

Integrating History of Mathematics in Teaching Cartesian Coordinate Plane: A Lesson Study¹

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The History of Mathematics (HOM) was integrated in teaching the Cartesian Coordinate Plane (CCP) to Grade Seven learners of Moonwalk National High School using Lesson Study. After the lesson was taught, there were three valuable issues emerged: (1) HOM is a Springboard and/or a Medium of Motivation in Teaching CCP; (2) The History of CCP Opened a Wider Perspective about Its Real-life Application in the Modern World (3) Integration of History Developed a Sense of Purpose and an Appreciation of Mathematics

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Among Learners. Feedbacks solicited from the learners showed that they have understanding of the importance of studying Mathematics after they learned the life and contributions of Rene Descartes to Mathematics. Hence, integration of history plays a vital role in developing positive attitudes among learners towards Math.

Keywords: lesson study, history of mathematics, Cartesian coordinate plane, learners, topic-demonstrator

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I. INTRODUCTION

“Mathematics is known as a discipline that is so difficult to teach that it has developed a specific educational sub-discipline with its own paradigms and characters (mathematical education) (Furinghetti, 2000; as cited in Burns, 2010).” Heiede (1992) stressed-out in his study that the main purpose why he teaches Math is not to save the world and make the lives of people easy, rather he teaches Math because of its own heritage, richness and beauty must be passed on to others. Also, Mitchell (1938) mentioned that the purpose of studying the history of mathematics “may or may not aid in predicting future developments and in devising directly prophylactic measures for the improvement of civilization; but it certainly does furnish an understanding of the present that can be obtained in no other way (p. 23).”

Educational researchers are fully doing their efforts to develop a meaningful Mathematics curriculum. Few among them strongly encouraged the inclusion of History of Mathematics (HOM) in some of the topics in Mathematics as a way of developing positive attitudes among the learners to Mathematics, as well as, a way of developing sense of purpose of studying Math among learners. Furthermore, The National Council of Teachers for Mathematics (NCTM, 2000) encourages the integration of HOM and pointing out its benefits among the learners. NCTM (2000) presented five reasons for including the HOM in Mathematics curriculum, these were:

- (1) Increasing motivation and creating a more positive approach to mathematics;
- (2) Obstacles to the development of mathematics in the past are a means to understanding the difficulties of learning in the present;
- (3) History raises and reveals the humanistic aspects of mathematical knowledge;
- (4) History provides an indicator and guide to teaching; and
- (5) Historical problems can help students develop mathematical thinking.

In the study conducted by Gazit (2012) he heightened that there is a rational justifica-

tion that emphasizes the HOM as a core part of determining the formation of learners' attitudes towards the field. Moreover, the inclusions of HOM will help the learners to develop significant appreciation about the abstract nature of Mathematics. Thus, giving them the notion that, regardless of its abstract nature, Mathematics "is a discipline that undergone an evolution and not something that has arisen out of thin air (Jankvist, 2009).

Swetz (1994) highlighted that the HOM "supplies human roots to the subject. It associates mathematics with people and their needs. It humanizes the subject and, in doing so, removes some of its mystiques." HOM promotes the appreciation of the great people who dedicated their whole life for the advancement of Mathematics. "Students who view mathematics as a set of discrete, unrelated topics may have difficulty in understanding the relational worth of each mathematical concept, its attachment and value to human life" (Panasuk & Horton, 2012). Hence, HOM may serve as a tool for teachers to inculcate the sense of appreciation among the learners about studying Mathematics, regardless of their notion that Mathematics is sometimes a hard-to-deal-with subject.

II. REVIEW OF RELATED LITERATURE

"A basic tenet of teachers of mathematics is that students [*sic*] attitudes toward mathematics are important and that a major task of the teacher is to promote positive attitudes (McBride & Rollins, 1977)." According to Middleton (1995) learners are motivated if the teacher integrates hands-on activities and real-life examples which challenge them to go beyond the regular structure of classroom discussion and activities. Learners' stimulation in Mathematics will greatly help them understand and appreciate the beauty of Mathematics.

Jugyoukenkyuu or in English translation *Lesson Study* (LS), is a professional development approach which originated from Japan and was considered as one of the trending tools in improving teaching and learning process among educators of the 21st century (Lewis, Perry & Murata, 2006). Teachers from different countries have started to acknowledge the positive impact the LS has in improving the teaching and learning process. This has been circulated in Japan and it is considered as the most crucial educational practice professional development of its teachers (Yoshida, 2005). In Japan, LS is conducted in different school areas. For example, teachers from different schools within the division area are invited to observe a class and to evaluate the teaching-learning process. After the actual teaching, a post-lesson discussion is conducted to share valuable insights for the improvement of teaching strategies and methodologies.

