# Perturbation in the Earth's Pole due to the Recent 31 Large Earthquakes of Magnitude over 8.0 

Sung-Ho Na ${ }^{1,3}$ and Jai-Bok Kyung ${ }^{2, *}$<br>${ }^{1}$ Space Geodetic Observation Center, 276-71 Wolsangondan-ro, Sejong 30060, Korea<br>${ }^{2}$ Department of Earth Science Education, Korea National University of Education, Heungduk-gu, Cheongju 28173, Korea<br>${ }^{3}$ University of Science and Technology, Yuseong-gu, Daejeon 34113, Korea


#### Abstract

We present our estimate of pole shift caused by the recent 31 largest earthquakes of magnitude over 8.0. After reviewing theory of perturbation in the Earth's rotation, each co-seismic as well as post-seismic pole shifts by the earthquakes are acquired and illustrated. A total co-seismic excitation due to these earthquakes is $\left(x_{1}, x_{2}\right)=(-3.35,5.89)$ milliarcsec, which increased about twice the initial estimation when the post-seismic deformation is considered. The single largest co-seismic excitation by 2011 Japan earthquake was $\left(x_{1}, x_{2}\right)=(-2.06,2.36)$ milliarcsec, which corresponds to 9.7 cm pole shift on the surface of the Earth.


Keywords: earthquake, pole shift, post-seismic deformation

## Introduction

The Earth moves around the Sun following a slightly elliptical orbit and completes each round in a period about 365.25 days. The plane of this orbital rotation called ecliptic has been maintained quite stable with only tiny variations for centuries. Not only the Earth undergoes the orbital rotation in the ecliptic plane, but also spins with apparently constant rate. Because of its consistency, the human beings have been using the Earth's spin rotation for time keeping.

The Earth's spin rotation is not strictly constant but shows subtle variations both in its speed and orientation. Due to torques exerted on the Earth's equatorial bulge from the Moon and the Sun (other celestial bodies do much smaller role as $\sim 10^{-5}$ ), observer in the space can see the Earth's precession which is a conical rotation of the Earth's spinning

[^0]This is an Open-Access article distributed under the terms of the Creative Commons Attribution Non-Commercial License (http:// creativecommons.org/licenses/by-nc/3.0) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.
axis. The precession rate is about 50 arcseconds per year, and the precession is accompanied with smaller oscillations called nutation. The position of spin rotation axis on the Earth's surface is called 'pole' or more specifically Celestial Intermediate Pole (CIP). Interestingly the pole of the Earth is also slowly changing with respect to the Earth's surface, and this movement has been termed as polar motion. It should be emphasized here that we are mainly concerned with the average pole (center of Chandler wobble circular motion $=$ the principal $\mathrm{x}_{3}$-axis of the Earth) in this report. While precession and nutation are driven motions caused by the luni-solar torque, the Earth's angular momentum remains unchanged with polar motion. Two main components of polar motion are Chandler and annual wobbles, and their amplitudes are a few hundred milliarcseconds, while their periods are 1.19 and 1.00 years, respectively. Chandler wobble is the free Eulerian nutation of the whole Earth. Suppose the Earth is perfectly rigid having the same shape and density structure, its period should be about 305 days. However Chandler wobble period is observed as about 433 days, and this difference is mainly due to elastic mantle and fluid ocean (Gross, 2009). One most reliable dataset of polar motion for last thirty five years is illustrated in Fig. 1.


Fig. 1. Polar motion time series IERS EOP C04 (Jan 1981Jun 2016) (Bizouard and Gambis, 2009). First shown as it appears on the Earth's surface; x -axis is directed along the Greenwich meridian while y-axis is along 90 W . Secondly, the each two components are separately shown.

