, dynamic errors, Figs. 10 and 11, are large. to reduce its peaks and errors, we replaced the PI power controller by Lyapunov controller.

7.3 Lyapunov controller associated to a sliding mode control in dynamic level

In this section, we replaced the PI controller with a Lyapunov controller associated to a sliding mode control to improve the results to previous results.

The sliding surface is defined by selected

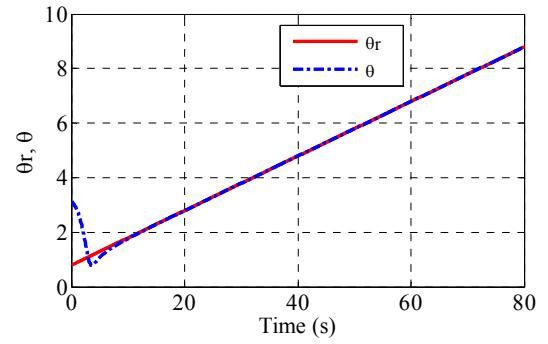


Fig. 8. The tracking θ_r and θ

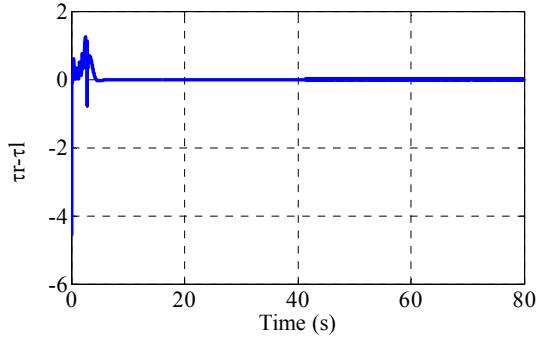


Fig. 9. The tracking error torque ($\tau_r - \tau_l$) (N^*m)

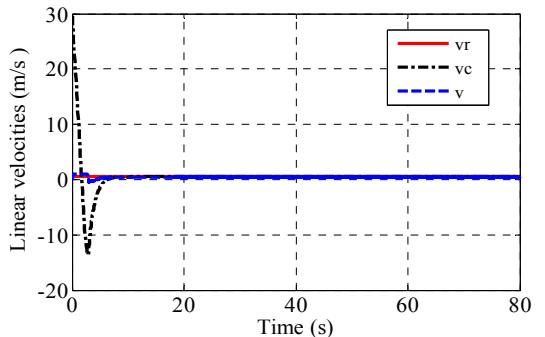


Fig. 10. Linear velocities

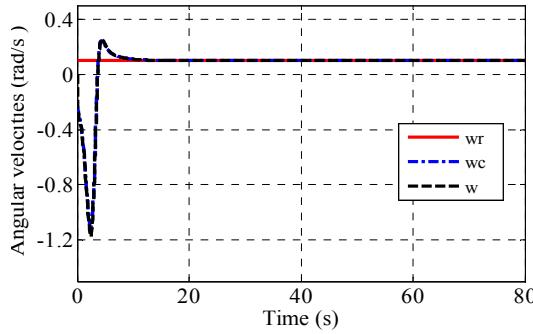


Fig. 11. Angular velocities

$$s(t) = ke + \dot{e}, \quad k > 0 \quad (15)$$

where

$$\mathbf{e} = \begin{bmatrix} e_v \\ e_w \end{bmatrix} = \begin{bmatrix} v_c - v_r \\ w_c - w_r \end{bmatrix} \quad (16)$$

For a good pursuit of velocity, it is important to make the invariant surface $\dot{S}(t) = 0$ and attractive $S^T \dot{S} < 0$

$$\dot{V}(t) = E\tau(t) + \Delta f(t) \quad (17)$$

where $\Delta f(t)$ present a parameter uncertainties and external disturbances.

