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Developing Metacognitive Skills of Mathematics Learners[1,2]

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Metacognition means "thinking about one's own thinking". There are generally two aspects of metacognition:
i) Reflection – thinking about what we know; and
ii) Self-regulation – managing how we go about learning.
Developing metacognitive abilities is not simply about becoming reflective learners, but about acquiring specific learning strategies as well. There are several strategies that may be used by teachers to develop metacognitive skills amongst learners. As part of a Professional Development project secondary school mathematics teachers have been developing their knowledge and skills to teach for metacognition. In this paper we analyze two lessons presented by groups of teachers in the project and tease out similarities and differences between the lessons that afford or hinder the development of metacognitive skills of learners.

Keywords: metacognition, mathematics learners, knowledge-building tasks
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INTRODUCTION

The framework of the school mathematics curriculum in Singapore (Ministry of Education, 2012) places emphasis on metacognition and clarifies it as follows: "Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning. To develop metacognitive awareness and strategies, and know when and how to use the strategies, students should have opportunities to solve non-routine and open-ended problems, to discuss their solutions, to think aloud and reflect on what they are doing, and to keep track of how things are going and make changes when necessary (Ministry of Education, 2012, p.17). Research has shown that good problem solvers have well developed metacognitive skills. They know how to recognise gaps in their own thinking, articulate their thought processes, and revise their efforts (Brown, Bransfold, Ferrara & Campione, 1983). Metacognition is most commonly broken down into two distinct but interrelated areas. John Flavell (1979), one of the pioneer researcher in metacognition and memory, defined these two areas as i) metacognitive knowledge – awareness of one's thinking, and ii) metacognition regulation – the ability to manage one's own thinking processes.

Brown and her colleagues (1983) describe three ways by which we direct our own learning. The ways are

i) Planning approaches to tasks – identifying the problem, choosing strategies, organising our thoughts, and predicting outcomes;
ii) Monitoring activities during learning – testing revising, and evaluating the effectiveness of our strategies; and
iii) Checking outcomes – evaluating the outcomes against specific criteria of efficiency and effectiveness.

Metacognition in the Classroom

Two key conditions support a metacognitive classroom environment. They are knowledge-centered and learner-centered learning environments. (Bransfold, Brown & Cocking, 2000). Knowledge-centered classrooms focus on meaningful and non-trivial activities where students are engaged in activities that build on their previous knowledge, challenge them with complex tasks, and require active sense-making. In such classrooms students need access to procedural knowledge – How are you going to do this and be successful? – as well as conditional knowledge – When is this going to be useful to you? Learner-centered classrooms take into account students' current knowledge, skills, attitudes, and beliefs. Metacognitive activities that ask students to reflect on what they know, care about, and are able to do not only help learners develop an awareness of themselves, but also give learner-centered teachers valuable information for their instruction.

Teachers who are developing metacognitive skills in the classroom help students' incorporate active reflection in their learning. They model and scaffold the processes of reflection, questioning, evaluating, and other thinking strategies that may not come naturally. According to Darling-Hammond et al., (2001) some of these strategies are predicting outcomes, evaluating work, questioning by the teacher, self-assessing, self-questioning, selecting strategies, using directed or selected thinking, using discourse, critiquing and revising. Problem posing is yet another strategy that engages students in active reflection (Brown & Walter, 2005).

THE EPMT PROJECT

Enhancing the Pedagogy of Mathematics Teachers (EPMT II) is a professional development project in which forty secondary mathematics teachers from seven secondary schools in Singapore are presently participating. The teachers in the project are developing their knowledge and skills to teach for metacognition in their mathematics lessons. The project comprises three phases and the phases are as follows: Phase I - In this phase of the project, the teachers met once a week for seven weeks for 3 hours each time. During the meetings the teachers examined performative and knowledge-building mathematics tasks, used typical textbook questions and modified them into knowledge-building tasks. They were also introduced to noticing using a four lens approach, classroom mathematical norms and the why, what and how of classrooms which develop metacognitive skills amongst learners. During the last two sessions, participants worked in groups (according to their schools) planning lessons that would teach for metacognition.

