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Expansion of Terzaghi Arching Formula to Consider an Arbitrarily Inclined Sliding Surface and Examination of its Effect

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ABSTRACT : This study expanded Terzaghi arching formula, which assumed a vertical surface as a sliding surface, to consider an arbitrarily inclined surface as a sliding surface and examined the effect of a sliding surface. This study firstly developed a formula to expand the existing Terzaghi arching formula to consider an inclined surface as well as a vertical surface as a sliding surface under the downward movement of a trap door. Using the expanded formula, the effect of excavation, ground, and surcharge conditions on a vertical stress was examined and the results were compared with them from Terzaghi arching formula. The comparison indicated that the induced vertical stress was highly affected by the angle of an inclined sliding surface and the degree of influence depended on the excavation, ground, and surcharge conditions. It is expected that the results from this study would provide a better understanding of various arching phenomenon in the future.

Keywords : Arching, Trap door, Inclined sliding surface, Excavation condition, Ground condition, Surcharge pressure

1. Introduction

When a part of the bearing zone yields on a soil stratum and the remaining parts are slightly displaced relatively, the soil that is adjoining the yielded part is displaced in relation to the adjacent soil that undergoes slight displacement. Here, the relative displacement is under the shear resistance of the surface that contacts the adjacent soil, which transfers the load from the yielded part to the adjacent parts to lower the pressure on the yielded part and increase the pressure on the adjacent parts. What occurs due to this mechanism is called the arching effect, and it is observed in various conditions. The two typical phenomena of the arching effect are caused by the horizontal displacement of the excavation wall and the deflection of the top of tunnel (Fig. 1), and a good understanding of these phenomena is very important for comprehending the load behaviors related to the excavation wall and the tunnel support.

Many studies on arching have been carried out, and early efforts were made by Engesser (1882), Bierbaumer (1913), Cain (1916), Marston (1930), Caquot (1934), Völlmy (1937), and Terzaghi (1943). Völlmy observed that an actual sliding surface can be closer to a vertical surface when the trap door is located in a shallow place, but it approaches 45+ϕ/2 on average as the depth of the trap door increases. Terzaghi (1943) first carried out a systematic study of the arching mechanism using trap door tests and investigated the change in the load in the upper part due to the deflection of the trap door. However, Terzaghi assumed that the sliding surface on top of the trap door is a vertical surface along the boundary of the trap door (lines ae and bf in Fig. 1). It was also assumed that the pressure on the trap door equals the difference between the weight of soil on top of the trap door and the shear resistance, which reaches its full capacity along the vertical surface. Since then, many studies have been conducted using the original or modified versions of the original trap door

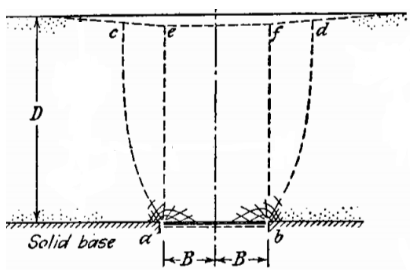

Fig. 1. Failure caused by downward movements of a trap door of the base of a layer of sand (Terzaghi, 1943)

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tests for different conditions, either theoretically (Nielson, 1966; Getzler et al., 1970; Spangler & Handy, 1982; Adachi et al., 1999), experimentally (McNulty, 1965; Ladanyi & Hoyaux, 1969; Ono & Yamaha, 1993; Paikowsky et al., 1993; Adachi et al., 2003; Chau & Bolton, 2006; Sardrekarimi & Abbasnejad, 2010), or numerically (Koutsabeloulis & Griffiths, 1989; Sakaguchi & Ozaki (1992); Chevalier & Otani, 2010; Chen et al., 2011). Moradi & Abbasnejad (2013) summarized arching studies and described many of them in detail.