Proposition:

Let the Lyapunov function candidate

$$L_1 = \frac{1}{2}e_v^2 + \frac{1}{2}e_w^2 > 0 \quad (18)$$

where ; the derivative \dot{L}_1 is given by:

$$\dot{L}_1 = \dot{e}_v e_v + \dot{e}_w e_w = (\dot{v}_c - \dot{v}_r)e_v + (\dot{w}_c - \dot{w}_r)e_w \quad (19)$$

Eq. (8) gives

$$\begin{cases} \dot{v}_c = E_{11}\tau_r + E_{12}\tau_l \\ \dot{w}_c = E_{21}\tau_r + E_{22}\tau_l \end{cases} \quad (20)$$

Eq. (19) becomes

$$\dot{L}_1 = (E_{11}\tau_r + E_{12}\tau_l - \dot{v}_r)e_v + (E_{21}\tau_r + E_{22}\tau_l - \dot{w}_r)e_w \quad (21)$$

If we put

$$\begin{cases} \tau_r = \frac{1}{E_{11}}(-E_{12}\tau_l + \dot{v}_r + k_a e_v) \\ \tau_l = \frac{1}{E_{22}}(-E_{21}\tau_r + \dot{w}_r + k_b e_w) \end{cases} \quad (22)$$

We find

$$\dot{L}_1 = k_a e_v^2 + k_b e_w^2 \quad (23)$$

The Lyapunov condition is satisfied for $k_a < 0$ and $k_b < 0$, where $k_a = -100$ and $k_b = -1000$

From the Eq. (17)

$$\begin{cases} \dot{v}_c = E_{11}\tau_r + E_{12}\tau_l + \Delta f_v \\ \dot{w}_c = E_{21}\tau_r + E_{22}\tau_l + \Delta f_w \end{cases} \quad (24)$$

The derivative of Lyapunov becomes

$$\begin{aligned} \dot{L}_2 &= (\dot{v}_c - \dot{v}_r)e_v + (\dot{w}_c - \dot{w}_r)e_w \\ &= (E_{11}\tau_r + E_{12}\tau_l + \Delta f_v - \dot{v}_r)e_v \\ &\quad + (E_{21}\tau_r + E_{22}\tau_l + \Delta f_w - \dot{w}_r)e_w \end{aligned} \quad (25)$$

If we put

$$\begin{cases} \tau_r = \frac{1}{E_{11}}(-E_{12}\tau_l + \dot{v}_r + k_c \text{sgn}(e_v) + k_c \Delta f_v) \\ \tau_l = \frac{1}{E_{22}}(-E_{21}\tau_r + \dot{w}_r + k_b e_w + k_d \text{sgn}(e_w) + k_d \Delta f_w) \end{cases} \quad (26)$$

We find

$$\begin{aligned} \dot{L}_2 &= (k_a e_v^2 + k_b e_w^2) + (k_c \text{sgn}(e_v) + \Delta f_v)e_v \\ &\quad + (k_d \text{sgn}(e_w) + \Delta f_w)e_w \end{aligned} \quad (27)$$

Substitution Eq. (23) in Eq. (27) gives

$$\dot{L}_2 = \dot{L}_1 + (k_c \text{sgn}(e_v) + \Delta f_v)e_v + k_d \text{sgn}(e_w) + \Delta f_w e_w \quad (28)$$

So for $\dot{L}_2 \leq 0$, it is necessary that

$$\begin{cases} (k_c \text{sgn}(e_v) + \Delta f_v)e_v \leq 0, & \forall e_v \\ (k_d \text{sgn}(e_w) + \Delta f_w)e_w \leq 0, & \forall e_w \end{cases}$$

And $\begin{cases} k_c > |\Delta f_v| \\ k_d > |\Delta f_w| \end{cases}$, where $k_c = -10$ and $k_d = -0,5$.

7.4 Simulation results

The same initial conditions and the same parameters of the WMR are used in this section. To test the robustness of the proposed approach, we have proceeded to two tests. Initially, a large variation in the mass m was introduced, a 50% increase between t=30s → 50s. The second test is the robustness of the controller against the change of the radius r, a 10% reduction of r is applied between t=60s → 70s. The following figures illustrate these results.