Phase II - In this phase in each of the seven schools the teachers worked together to plan and enact a lesson that was learner-centered and knowledge-centered with the goal of developing metacognitive skills amongst the learners. Two project meetings were held during the phase. During the meetings the school teams showcased their lessons, presenting video-recorded segments of the lessons that "developed metacognition". Peer feedback was also gathered from the teachers who were not in the team presenting using the 4-lens noticing approach. The research team collated the peer feedback for the respective schools and sent to them. Following which, individual school based meetings were held by the researchers. During these meetings the project teams in the school were guided in reviewing the enactment of their planned lesson and writing a narrative of the lesson highlighting the metacognitive strategies that were evident and also missed opportunities. Teachers who planned, enacted and reviewed their lessons were also encouraged to write a journal about their learning journey. Teachers in the project are now in Phase II..

During the meetings the data collected are reviewed and implications discussed. The aims and objectives During Phase III, the project participants will continue to work with their project mates and other teachers in their respective schools to advance the knowledge and skills they have gained from the first two phases. Periodic project meetings will be held for the teachers to share their lessons and invite critique from their peers to improve on their lessons that develop metacognitive skills for mathematics learners. In this paper we examine two lessons presented by two schools during phase II of the project.

A study of two lessons conducted by teachers in the EPMT project

Lessons presented by schools S1 and S2, in the second phase of the EPMT project, are examined for similarities and differences to tease out what afforded or hindered the development of metacognitive skills amongst the learners during their enactment. Our framework of analysis is informed by conditions that support a metacognitive environment in mathematics classrooms, i.e. classroom discourse that is learner-centered and knowledge-centered. Therefore we examined the following aspects of the lessons: the learning tasks, enactment of the learning tasks, opportunities for the development of metacognitive skills and opportunities for students to demonstrate their engagement in metacognition, such as self-questioning or critiquing. Table 1 shows the brief outline of the two lessons.

From Table 1, we can infer that the learning task used in school 1 was of the type knowledge building while those used in school two were routine performative learning tasks to work on the areas of triangles. From the flow of the lessons we can also infer that guided practice dominated the learning in school 2 while in school 1, problem posing guided by a rubric that delineated the criteria of the problems to be posed formed the bedrock of the lesson.

Table 1. Outline of the two lessons

	School	
	S1	**S2**
Topic	Application of trigonometry – Simple 3D problems	Trigonometry - Area of a Triangle
Level	Sec 3 Express Course	Sec 3 Normal Academic Course
Duration	70 minutes	50 minutes
Lesson Objectives	1. Apply trigonometry to solve simple 3D problems. 2. Problem posing.	To derive the formula for area of a triangle.

Table 1 (cont.). Outline of the two lessons

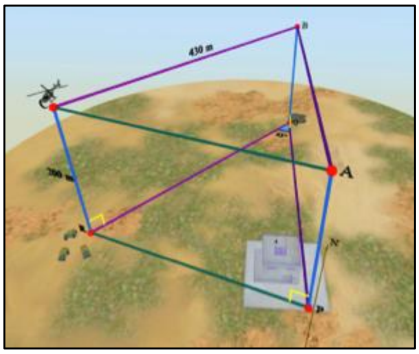
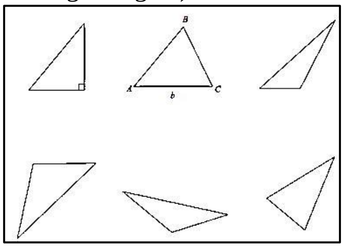
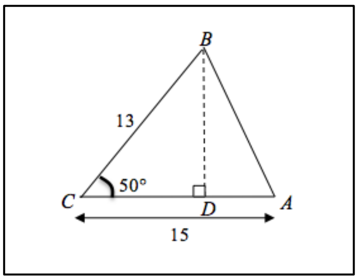
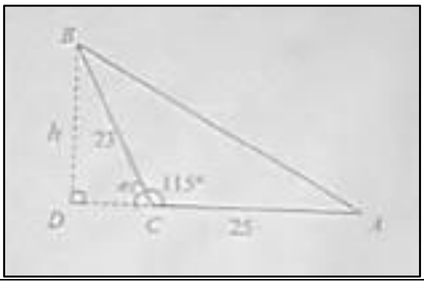
School	S1	S2
Learning Task(s)	Task 1.1 Pose problems using the given task. A helicopter flies in a triangular circuit *ABC* at a constant height of 200 m above the ground. *A*, *B*, and *C* are directly above the points *P*, *Q*, and *R*, which are markings on level ground. *P* is due east of *R*, the bearing of *Q* from *P* is *342°*, *BC* = 430 m and $\angle PQR=43°$. 	Task 2.1 Identify the base and the height of the given triangles and draw the height of the various triangles (right-angled and non-right-angled).  Task 2.2 Case 1: (Included angle is acute) Suppose Δ*ABC* has sides *AC* = 15 cm and *BC* = 13 cm, and the included angle, $\angle C$ =50°, is acute. Find Base b, Height h and area of Δ*ABC*.  Case 2: (Included angle is obtuse) Suppose Δ*ABC* has sides *AC* = 25 cm and *BC* = 23 cm, and the included angle, $\angle C$ =115°, is obtuse. Find Base b, Height h and area of Δ*ABC*. 