With the positive impacts of LS, Maddern (2000) "expressed that the rest of the world has started taking notice of this Japanese innovation." "People have realized that it con-

tributes to the high standards of educational achievement in the country (Maddern, 2000).” “After years of conducting LS, the Japanese mathematics education community has implicitly but strongly linked good mathematics instruction with the problem-solving approach (Elipane, 2012).” Also, he added that Japanese lesson study presents an educational practice that may lead to the achievement of the standards for the teacher education in the Philippines. Thus, many countries have started to make their own version of LS in their respective countries. To name a few, we have China, Indonesia, Iran, Germany and Singapore (Sarkar Arani, Fukaya&Lassegard, 2010).

Lewis (2002, p. 1) mentioned that LS provides an ongoing method to improve instruction on careful observation of students and their work. She also reiterated that LS has four cycles where teachers work together to:

- (1) Formulate goals for student learning and long-term development;
- (2) Collaboratively plan a research lesson;
- (3) Conduct the research lesson, with one team member teaching and others gathering evidence on student learning and;
- (4) Discuss the evidence gathered during the lesson and use it to improve the lesson.

Despite of many cycles or stages that the teacher should undergone to be able to conduct a LS, it is still being known and becoming a trend worldwide. It is for the fact that the basic foundation behind conducting LS is to improve teaching, “the most effective place to do so would be in the context of a classroom lesson (Stigler & Hiebert, 1999).”

Using LS, this study aims to determine if integration of history of mathematics will help improve the teaching of CCP. Moreover, it aims to develop a sense of purpose among learners as to why there is a prevalent need to study Mathematics.

III. METHODOLOGIES

The authors are taking up Masters of Science in Teaching Mathematics (MST-Mathematics) at De La Salle University-Manila (DLSU-Manila), and the LS was a part of their requirements in the academic course “History and Philosophy of Mathematics” in the third term of the academic year 2014–2015. The academic course has a total of 42 hours of lectures, reporting, and preparation and conducting the LS. In one session of the academic course, the professor invited the class to watch a video of a Japanese teacher executing LS so the class will become familiar about how it should be executed and the benefits of conducting it.

To reach the goal of conducting an LS, the authors followed certain steps to achieve a unified effort in collaborative lesson study by using the suggested procedures of Stigler &

Hiebert (1999; as cited in Artista et al., 2014). Due to minimal time for preparation, the goal of the authors was to conduct one cycle of LS, and to join the class to critique and reflect on its execution.

1. Definition of a problem

The authors had a lot of options but eventually decided to have CCP as the topic for their LS. The same topic was the topic demonstrated by one of the authors during his final demonstration teaching back in college. That idea gave the authors an opportunity to assess and to eventually improve what has been done before. At the start, the authors let their co-author to point out the strengths and weaknesses of his teaching demonstration. Based on that, the authors suggested a better way of demonstrating the topic, as well as, elicited possible problems that will arise during the conduct of their LS. The authors worked hand-in-hand, contributed ideas, and capitalized on each other strengths to make their LS successful.

2. Plan the mathematics lesson

The authors believed that as early as Grade 7, the learners should have an understanding about the importance of familiarization among the important people who dedicated their lives so we could have a better foundation of our Modern Mathematics. Moreover, they should also be encouraged to continue innovating new ideas for Mathematics to flourish. To establish a link between the academic course and the LS, the authors considered in choosing a topic was the integration of HOM. Among their options were Pythagorean Theorem—to introduce the life and contributions of Pythagoras and the Babylonians, and CCP—to appreciate the works of Rene Descartes in Mathematics. Thus, the authors decided to choose CCP as the topic of their LS. Besides, the introductory lesson about CCP is a part of the lesson framework for Grade 7 Mathematics. In accordance, the CCP is one of the lessons in Mathematics that has numerous real-life applications, and one of these is the idea of location.