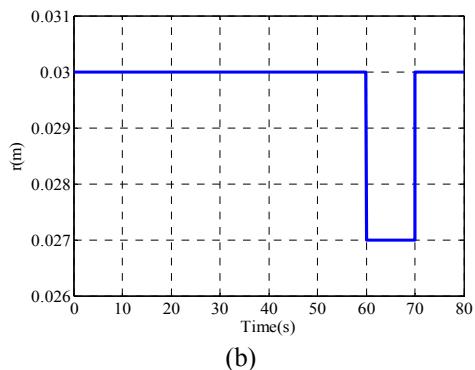
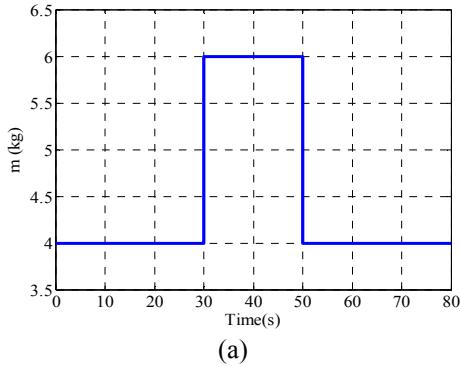


Fig. 12. Variation parameters: (a) Mass of the WMR; (b) Radius of the wheel

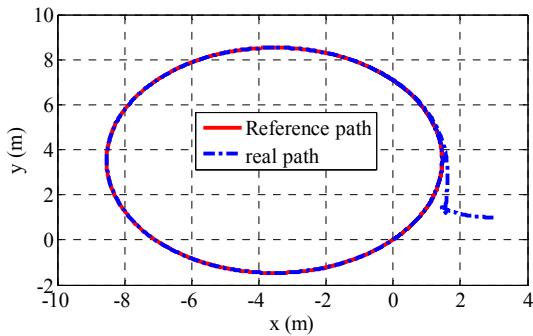


Fig. 13. The robot trajectory in X-Y plane

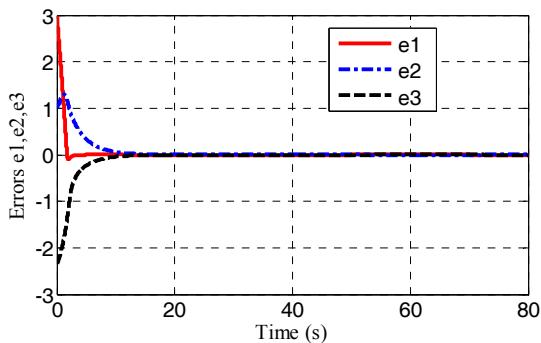


Fig. 14. Tracking errors trajectory e_1 , e_2 and e_3

Figs. 13-20 shows clearly that the proposed control is widely robust against the parameters variations. By