Table 1 (cont.). Outline of the two lessons

School	S1	S2
Flow of the Lesson	Introduction of lesson objectives → Recap →Task guidance (Introduction of Rubrics) → Pair Work (Pose problems) → Think - Pair – Share → Class Discussion → Closure	Introduction of lesson objectives → Recap → Guided Practice (Task 1) → Guided Practice (Task 2) → Conclusion → Practice

An analysis of the lessons was carried out by the researchers together with the teachers of the respective schools that planned and enacted the lessons. The lesson narratives were constructed and episodes in the lessons that provided students with opportunities to develop metacognitive skills were examined. Table 2 lists the findings of the analysis.

Table 2. Learning Strategies for the Development of Metacognitive Skills

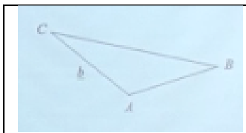
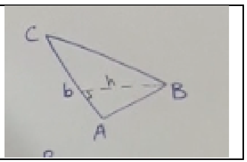
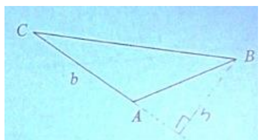
Learning Strategy: Questioning by the teacher	
S1	Teacher asks thought provoking questions at various junctions to get students to think. Examples: T: "What does it mean by P is due east of R, the bearing of Q from P is 342°?" (A student demonstrates what it means by drawing it on the whiteboard.) T: "Do you think it is possible to form right-angled triangles based on the 3D drawing?" "Name some right-angled triangles." S: Yes. Triangles *CRQ*, *CRP*, and *APQ*. T: "Are the requirements in the rubrics met?" T: "Is the solution accurate and presented clearly?" T: "Any other approaches to solve the problem?"(to challenge them to come up with alternative (better) solutions for the question)
S2	Questions asked by the teacher have low levels of demand for thinking. They are mainly to elicit information. Examples: T noticed an error (perpendicular drawn incorrectly) in the drawing and asks whether anyone has a different answer.   T: Can anybody make it better? What is the definition? S: It is wrong. T: How it is wrong? S: It is not perpendicular. T: How to make it perpendicular? S: Extend another right angle. T: How do I extend another right angle? Can you show me? (Student draws the perpendicular outside the triangle). T: So what is the base now? Can anybody tell me? S: Same T: Ah the base is still the same. So the base is AC. 

