A Perspective on Teaching Mathematics in the School Classroom[1]

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WHAT we teach, and HOW students experience it, are the primary factors that shape students' understanding and beliefs of what mathematics is all about. Further, students pick up their sense of mathematics from their experience with it. We have seen the results of the approach to "break the subject into pieces and make students master it bit by bit. As an alternative, we strive to create a teaching environment in which students are DOING mathematics and thereby engender selected aspects of "mathematical culture" in the classroom. The vehicle for doing this is the so-called Japanese Open-ended approach to teaching mathematics. We will discuss three aspects of the open-ended approach – process open, end product open, formulating problems open – and the associated approach to assessing learning.

INTRODUCTION

Vision without action is merely a dream.
Action without vision just passes the time.
Vision with action can change the world.

— **Joel Arthur Barker, Futurist**

So, what might be a vision?

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AN OPEN APPROACH TO TEACHNG MATHEMATICS

The objectives in this talk are:

1. To describe an "open approach" to teaching mathematics;
2. To describe lesson plans and lesson records and point out their crucial roles in this approach to teaching;
3. To illustrate the approach with some comfortable examples which I hope you will find indulging;
4. To illustrate an approach to assessing student learning using this teaching approach.
5. To exhibit some research findings regarding the effects of teaching using the "Open Approach."
6. Reforming school mathematics is an important goal.

Calls for reform are urgent, but not new.

In many countries, official reform *writers* have prepared authoritative papers, reform *documents* have been published, reform *projects* have been started, and reform *movements* have been launched. In the U.S., driving the reform are official documents such as the Curriculum & Evaluation Standards (NCTM, 1989), the Professional Standards for Teaching Mathematics (NCTM, 1991), and the Assessment Standards (NCTM, 1995), and these were integrated into the Principles and Standards for School Mathematics (NCTM, 2000). And these, and more, have recently led to the Common Core State Standards for school mathematics (CCSSM) - highly controversial due mainly to the associated changes to *assessment* (*standardized testing*). The common "philosophy" in all these documents represents a shift in paradigms, namely, to no longer view "teaching as treatment" and learning as effect." Rather, students are viewed as learners who actively construct mathematics. An open approach to teaching mathematics is proposed to complement the established curriculum, not replace it. The aim is to make this an indispensable part of school teaching. A multitude of problems, lesson plans, lesson records at all levels have been developed to

(1) Draw on students' natural ways of thinking,
(2) Mathematize situations,
(3) Find rules and patterns in solving problems,
(4) See other students' results and discoveries,
(5) Examine, compare and evaluate ideas of different students through discussion,
(6) Modify and further develop ideas.

The approach is rich for developing mathematical thinking.

UNDERLYING ASSUMPTION

<u>What</u> we teach, and <u>how</u> students experience it, are the primary factors that shape students' understanding and beliefs of what mathematics is all about (NCTM, 2000).

PEDAGOGICAL STANCE

Students pick up their sense of a domain, such as mathematics, from their experience with it. We've seen the results of the approach to "break the subject into pieces and make students master it bit by bit." Here's an alternative: Create an instructional environment in which the students are – at a level appropriate for them – <u>doing</u> mathematics.   That is, engender selected aspects of "mathematical culture" in the classroom. (Schoenfeld, 1991) This stance fits the recommendations of the reform movement in the U.S.

OPENNESS IN MATHEMATICS EDUCATION

An open approach to teaching mathematics is proposed to complement the established curriculum, not replace it. The aim is to make this an indispensable part of school teaching. A multitude of problems, lesson plans, lesson records at all levels have been developed to:

- Draw on students' natural ways of thinking,
- Mathematize situations,
- Find rules and patterns in solving problems,
- See other students' results and discoveries,
- Examine, compare and evaluate ideas of different students through discussion, led by the teacher, and
- Modify and further develop ideas.