Though many studies have been performed, most studies of the arching mechanism using the trap door tests assumed that the sliding surface on top of the trap door is vertical, which can differ from actual field conditions where the sliding surface can be inclined (Costa et al., 2009; Pardo & Sáez, 2014). To overcome the limitation of the assumption of a vertical sliding surface this study expanded the Terzaghi arching formula, which is based on a vertical sliding surface, to consider an arbitrarily inclined sliding surface and examined the effect of an inclined sliding surface. The expanded formula was used to study the change in vertical stress under various conditions such as excavation condition, ground condition, and surcharge pressure and to compare with the results using the Terzaghi arching formula. The results of this study would be helpful for understanding various arching phenomena in more actual field conditions.

2. Terzaghi Arching Formula

Terzaghi (1943) developed an arching formula based on the assumption of Cain (1916) among the various arching theories. Cain assumed that 1) the soil is homogeneous, isotropic, and in the semi-infinite state; 2) the shear resistance of soil is represented by $\tau = c + \sigma\tan\phi$; 3) the sliding surface is vertical and adjoining the external boundary of deflection zone; 4) the vertical sliding surface is under the greatest shear resistance; and 5) the coefficient of earth pressure, which is the ratio between vertical and horizontal earth pressures, is consistent. Based on these assumptions, Terzaghi developed an arching formula considering the force equilibrium of the differential area between two vertical surfaces as shown in Fig. 2.

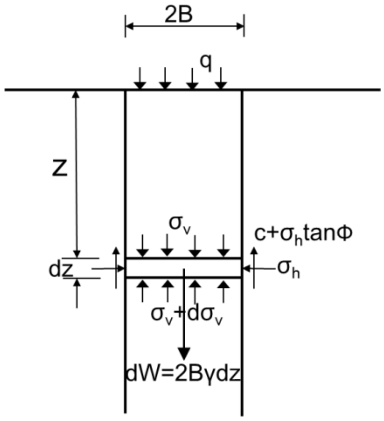


Fig. 2. Diagram illustrating assumptions on which computation of pressure between two vertical surfaces of sliding is based (Terzaghi, 1943)

$$\begin{aligned} & 2B\cdot\gamma dz = 2B(\sigma_v + d\sigma_v) - 2B\sigma_v + 2cdz + 2K\sigma_v\tan\phi dz \\ & \Rightarrow \frac{d\sigma_v}{dz} = \gamma - \frac{c}{B} - K\sigma_v\frac{\tan\phi}{B} \\ & \sigma_v = q \ \text{ for } \ z = 0 \end{aligned} \tag{1}$$

$$\therefore \sigma_v = \frac{B\cdot\gamma - c}{K\cdot\tan\phi}(1 - e^{-K\tan\phi\frac{z}{B}}) + qe^{-K\tan\phi\frac{z}{B}} \tag{2}$$

For $z = \infty$ and surcharge pressure on the ground surface, $q = 0$:

$$\sigma_v = \frac{B\cdot\gamma - c}{K\cdot\tan\phi} \tag{3}$$

(where, $2B$ = width of deflection, γ = unit weight, c = cohesion, ϕ = friction angle, K = earth pressure coefficient, q = surcharge pressure on ground surface)

3. Derivation of an Arching Formula to Consider an Arbitrarily Inclined Sliding Surface

The aforementioned arching formula of Terzaghi (1943) is based on only the vertical sliding surface and may be significantly different from the actual sliding surface. To examine the actual effect of this issue, this study expanded Terzaghi's arching formula to consider an arbitrarily inclined sliding surface. In fact, a sliding surface does not generally form a consistent sliding angle from the part of deflection to the ground surface. Nevertheless, this study assumed that the inclined sliding angle is consistent regardless of the depth with a view to examining the effect of inclined sliding surface conditions. Although an arching theory which can consider the effect of depth of sliding angle can be developed considering a function reflecting the influences of depth, it would be far more complicated and require a numerical method, and therefore this study leaves it for future work.