Unlike annual wobble, which can be readily interpreted as a forced motion driven by various geophysical phenomena having seasonal fluctuations, Chandler wobble has been on long debate about its energy source. Smylie pondered on seismic activity to be one possible energy source of Chandler wobble (Smylie and Manshina, 1968 and Smylie and Zuberi, 2009). Recently Chung and Na (2016) carried least square fit model analysis on the polar motion time series and concluded that there is no appreciable precursory or similar behavior of polar motion at around times of each six largest earthquakes of magnitude over 8.5 . However, there exist studies about
quantitative relation between seismic moment and pole excitation (for example, see Gross and Chao, 2006). It is our objective in this report to estimate the pole excitation caused by the recent 31 largest earthquakes. We also attempt to assess the amount of excitations associated with post-seismic deformation by using a simplified empirical model. It is noted here that although seismic excitation on the pole is quite small and so not easily detectable, the average pole should jump by that amount at time of large earthquake occurrence. Another difficulty for observing seismic excitation is that the locus of instantaneous pole never shows discontinuity for a step-function type seismic event (Smylie, 1968; Lambeck, 1980). Finally it should be made clear that, the major cause of polar motion excitation is not the seismic activity but the perturbations through the fluid outer layers of the Earth, i.e., atmosphere and hydrosphere (Gross, 2000).

## Theory Outline

The angular momentum of deformable Earth is expressed by product of Earth's inertia tensor and its angular velocity as follows (Plag et al., 2005; Gross, 2009).

$$
\left[\begin{array}{l}
L_{1} \\
L_{2} \\
L_{3}
\end{array}\right]=\left[\begin{array}{ccc}
A+\Delta I_{11} & \Delta I_{12} & \Delta I_{13} \\
\Delta I_{21} & A+\Delta I_{22} & \Delta I_{23} \\
\Delta I_{31} & \Delta I_{32} & C+\Delta I_{33}
\end{array}\right]\left[\begin{array}{c}
m_{1} \\
m_{2} \\
1+m_{3}
\end{array}\right] \omega_{0},(2-1)
$$

where $\Delta I_{i}$ and $m_{i} \omega_{0}$ are small changes in the inertia tensor and angular velocity. $C$ and $A$ are two principal moments of inertia of the Earth, and $\omega_{0}$ is the average spin angular velocity of the Earth. After some algebra through the law of angular momentum conservation in the rotating frame; $\frac{d L}{d t}+\vec{\omega} \times \vec{L}=0$, the pole position change on the Earth's surface can be expressed as the two following differential equations.

$$
\begin{align*}
& \frac{1}{\Omega} \frac{d m_{1}}{d t}+m_{2}=x_{2}-\frac{1}{\omega_{0}} \frac{d x_{1}}{d t}  \tag{2-2a}\\
& \frac{1}{\Omega} \frac{d m_{2}}{d t}-m_{1}=-x_{1}-\frac{1}{\omega_{0}} \frac{d x_{2}}{d t}, \tag{2-2b}
\end{align*}
$$

where $x_{i}$ and $\Omega$ are the excitation function and Chandler wobble frequency, and they are defined as follows (Gross, 2009).

$$
\begin{align*}
& x_{i}=\frac{1.100 \Delta I_{i 3}}{C-A}(i=1 \text { or } 2), \\
& \Omega=\frac{\omega_{0}}{433.5}\left(1+\frac{i}{2 Q}\right) . \tag{2-3}
\end{align*}
$$

We recommend Gross (2009) for more detailed information.

Dahlen did pioneering works to derive the relation between seismic moment and resultant pole excitation (Dahlen, 1971 and 1973). Based on Dahlen's formulation, Gross (1986) re-wrote the seismic pole excitation using a slightly more concrete notation as follows.

$$
\begin{aligned}
\Delta I_{13} & =\tilde{\Gamma}_{1}(\eta)\left[\frac{1}{2}\left(M_{\theta \theta}-M_{\phi \phi}\right) \sin 2 \theta \cos \phi-2 M_{\theta \phi} \sin \theta \sin \phi\right] \\
& -\tilde{\Gamma}_{2}(h) M_{r r} \sin 2 \theta \cos \phi \\
& +\tilde{\Gamma}_{3}(h)\left[-M_{r \theta} \cos 2 \theta \cos \phi+M_{r \phi} \cos \theta \sin \phi\right],(2-4 \mathrm{a})
\end{aligned}
$$