3. Teach and observed the lesson in the classroom

One member of the team was tasked to deliver the lesson to the invited participants, while the others were tasked to address pertinent matters during the LS. The authors also used the recorded videotapes to keenly evaluate the learning process that transpired through teacher-learner's and learner-learner's interaction during the lesson. Also, the authors jotted down in verbatim form all the valuable constructive criticisms, suggestions and commendation shared by the observers during the post conference. Moreover, the learners' behaviors, actions and form of conversations were also analyzed in order to de-

termine how the effect of HOM in learning the CCP lesson.

4. Critique and reflectively discuss the lesson after classroom observations

The post-lesson discussion was started by inviting the teacher, the authors, the teacher-observers, and the professor to form a huddle right after the topic demonstration. Also, the authors provided two evaluation forms, one for the teacher-observers and the other for the learners. The teacher-observers were asked to evaluate the effectiveness of integrating HOM and applying real-life situations in teaching CCP using LS. On the other hand, the learners were asked to evaluate the impact of the lesson to them, as well as, to determine their stands about the need and impact of integrating HOM in teaching of Mathematics.

5. Share the results

The authors merged all the inputs into a research output which describes the impact of LS and integration of HOM in improving the lesson on CCP, which is the main focus of this paper.

Research Respondents and Locale

The introductory topic about the CCP is a part of the Mathematics Curriculum for Grade 7. Thus, the authors invited 20 Grade 7 learners from Moonwalk National High School located in Pasay City, Metro Manila to become the participants of this study. The authors chose the learners through purposive sampling method. The authors made sure that these learners were not yet exposed on the topic to be carried-out by one of the authors. This is to ensure they will meet the objectives of the topic.

Presentation of the Lesson

At the beginning, the teacher asked the learners if they have tried to watch a movie in a movie house. In this manner, the teacher established the idea of location based on the number of their seats in the movie house. Then, the teacher used this idea to introduce Rene Descartes, a French Mathematician who discovered the CCP. The teacher told the story of how Rene Descartes discovered the CCP through the fly on the ceiling while he was lying on his bed. Similarly, the teacher asked the learners to find the attached cut-out picture of a fly inside the classroom.

The teacher prepared an improvised CCP on the floor with balloons, colored papers and cut-out pictures—these represented the points on the CCP. This was the main instructional material of the activity called “Save the Earth.” The teacher brought the learners into a situation where they needed to save our planet by detonating the bombs planted by the terrorists all over the world. The balloons served as the bombs while the improvised

CCP as the Earth itself. The idea of locating the bombs helped the learners discover how to locate points on the CCP.

The authors maximized the use of the improvised CCP in the activity by pair. The first part of the activity required each pair to locate the points on the CCP by popping out each balloon. While on the other part, the learners were asked to give the ordered pairs of the cut-out pictures (crescent moon, stars, box, etc.) attached on the CCP. The first in each pair who popped out the balloon, and gave the correct answer received a chocolate bar as a reward.

To establish the application of locating of points on the CCP to real-life situations, the learners were asked to locate the positions of certain establishments on the city map. The establishments represented the ordered pairs while the city map as the CCP.

On the assessment part of the lesson, the learners were asked to reveal the mystery picture by plotting the given ordered pairs and connecting them in parallel to the instructions provided on the assessment sheet. They performed this assessment by group. The learners had difficulties in revealing the mystery picture due to ambiguity of where to connect each point and when to stop connecting points. Thus, among the three groups, only one group successfully revealed the picture of a butterfly on the CCP on a given time. Afterwards, the learners were asked to color their work.

To synthesize the topic, each of the learners was asked to pick a piece of paper in a jar, each paper has a concept and what they need to do is to share the learnings they gained from that. Lastly, they were also asked about the practical applications of the topic into the real-world setting.

IV. RESULTS AND FINDINGS

After the lesson was taught, there were three valuable issues emerged pertaining to the integration HOM in teaching CCP, these were:

4.1. HOM is a Springboard and/or a Medium of Motivation in Teaching CCP

Silence was observed among the learners when they had a face encounter with their teacher. This silence was turned into an exciting learning atmosphere when the teacher started the lesson by asking the students to look for a printed picture of a fly inside the classroom. This engaged the learners more as they wondered why the fly was posted on the ceiling. The teacher told the class about the history of CCP. She said that there was a sick man lying on his bed named Rene Descartes. He noticed a fly on the ceiling which was exactly located on the intersection of lines. He wondered if he could use these lines to determine the exact location of the fly and he did. The teacher asked the learners to

locate the fly on the ceiling as to how Rene Descartes did. Expectedly, the learners were able to locate the said fly by using the grids on the ceiling. In relation to this, one of the teacher-observers commented:

“The students are very attentive in the discussion part of HOM, which HOM served as a motivation part, which is very interesting and very effective for the students to get introduced to the topic.”