To derivate the expanded arching formula due to the deflection of trap door, this study used the assumptions that Terzaghi applied except for the fact that the angle of sliding surface is not limited to vertical. As shown in Fig. 3, this study considered the force equilibrium in the differential area between the sliding surfaces with an angle (α) to derive an arching formula that considers the arbitrarily inclined sliding surfaces as follows.

Considering the force equilibrium of the differential zone in the direction of depth (z),

$$\gamma\cdot\left[2Bdz+\frac{2(H-z)dz}{\tan\alpha}-\frac{dz^2}{\tan^2\alpha}\right]dz$$
$$=\left[2B+\frac{2[(H-(z+dz))]}{\tan\alpha}\right]\cdot(\sigma_v+d\sigma_v)$$
$$-\left[2B+\frac{2(H-z)}{\tan\alpha}\right]\sigma_v+2\sigma_n\frac{dz}{\tan\alpha}+2\tau_n$$
$$\Rightarrow\left[2B\gamma dz+\gamma\cdot\frac{2(H-z)dz}{\tan\alpha}+\frac{2\sigma_v}{\tan\alpha}-2\sigma_n\frac{1}{\tan\alpha}\right.$$
$$\left.-2c-2\sigma_n\tan\phi\right]dz==\left[2B+\frac{2(H-z)}{\tan\alpha}\right]d\sigma_v \tag{4}$$

$$\therefore\frac{d\sigma_v}{dz}=\frac{\left[2B\gamma dz+\gamma\cdot\frac{2(H-z)dz}{\tan\alpha}+\frac{2\sigma_v}{\tan\alpha}-2\sigma_n\frac{1}{\tan\alpha}-2c-2\sigma_n\tan\phi\right]}{\left[2B+\frac{2(H-z)}{\tan\alpha}\right]}$$

$\sigma_n=\frac{\sigma_v+K\sigma_v}{2}+\frac{\sigma_v-K\sigma_v}{2}\cos2\alpha$, where K = Earth pressure coefficient

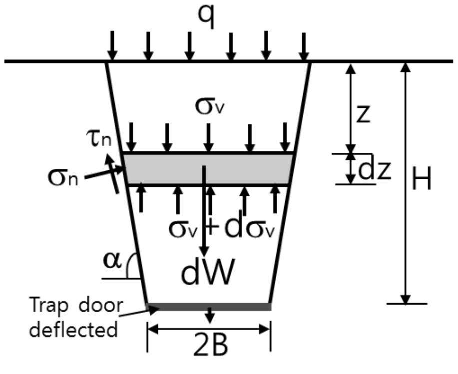


Fig. 3. Diagram illustrating assumptions on which computation of pressure between two inclined surfaces of sliding is based

$$\frac{d\sigma_v}{dz}=$$
$$\frac{\left[2B\gamma dz+\gamma\cdot\frac{2(H-z)dz}{\tan\alpha}-2c+[\frac{(1-K)}{\tan\alpha}(1-\cos2\alpha)-(1+K+(1-K)\cos2\alpha)\tan\phi]\sigma_v\right]}{\left[2B+\frac{2(H-z)}{\tan\alpha}\right]}$$

$$\Rightarrow\frac{d\sigma_v}{dz}=$$
$$\frac{d\sigma_v}{\left[2B\gamma dz+\gamma\cdot\frac{2(H-z)dz}{\tan\alpha}-2c+[\frac{(1-K)}{\tan\alpha}(1-\cos2\alpha)-(1+K+(1-K)\cos2\alpha)\tan\phi]\sigma_v\right]}$$
$$=\frac{dz}{\left[2B+\frac{2(H-z)}{\tan\alpha}\right]} \tag{5}$$

Let's put A as $[\frac{(1-K)}{\tan\alpha}(1-\cos2\alpha)-(1+K+(1-K)\cos2\alpha)\tan\phi]$