$$
\begin{aligned}
\Delta I_{23} & =\tilde{\Gamma}_{1}(h)\left[\frac{1}{2}\left(M_{\theta \theta}-M_{\phi \phi}\right) \sin 2 \theta \cos \phi-2 M_{\theta \phi} \sin \theta \sin \phi\right] \\
& -\tilde{\Gamma}_{2}(h) M_{r r} \sin 2 \theta \sin \phi \\
& -\tilde{\Gamma}_{3}(h)\left[M_{r \theta} \cos 2 \theta \sin \phi+M_{r \phi} \cos \theta \cos \phi\right], \quad(2-4 \mathrm{~b})
\end{aligned}
$$

where $M_{r r}, M_{r \theta}, M_{r \phi}, M_{\theta \theta}, M_{\theta \phi,}, M_{\phi \phi}$, are seismic moment tensor in the Earth's spherical coordinates, and $\phi, \theta, h$ are the longitude, colatitude, and depth of earthquake, and $\tilde{\Gamma}_{i}$ are the parameter determined by the Earth's interior mechanical structure.

We adopted one simple empirical relation for seismic displacement as

$$
\begin{equation*}
u(t)=u_{0}+u_{1}\left(1-e^{-t / \tau}\right), \text { for } t \geq 0 \tag{2-5}
\end{equation*}
$$

where $u_{0}, u_{1}$, and $\tau$ are the co-seismic displacement, post-seismic displacement at $t=\infty$, and decay time. The decay time $\tau$ can be varied between 80 and 200 days for different earthquakes (Hearn, 2003). We took the result of a post-seismic deformation analysis;
about 43-48\% amount of post-seismic deformation after 162 days (Baek et al, 2012). By using this data, $u_{1}$ of eq. (2-5), is determined as $u_{1}=0.524 u_{0}$ for $\tau=80$ days and $u_{1}=0.820 u_{0}$ for $\tau=200$ days respectively. For simplicity, we applied these two limiting sets of numbers to all the 31 earthquakes in this study. Also we assumed that post-seismic deformation occurs homogeneously so that post-seismic pole shift would occur following the same relation as eq. (2-5).

## Recent 31 largest earthquakes of magnitude over 8.0 since 1981

We acquired the necessary information of recent large earthquakes from the USGS internet open site (http://earthquake.usgs.gov/earthquakes/browse/), and briefly summarized them into Table 1 . The only procedure needed before estimation of seismic pole excitation is the change of representation of each seismic moment tensor from the given ones (three principal moments) by a straightforward coordinate transformation.

$$
\begin{equation*}
M_{i j}=\sum_{k=1}^{3} A_{i k} A_{j k} M_{k} \tag{2.6}
\end{equation*}
$$

where $M_{k}$ is the given principal seismic moment, and the matrix $A$ is a transformation matrix, of which column vectors consist of the direction cosines of principal directions as
$\left[\begin{array}{l}A_{11} A_{12} A_{13} \\ A_{21} A_{22} A_{23} \\ A_{31} A_{32} A_{33}\end{array}\right]=\left[\begin{array}{ccc}-\sin p l_{1} & -\sin p l_{2} & -\sin p l_{3} \\ -\cos p l_{1} \cos a z_{1} & -\cos p l_{2} \cos a z_{2} & -\cos p l_{3} \cos a z_{3} \\ \cos p l_{1} \cos a z_{1} & \cos p l_{2} \cos a z_{2} & \cos p l_{3} \cos a z_{3}\end{array}\right]$
where $a z_{i}$ and $p l_{i}$ are the azimuth and plunge angles of the $i$-th principal direction. $M_{i j}$ is the component of seismic moment tensor in the local coordinate frame $(\hat{r}, \hat{\theta}, \hat{\phi})$. The azimuth and plunge angles of USGS earthquake moment tensor are given in $(\hat{n}, \hat{e}, \hat{z})$ frame, where $z$-axis is along downward vertical. Two local frames are related as $(\hat{r}, \hat{\theta}, \hat{\phi})=(\hat{-z}, \hat{-n}, \hat{e})$.