The integration of HOM particularly of CCP in the discussion added interest to the learning process. In connection to this, one of the teacher-observers during the post conference actively shared:

“That’s the most interesting part to me because there are some teachers who cannot integrate because we are more exposed into lectures and class discussion. The LS served as a realization for me to become open to the possibility that HOM can be a good motivation.”

In relation to the totality of the lesson discussion, one of the teacher-observers mentioned:

“Overall, the class was meaningful and interesting. The lesson is simple yet the students had so much fun in learning. The activities were attractive and encouraging that makes teaching-learning in a light mood.”

On the other hand, to make the lesson in higher-order level one general suggestion came from one of the teacher-observers, he suggested:

“To make the lesson in higher-order problematized everything. It should be higher order because I believe that we can still push them because I believe one point of Mathematics education is to push the students to their limits and not to teach them what they only need to know.”

4.2 The History of CCP Opened a Wider Perspective about Its Real Life Application in the Modern World

The learners were asked to locate the positions of grocery, bank, city hall, library, theater and gas station. With this activity, the learners were surprised when they discovered that they were actually using the idea of locating of points on the CCP even before they were not yet exposed to the topic. Thus, in that happenstance they saw with their own eyes the beauty of Mathematics. Aside from this, there was also a part where the learners were put in situations which they need to provide assistance to their friends by providing accurate direction. Taking note that one unit on the CCP is equivalent to one meter, the learners were asked to provide directions on how to reach City Hall from the bank and how to get to the grocery from the library. This what transpired:

Teacher: “What will you tell to your friend if he/she asks you about the directions from the city hall going to the bank?” (Taking note that each unit in our Cartesian plane represents one meter.)

Learner: “Four meters northeast.” (A learner from the class responded right away).

To make the response of the learners clear and accurate, the teacher had a clarification:

Teacher: “How many meters going to north?”

Learners: “Four going north” (Altogether).

Teacher: “Then, after that?”

Learners: “Four going east” (In chorus).

The teacher also asked the learners about other applications of CCP in the modern world. One of them said that aside from a simple map for locating places, it can also be used in locating countries through the use of world map. While another learner mentioned that navigation is another application of CCP nowadays.

In post-lesson discussion everyone agreed that integrating HOM and applying real-life situations in teaching CCP made the learning process more interactive and easy.

4.3. The Integration of History Developed Sense of Purpose and Appreciation to Mathematics among Learners

After the discussion, the learners were invited to answer an evaluation form. Here were the feedbacks of the learners pertaining to the effectiveness of integration of HOM in class discussion:

“It is easier to understand the lesson when there is a historical background.”

“It makes the discussion more interesting.”

“It shows the sense of the Mathematics lesson.”

Furthermore, in the question about whether to or not to include HOM in class discussion, here were the responses of majority of the learners:

“Yes, so we will know the reasons why we study Math.”

“Yes, so that the students like us will appreciate the lesson more effectively.”

“Yes, to make it more interesting because it is interesting that Rene Descartes learned the Cartesian plane by the location of a fly on the ceiling.”

In this way, it was evident that the learners appreciated the integration of HOM in the discussion. It did not only make the learning interactive and entertaining. Also, it served its purpose that is to develop a sense of appreciation and a sense of connection to the significant reasons why we study Math.

V. RECOMMENDATIONS

Positive feedbacks solicited from teacher-observers about integrating HOM and applying real-life situations in teaching CCP proved its positive effects in transferring knowledge among the learners. Feedbacks solicited from the learners showed that they appreciated the integration of HOM in the topic and developed the sense of purpose of CCP in real-life.

With LS as a tool of this research, emergent issues were identified as follows:

- (1) To make a LS as a way of life of the teaching community, educational institutions should provide a timetable which can address the rigorous process of LS;
- (2) Teachers should recognize and support the learner's ability to construct knowledge by making the lesson in higher-order;
- (3) Teachers must integrate HOM in teaching Mathematics lessons to develop a sense of purpose of studying the subject among the learners; and
- (4) Teachers can use HOM as a teaching strategy that can make a smooth transition of the lesson from motivational phase towards the lesson proper.

In general, the authors propose a research and development system for improving teaching practices that shapes on the collaborative practice of LS and integration of HOM.

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