Let's put D as $\frac{-2}{\tan\alpha}$

$$\int A\times D\times\frac{d\sigma_v}{\left[2B\gamma dz+\gamma\cdot\frac{2(H-z)dz}{\tan\alpha}-2c+[\frac{(1-K)}{\tan\alpha}(1-\cos2\alpha)-(1+K+(1-K)\cos2\alpha)\tan\phi]\sigma_v\right]}$$
$$=\int A\times D\times\frac{dz}{\left[2B+\frac{2(H-z)}{\tan\alpha}\right]}$$
$$\Rightarrow-\frac{2}{\tan\alpha}\ln\left[2B\gamma+\frac{2(H-z)\gamma}{\tan\alpha}+-2c+A\sigma_v\right]=A\ \ln[2B+\frac{2(H-z)}{\tan\alpha}]$$
$$\Rightarrow C_1e^{A\times-\frac{\tan\alpha}{2}\cdot\ln[2B+\frac{2(H-z)}{\tan\alpha}]}=2B\gamma+\frac{2(H-z)\gamma}{\tan\alpha}-2c+A\sigma_v \tag{6}$$

If $z=0$, $\sigma_v=q=$ surcharge pressure on ground surface

$$C_1e^{A\times-\frac{\tan\alpha}{2}\cdot\ln\left[2B+\frac{2H}{\tan\alpha}\right]}=2B\gamma+\frac{2H\cdot\gamma}{\tan\alpha}-2c+Aq$$
$$\Rightarrow C_1=[2B+\frac{2H}{\tan\alpha}]^{\frac{A\tan\alpha}{2}}\times\left[2B\gamma+\frac{2H\cdot\gamma}{\tan\alpha}-2c+Aq\right] \tag{7}$$

$$\therefore\sigma_v=\frac{1}{A}\left[(2B+\frac{2H}{\tan\alpha})^{\frac{A\tan\alpha}{2}}\times(2B+\frac{2(H-z)}{\tan\alpha})^{-\frac{A\tan\alpha}{2}}\right.$$
$$\left.\times(2B\gamma+\frac{2H\gamma}{\tan\alpha}-2c+Aq)-(2B\gamma+\frac{2(H-z)\gamma}{\tan\alpha}-2c)\right] \tag{8}$$

(Where, A = variable, $2B$ = width of deflection, γ = unit weight, c = cohesion, ϕ = friction angle, K = earth pressure coefficient, q = ground surcharge pressure, α = inclination angle of sliding surface)

Verification 1 :

If $z=0$, $\alpha=90°$, surcharge pressure $=q$

$$\therefore\sigma_v=\frac{1}{-2K\cdot\tan\phi}(-2K\cdot\tan\phi\cdot q)=q,\ o.k \tag{9}$$

Verification 2 :

If $z=H=\infty$, $\alpha=90°$, surcharge pressure $=q$

$$\therefore\sigma_v=\frac{-2B\gamma+2c}{-2K\cdot\tan\phi}=\frac{B\gamma-2c}{K\cdot\tan\phi},\ o.k \tag{10}$$

sliding surface is always 90° when considering the Terzaghi's formula.

As shown in the figure, the vertical stress decreased as the cohesion value increased. Similar to the above discussion, the vertical stress increased with a deeper excavation depth and it was below 0 value under a certain inclination angle implying unstable stress condition. This is attributable to the fact that the cohesion of ground directly affects the maximum shear strength induced on the sliding surface and therefore the force equilibrium under different inclination angle of sliding surface.

4. Analysis of the Effect of Excavation Condition, Ground Condition and Surcharge Pressure and Comparison of Results

4.1 Analysis of the Effect of Excavation Condition

Fig. 4 comparatively shows the vertical stresses induced with the different excavation width and depth under varying inclination angle of sliding surface. The inclination angle of sliding surface is always 90° when considering the Terzaghi's formula.