The approach is rich for developing mathematical thinking. Below we see the components of openness:
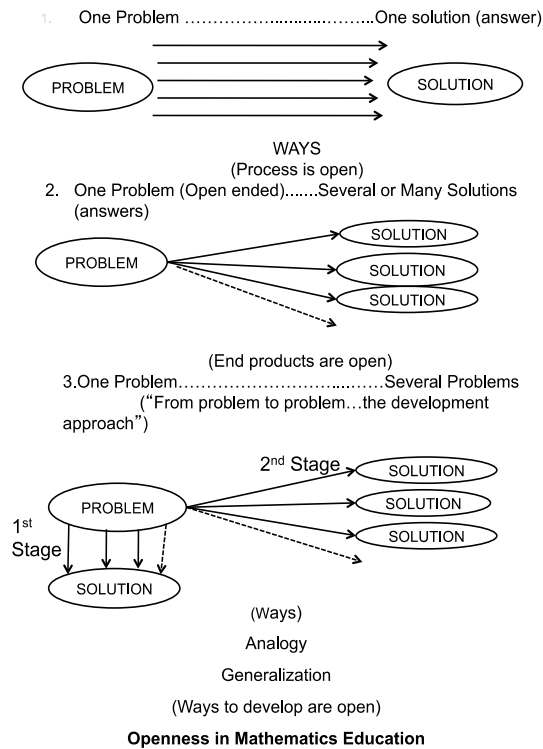
Figure 1. Openness in Mathematics Education

IMPLEMENTING THE APPROACH

Detailed *lesson plans* are crucially important:

1. Choose or develop a good problem situation,
2. Develop a DETAILED lesson plan for teaching,
3. Write down all anticipated student responses to help the teacher to a good understanding of the problem, be prepared to respond to students' responses and assess student learning, and
4. Provide opportunity for students to use their own natural mathematical ways of thinking.

Use of *lesson records* is also important. Together the two contribute to **Teacher** and **Curriculum** improvement. This approach calls for a different approach to *assessment*, so by analyzing students' worksheets, observing students when e gaged solving and noticing students during discussion. Teachers can assess students' *Fluency*, *Flexibility*, *Originality* and *Elegance*. Lessons are organized in a, say, 45-minute class period as follows:

- **Introduce** the problem situation,
- **Understand** the problem,
- **Problem Solving** by individual students [the heart of the approach],
- **Comparing and Discussing** students' productions and
- **Summing Up** by the teacher.

A teacher may also ask students to write down what they have learned and to formulate new problems *of their own* as a means of evaluating a lesson.

RESEARCH RELATED TO AN "OPEN APPOACH" TO TEACHING MA THEMATICS

It is appropriate to dwell on open approach research and other related work in four parts:

I. Background to Research Done by **Japanese Mathematics Educators** [that is substantial],

II. Research by **Evans** (1964) – elementary and lower junior high school students … creativity,

III. Doctoral Dissertations done by my Students: e.g., **Tougaw** (1993), **Conway** (1996) and **Walton** (2002), and

IV. Evaluation of a large NSF-Funded Teacher Enhancement Project that focused on the Open Approach.

In general, findings in this research can be characterized as follows: evidence of significant changes in attitudes towards problem solving, attitudes towards mathematics, students' beliefs about mathematics, seeing mathematics as an activity, a shift towards *higher order thinking* and lowering students' anxiety about mathematics. Also, Walton found that changes in teaching behavior persisted significantly beyond the period of time teachers were participating in the project. The results were based on both teacher and school-student participants in the research. In addition, the Teacher Enhancement Project served as a model for substantial teacher enhancement with urban-based teachers in the Chicago Public Schools.

CONCLUDING MORAL

Based on the work and research described and reported above, and that of others, we formulate and conclude a moral:

Theorem I. Mathematics is a living, breathing and exciting discipline of sense-making.

Theorem II. Students will come to see it that way if and only if they experience it that way in their classrooms.

Corollary: Virtually all standard classroom instruction should be enhanced by courses in which students grapple with subject matter in intellectually honest ways (Schoenfeld, 1991).

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