Table 1. Recent 31 largest earthquakes (from the USGS earthquake dataset) and each of their calculated co-seismic pole shift in [milliarcsec]

| id | date | place | mag, h $(\mathrm{km})$ | $($ lat, lon $)$ | coseismic excitation $\left(x_{1}, x_{2}\right)$ |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $1985-03-03$ | Chile | $7.9,40.7$ | $33.14 \mathrm{~S}, 71.87 \mathrm{~W}$ | -0.044 | 0.124 |
| 2 | $1985-09-19$ | Mexico | $8.0,21.3$ | $18.19 \mathrm{~N}, 102.53 \mathrm{~W}$ | 0.005 | -0.065 |
| 3 | $1986-05-07$ | Aleutian | $7.9,31.3$ | $51.52 \mathrm{~N}, 174.78 \mathrm{~W}$ | -0.094 | 0.009 |
| 4 | $1989-05-23$ | Australia | $8.2,15.0$ | $52.34 \mathrm{~S}, 160.57 \mathrm{E}$ | 0.061 | -0.047 |
| 5 | $1994-06-09$ | Bolivia | $8.2,651$. | $13.84 \mathrm{~S}, 67.55 \mathrm{~W}$ | -0.043 | 0.068 |
| 6 | $1994-10-04$ | Kuril | $8.3,60.5$ | $43.77 \mathrm{~N}, 147.32 \mathrm{E}$ | -0.188 | 0.215 |
| 7 | $1995-07-30$ | Chile | $8.0,30.5$ | $23.34 \mathrm{~S}, 70.29 \mathrm{~W}$ | -0.046 | 0.112 |
| 8 | $1995-10-09$ | Mexico | $7.9,13.5$ | $19.06 \mathrm{~N}, 104.21 \mathrm{~W}$ | 0.006 | -0.042 |
| 9 | $1996-02-17$ | Indonesia | $8.1,11.5$ | $0.89 \mathrm{~S}, 136.95 \mathrm{E}$ | -0.014 | -0.032 |
| 10 | $1998-03-25$ | Australia | $8.1,17.5$ | $62.88 \mathrm{~S}, 149.52 \mathrm{~W}$ | -0.049 | -0.117 |
| 11 | $2000-11-16$ | Papua N.G | $8.0,23.5$ | $3.98 \mathrm{~S}, 152.17 \mathrm{E}$ | 0.020 | 0.032 |
| 12 | $2001-06-23$ | Peru | $8.4,23.5$ | $12.27 \mathrm{~S}, 73.64 \mathrm{~W}$ | 0.063 | 0.234 |
| 13 | $2003-09-25$ | Japan | $8.2,23.5$ | $41.82 \mathrm{~N}, 143.91 \mathrm{E}$ | -0.083 | 0.129 |
| 14 | $2004-12-23$ | Australia | $8.1,13.5$ | $49.31 \mathrm{~S}, 161.35 \mathrm{E}$ | 0.091 | 0.055 |
| 15 | $2004-12-26$ | Sumatra | $9.1,13.5$ | $2.30 \mathrm{~N}, 95.98 \mathrm{E}$ | -0.623 | 0.561 |
| 16 | $2005-03-28$ | Sumatra | $8.6,30.5$ | $2.09 \mathrm{~N}, 97.11 \mathrm{E}$ | -0.180 | 0.082 |
| 17 | $2006-05-03$ | Tonga | $8.0,60.5$ | $20.19 \mathrm{~S}, 174.12 \mathrm{~W}$ | 0.069 | 0.031 |
| 18 | $2006-11-15$ | Kuril | $8.3,11.5$ | $46.59 \mathrm{~N}, 153.27 \mathrm{E}$ | -0.227 | 0.186 |
| 19 | $2007-01-13$ | Kuril | $8.1,11.5$ | $46.24 \mathrm{~N}, 154.52 \mathrm{E}$ | 0.120 | -0.113 |
| 20 | $2007-04-01$ | Solomon | $8.1,21.5$ | $8.47 \mathrm{~S}, 157.04 \mathrm{E}$ | 0.030 | -0.992 |
| 21 | $2007-08-15$ | Peru | $8.2,25.5$ | $13.39 \mathrm{~S}, 76.60 \mathrm{~W}$ | 0.015 | 0.778 |
| 22 | $2007-09-12$ | Sumatra | $8.4,30.5$ | $4.44 \mathrm{~S}, 101.37 \mathrm{E}$ | -0.099 | -0.026 |
| 23 | $2099-09-29$ | Sumatra | $8.1,15.5$ | $15.49 \mathrm{~S}, 172.10 \mathrm{~W}$ | -0.106 | 0.042 |
| 24 | $2010-02-27$ | Chile | $8.8,30.5$ | $36.12 \mathrm{~S}, 72.90 \mathrm{~W}$ | -0.776 | 2.028 |
| 25 | $2011-03-11$ | Japan | $9.1,11.5$ | $38.30 \mathrm{~N}, 142.38 \mathrm{E}$ | -2.064 | 2.356 |
| 26 | $2012-04-11$ | Sumatra | $8.6,30.5$ | $2.33 \mathrm{~N}, 93.06 \mathrm{E}$ | 0.707 | 0.046 |
| 27 | $2012-04-11$ | Sumatra | $8.2,53.7$ | $0.80 \mathrm{~N}, 92.46 \mathrm{E}$ | 0.196 | -0.006 |
| 28 | $2013-02-06$ | Solomon | $8.0,15.0$ | $10.80 \mathrm{~S}, 165.11 \mathrm{E}$ | 0.027 | -0.036 |
| 29 | $2013-05-24$ | Okhotsk | $8.3,610$. | $54.90 \mathrm{~N}, 153.22 \mathrm{E}$ | -0.002 | -0.317 |
| 30 | $2014-04-01$ | Chile | $8.2,25.5$ | $1.61 \mathrm{~S}, 70.77 \mathrm{~W}$ | -0.021 | 0.092 |
| 31 | $2015-09-16$ | Chile | $8.3,25.5$ | $31.57 \mathrm{~S}, 71.67 \mathrm{~W}$ | -0.102 | 0.299 |