As shown in the figure, the vertical stress increased as the excavation width and depth increased. The effect of the inclination angle became more prominent as the excavation width increased and the excavation depth decreased. When the inclination angle of sliding surface was smaller than a certain angle, the vertical stress was less than 0 implying unstable stress condition. In this study, any vertical stress less than 0 was represented as 0 value.

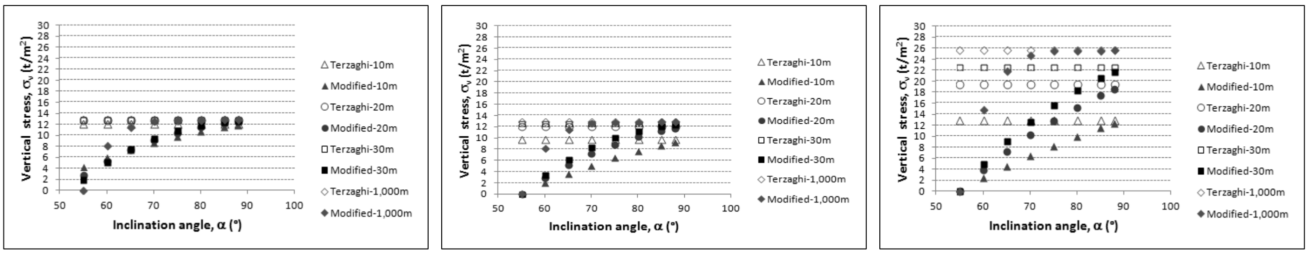
4.2 Analysis of the Effect of Cohesion

Fig. 5 comparatively shows the vertical stresses induced with the different cohesion and excavation depth under varying inclination angle of sliding surface. The inclination angle of sliding surface is always 90° when considering the Terzaghi's formula.

4.3 Analysis of the Effect of Friction Angle

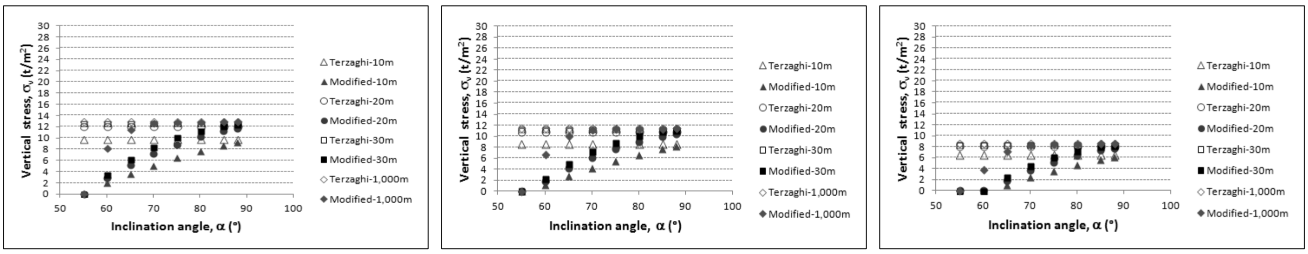
Fig. 6 comparatively shows the vertical stresses induced with the different friction angle (ϕ) and excavation depth under varying inclination angle of sliding surface. The inclination angle of sliding surface is always 90° when considering the Terzaghi's formula.

As shown in the figure, as the friction angle increased the vertical stress decreased and the effect of inclination angle was also decreased. This is attributable to the friction angle of the ground directly affecting the maximum shear strength induced on the sliding surface. The vertical stress was less than 0 at low inclination angle and friction angle. However, the condition disappeared when the inclination