Table 2 (cont.). Learning Strategies for the Development of Metacognitive Skills

Learning Strategy: Critiquing	
S1	Teacher instructs pairs of students to exchange their work. They next evaluate their classmates' problem posed and its solution. They give feedback according to the criteria listed in the given rubric. Using the feedback students attempt to improve the problems posed. By critiquing one another's work students get a chance to look at alternative solutions or provide more efficient method of getting solutions. Students get to communicate their sense making when they were developing solution to their problems and justify their choice of the strategy to their peers. It also allows students (who are receiving feedback) to identify the gaps in their solution and to improve their own thinking process. Examples: Students "critique" other students' work, spot errors and explain their choice of strategy. S1: You can use hypotenuse then times 2. S2: But it's not times 2. S3: It's the wrong answer. S4: This angle and this side, can we find Q? And we also have BA. We have QA and the angle BQA. So we can find angle BQA.
S2	Critiquing is not observed during any of the learning episodes.
Learning Strategy: Self-questioning	
S1	Teacher provides opportunities for self-questioning by giving the students a rubric to ask themselves a series of questions while they work. The rubric is given before the task, with elaboration by the teacher so that students are guided on what they should look out for when posing problems. T explains the following deliverables to be produced by saying, "There are 4 things I want you to think about": i) Topic relevance. "T: you are learning trigonometry but talk about volume then you are digressing the topic." ii) Inter-dependency. "T: A good indicator of inter-dependency is that, let's say, if you use answer from part A to solve the subsequent parts of the question." (Highlighted that a good indicator is when the previous answer is applied to obtain the solution of the subsequent part of the question) iii) Mathematical errors (e.g. rounding off error, units). "T: Of course, your solutions cannot have any errors. Please figure it out within your pairs if you notice any errors." iv) Use of mathematical terminology "T: Please try to phrase your question appropriately such that everyone in the class will be able to understand."
S2	Students do not get many opportunities to reflect on their own thinking process because the task was highly scaffolded and it gets students to follow only the 'targeted' method.
Learning Strategy: Using directed or selective thinking	
S1	While students work on the 3D problem, the teacher asks them to break up the3D problem into 2D planes and recognise angles. This process helps students to understand the problem, identify the given information and plan the next/series of step(s) to take.
S2	The teacher does not use this learning strategy in the lesson.

Table 2 (cont.). Learning Strategies for the Development of Metacognitive Skills

Learning Strategy: Problem Posing	
S1	Students are involved in specific learning process by shifting of responsibility from teachers to students, generating new problem through "Inquiry - based -learning" environment. Teacher instructs the students to work in pairs and assigns them to craft problems based on the given knowledge building task. A rubric is given to the students before the task, with elaboration by the teacher. The teacher explains the deliverables to be produced so that students are aware of what they should look out for when posing problems.
S2	The teacher does not use this learning strategy in the lesson.

From Table 2, it is apparent that in school 1, the teachers planned and enacted a lesson that provided several opportunities for students to develop their metacognitive skills. However, the same was not the case for school 2 where the teachers relied totally on one strategy – questioning by the teacher and failed. This was because the questions asked were merely to recall and comprehend knowledge.

DISCUSSION AND NEXT STEP

Two key conditions that support a metacognitive classroom environment are knowledge-centered and learner-centered tasks and activities. In school 1, our analysis shows that the teachers planned and enacted a lesson using a knowledge building task that was the bedrock of the learner-centered activities that the teacher engaged his students with. An attempt was also made to use five learning strategies, namely – questioning by the teacher, critiquing, self-questioning, and using directed or selective thinking and problem posing. However, in school 2 the teachers planned and enacted a rather different sort of lesson. Instead they used a task that lacked cognitive demand and may be referred to as a performative task. This task was used to engage students in "hands-on" work that was directed by the teacher closely. Therefore it appears that "knowledge-building" by students was not the focus. The task and its enactment failed to create an environment that was both knowledge-centered and learner-centered. Furthermore, the only learning strategy planned and used for developing metacognitive skills was 'questioning by the teacher'. This strategy also failed to achieve its objective as the questions asked by the teacher were merely for recall and comprehension of knowledge.

During the meeting with teachers in school 2, the teachers having realised that their lesson did not teach for metacognition, revisited their learning about teaching for metacognition and re-planned the lesson for "next time". They now re-crafted the learning task so that it asks students to infer what they know, and can use to find the area of a triangle when they are not able to use the formula half base times height. Triangles of several types would be part of the task. They also planned to widen the scope of strategies to develop metacognitive skills by facilitating questioning by teacher, self-questioning by students and critiquing.

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