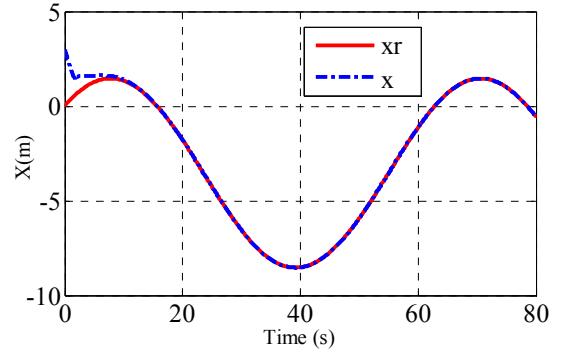


Fig. 15. The tracking errors in X-coordinate

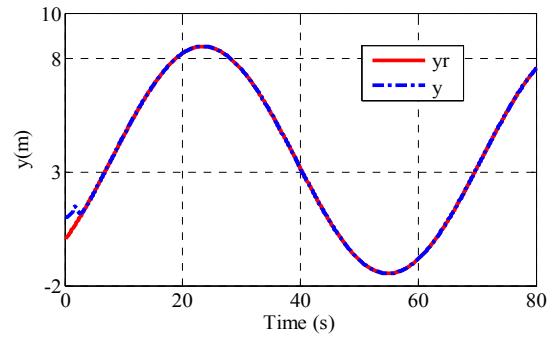


Fig. 16. The tracking errors in Y-coordinate

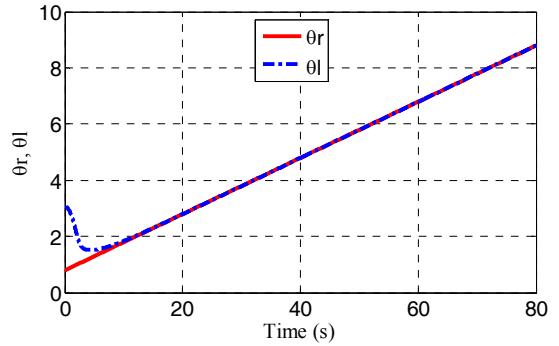


Fig. 17. The tracking θ_r and θ_l

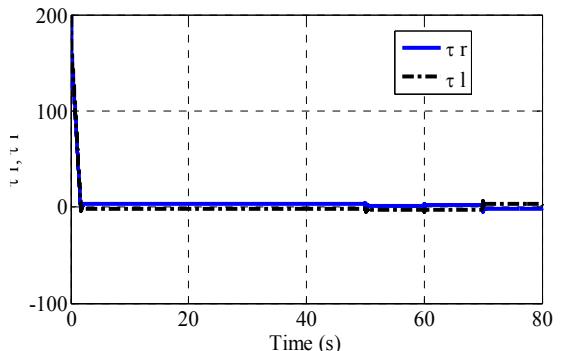


Fig. 18. The tracking error torque ($N \cdot m$)

comparison, these results with previous results, we note that the oscillations of e_1 , e_2 and e_3 are reduced, which gives us a good improvement of dynamic and static errors,

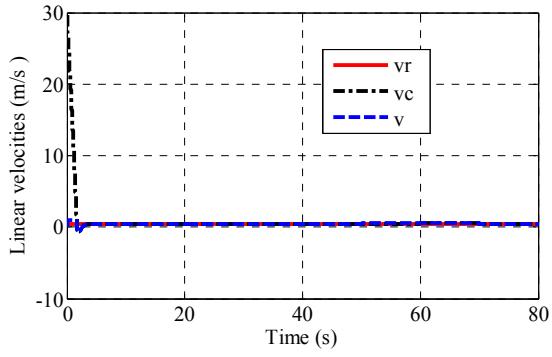


Fig. 19. Linear velocities

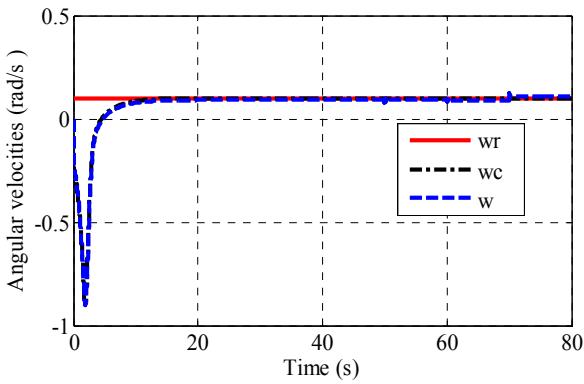


Fig. 20. Angular velocities

the same for the linear velocities and angular velocities. Tracking in y-coordinate is better and the dynamic error decreases, Fig. 17 shows the θ follows θ_r with less oscillations. Employing nonlinear method for controlling the WMR gives a good trajectory tracking and velocity, and confirms the robustness of the control.

8. Conclusion

This paper focuses on the design of a nonlinear tracking controller for a nonholonomic mobile robot with unknown parameters, we proposed a controller based on Lyapunov theory associated with the sliding mode control. The stability of the system was proved by Lyapunov theory which satisfies a good performance tracking position control. Simulation results demonstrate that the proposed controller is effective.

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