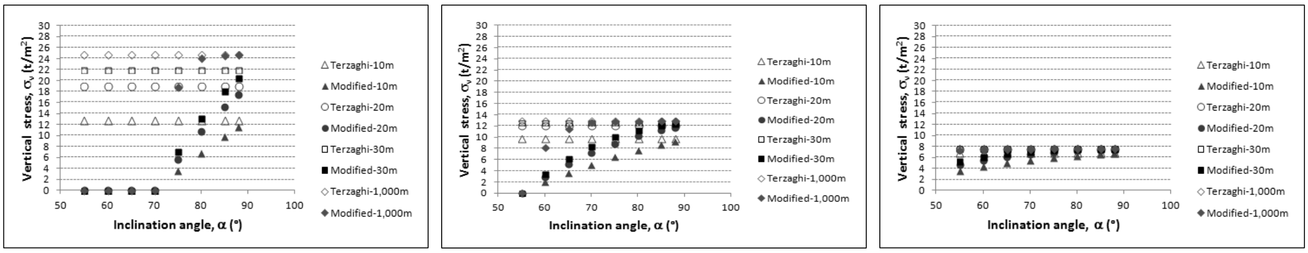
(a) 2B = 2 m (b) 2B = 10 m (c) 2B = 20 m

Fig. 4. Effect of excavation width and depth under varying inclination angle (γ = 1.8 t/m^3, c = 0, ϕ = 35°, q = 0, K = 1)



(a) c = 0 (b) c = 1 t/m^2 (c) c = 3 t/m^2

Fig. 5. Effect of cohesion and excavation depth under varying inclination angle (γ = 1.8 t/m^3, 2B = 10 m, ϕ = 35°, q = 0, K = 1)



(a) $\phi = 20°$ (b) $\phi = 35°$ (c) $\phi = 50°$

Fig. 6. Effect of friction angle and excavation depth under varying inclination angle ($\gamma = 1.8$ t/m^3, 2B = 10 m, c = 0, q = 0, K = 1)

angle and friction angle increased. The results indicate that the vertical stress in a trapdoor problem is interactively affected by the combination of the sliding and frictional conditions.

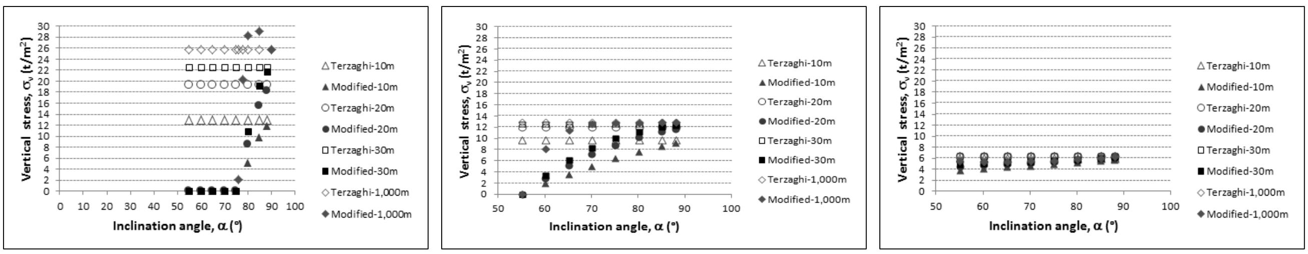
4.4 Analysis of the Effect of Earth Pressure Coefficient

Fig. 7 comparatively shows the vertical stresses induced with the different earth pressure coefficient (K) and excavation depth under varying inclination angle of sliding surface. The inclination angle of sliding surface is always 90° when considering the Terzaghi's formula.

As shown in the figure, as the earth pressure coefficient increased, the vertical stress decreased and the effect of inclination angle was also decreased. If the earth pressure coefficient was decreased to 0.5 the vertical stress was less than 0 when the inclination angle was smaller than 75°. In addition the vertical stress did not show a consistent pattern with the inclination angle near 80°. when the excavation depth was very deep. On the other hand, if the earth pressure coefficient was increased to 2, the vertical stress was relatively small and there was a little change in the vertical stress with the inclination angle. This results show that the earth pressure coefficient of ground directly affects the maximum shear strength induce on the sliding surface and therefore the force equilibrium under different inclination angle of sliding surface.