## Result and Conclusion

The seismic excitation due to the 31 large earthquakes (Table 1) were acquired through the procedure described in the former section. The individual excitations are shown as histogram in Fig. 2. Obviously the excitations of the six largest earthquakes of magnitude over 8.5 are prominent in the histogram. Particularly the contributions from the two largest ones (2010 Chile and 2011 Japan earthquakes) take more than half of the total excitation. The largest single co-seismic pole shift $\left(x_{1}, x_{2}\right)=(-2.064,2.356)$ milliarcsec was due to the 2011 Japan earthquake, and it corresponds to 9.67 cm on the Earth's surface. To
show clearly the contributions of post-seismic deformation, cumulative pole excitation is repeatedly acquired and illustrated for (i) co-seismic only, (ii) and (iii) including post-seismic with decay time $\tau=80$ days and $\tau=200$ days. (Fig. 3). In addition, the same result is shown again on the Earth's surface (Fig. 4). On this type of figures, one milliarcsec shift corresponds to 3.1 cm displacement on the ground. In fact, recently almost identical result as ours about co-seismic pole excitation has been presented (Chao, 2016). Though our model is not quite sophisticated, to the best knowledge of the authors, post-seismic polar motion excitation has not been reported in any former articles.
The calculated seismic excitation is compared with


Fig. 2. Calculated co-seismic pole shift. Each contributions due to 31 large earthquakes are illustrated as histogram. Two components $x_{1}, x_{2}$ are shown separately.


Fig. 3. Cumulative seismic pole shift due to the recent 31 largest earthquakes. Two limiting cases of pole excitations for post-seismic deformation with different decay time ( $\tau=$ 80 or 200 days) are illustrated together with the co-seismic excitation only.
the average pole movement in the same time span (Fig. 5). Normally calculated polar motion excitation function is not compared with polar motion but with observed polar motion excitation. The average pole in Fig. 5 was acquired by successive least square error fittings with sliding time window on a modified EOP C04 dataset, which does not contain annual wobble and linear trend, and, therefore, can be regarded as the time series of the Chandler wobble center (Chung and


Fig. 4. Cumulative seismic pole shift due to 31 large earthquakes shown two dimensionally on the Earth's surface. Two limiting cases for post-seismic deformation with different decay time ( $\tau=80$ or 200 days) are illustrated together with the co-seismic excitation.
$\mathrm{Na}, 2016$ ). One can readily realize that pole excitation by earthquakes are much smaller compared with the variation of the coordinates $\left(x_{0}, y_{0}\right)$ of the average pole, although the each amounts of energy released with these 31 largest earthquake should be devastating. This reflects that (i) the Earth's spin rotational inertia associated with the equatorial bulge is enormously large (i.e., $C-A=2.631 \times 10^{35} \mathrm{kgm}^{2}$ ) so that even the world largest earthquake does not lead to a large shift of average pole, and (ii) pole excitations by other sources (such as, atmosphere or ocean) are much larger and dominant in the observed average pole change. However, a few centimeters movement of the average pole should exist with occurrence of largest earthquakes as listed in Table 1. Such movement cannot be easily isolated in observation, because the pole path alteration with a new center (shifted average pole) do not accompany any sudden jump in the instantaneous pole (CIP) path itself (polar motion).