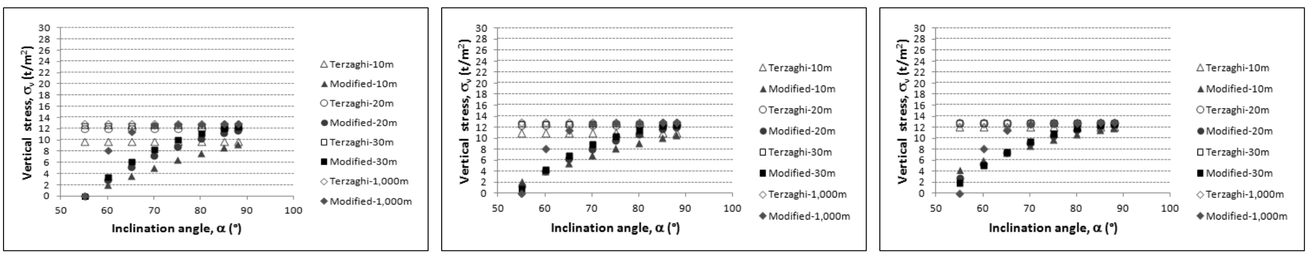
4.5 Analysis of the Effect of Surcharge Pressure

Fig. 8 comparatively shows the vertical stresses induced with the different surcharge pressure (q) and excavation depth under varying inclination of sliding surface. The inclination angle of sliding surface is always 90° when considering the Terzaghi's formula.



(a) K = 0.5 (b) K = 1 (c) K = 2

Fig. 7. Effect of earth pressure coefficient and excavation depth varying inclined angle ($\gamma = 1.8$ t/m^3, 2B = 10 m, c = 0, $\phi = 35°$, q = 0)



(a) q = 0 (b) q = 5 t/m^2 (c) q = 10 t/m^2

Fig. 8. Effect of surcharge pressure and excavation depth varying inclined angle ($\gamma = 1.8$ t/m^3, 2B = 10 m, c = 0, $\phi = 35°$, K = 1)

As shown in the figure, the vertical stress increased overall as the ground surcharge pressure increased and the effects of the excavation depth and inclination angle was decreased. The change in the vertical stress was increased when the excavation depth and inclination angle were decreased. When the surcharge pressure was small or the excavation depth was big, the vertical stress was less than 0 in the range of inclination angle smaller than 55°. This is attributable to the fact that the ground surcharge pressure directly affects the maximum shear strength induce on the sliding surface and therefore the force equilibrium under different inclination angle of sliding surface.

5. Conclusions

This study expanded the Terzaghi arching formula, which is based on a vertical sliding surface, to consider an arbitrarily inclined sliding surface and examined the effect of an inclined sliding surface. The expanded formula was used to study the change in vertical stress under various conditions such as excavation condition, ground condition, and surcharge pressure and to compare with the results using the Terzaghi arching formula. The following conclusions were drawn from this study.

(1) Terzaghi arching formula assumes only a vertical sliding surface in 2D plane strain conditions and has some limitations. The formula was expanded to consider an inclined sliding surface and an arching formula to consider inclined sliding surfaces was derived.

(2) Extended parametric studies were conducted to examine the change in vertical stress under various inclination angles of the sliding surface, excavation width, ground condition, and surcharge pressure condition. The results indicated that a vertical stress can be significantly affected by the inclination angle of the sliding surface and the excavation condition. The degree of influence varied according to both the ground and surcharge pressure conditions interacting with excavation and sliding conditions.

(3) The arching phenomenon based on the new formulation could be more realistic for the conditions that may have inclined sliding surfaces. It is expected that the results from this study will provide better understanding of various arching phenomena in the future.

(4) A sliding surface may not form a consistent sliding angle from the part of deflection to the ground surface. In addition, the excavation length in a longitudinal direction can be an important factor to affect the arching phenomena in actual field conditions. These factors are left for future works.

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