## Acknowledgments

The authors thank three reviewers for their both


Fig. 5. Comparison of the calculated seismic pole shift with the observed average pole movement in the same time span. The average pole movements were determined by successive least square fittings on the modified polar motion time series.
kind and harsh comments, which led valuable enhancement of this article. They also thank all in the Korea Earth Science Society. Sung-Ho Na should like to acknowledge spiritual help of the Lord Jesus Christ.

## References

Baek, J., Shin, Y.H., Na, S.H., Shestakov, N.V., Park, P.H., and Cho, S., 2012, Coseismic and postseismic crustal deformations of the Korean peninsula caused by the 2011 M 9.0 Tohoku earthquake, Japan from GPS data, Terra Nova, 24, 295-300, doi:10.1111/j.1365-3121.2012. 01062.x.

Bizouard, C. and Gambis, D, 2009, The combined solution C04 for Earth Orientation Parameters consistent with

International Terrestrial Reference Frame 2005. IAG Symposium 134, 265-270, doi:10.1007/978-3-642-008603_41.
Chao, B.F., 2016, Rotational normal modes in classical mechanics: resonance, excitation, convolution/deconvolution etc. Lecture of GAGER 2016, July 18-24, Wuhan, China.
Chung, T.W. and Na, S.H., 2016, A least square fit analysis on the Earth's polar motion time series: implication against Smylie's conjecture. Jigu-Mulli-wa-Mulli-Tamsa, 19, 91-96, doi:10.7582/GGE.2016.19.2.091.
Dahlen, F.A., 1971, The excitation of the Chandler wobble by earthquakes, Geophysical Journal Royal Astronomical Society, 25, 157-206.
Dahlen, F.A., 1973, A correction to the excitation of the Chandler wobble by earthquakes. Geophysical Journal Royal Astronomical Society, 32, 203-217.
Gross, R.S., 1986, The influence of earthquakes on the Chandler wobble during 1977-1983. Geophysical Journal Royal Astronomical Society, 85, 161-177.
Gross, R.S., 2000, The excitation of the Chandler wobble. Geophysical Research Letters, 27, 15, 2329-2332, doi:10.1029/2000GL011450.
Gross, R.S., 2009, Earth rotation variation - long period. In Schubert, G. (ed.), Trietise of Geophysics vol. 3 Geodesy, Elsevier, Amsterdam, 239-294.
Gross, R.S. and Chao, B.F., 2006, The rotational and gravitational signature of the December 26, 2004 Sumatran earthquake. Surveys in Geophysics, 27, 615632.

Hearn, E.H., 2003, What can GPS data tell us about the dynamics of post-seismic deformation? Geophysical Journal International, 155, 753-777.
Lambeck, K., 1980, The Earth's variable rotation: geophysical causes and consequences. Cambridge University Press, Cambridge, 449 p.
Plag, H.P., Chao, B., Gross, R, and van Dam T. (eds.), 2005, Proceedings of the workshop: forcing of polar motion in the Chandler frequency band: a contribution to understanding interannual climate variation. April 2123, 2004, Luxemburg, European Centre for Geodynamics and Seismilogy, 165 p.
Smylie, D.E. and Manshina, L., 1968, Earthquakes and observed motion of the rotation pole. Journal of Geophysical Research, 73, 7661-7673.
Smylie, D.E. and Zuberi, M., 2009, Free and forced polar motion and modern observation of the Chandler wobble. Journal of Geodynamics, 48, 226-229, doi:10.1016/ j.jog.2009.09.028.


[^0]:    *Corresponding author: jbkyung@knue.ac.kr
    Tel: +82-43-230-3742
    Fax: +82-43-232-7176

