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AN EXTENSION OF SOFT ROUGH FUZZY SETS

Ismat Beg and Tabasam Rashid

Abstract. This paper introduces a novel extension of soft rough fuzzy set so-called modified soft rough fuzzy set model in which new lower and upper approximation operators are presented together their related properties that are also investigated. Eventually it is shown that these new models of approximations are finer than previous ones developed by using soft rough fuzzy sets.

1. Introduction

The management of uncertainty in real-world problems is always a complex task and, in many situations classical mathematical tools and models cannot deal with the uncertainty involved with the information. Hence many theories have been presented in the literature to cope with the uncertainty, vagueness and ambiguity, like fuzzy set theory [31], rough set theory [19, 20], soft set theory [17] and many other mathematical tools. Each of these theories has its inherent difficulties as pointed out in [17]. Fuzzy set theory has thus been used to handle imprecision in decision making problems to take care of the ambiguity in information [3–6, 32]. Significant applications of rough sets in various fields can also be seen in [10, 12, 13, 20–23]. Molodtsov [17] introduced the concept of soft set theory as a new mathematical tool to deal with uncertainty. Maji et al. [15] further developed the theoretical concepts of soft set theory. This theory has been widely applied to many real world

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problems [27–29] and in the development of new mathematical structures [1, 2, 9, 11, 14, 18, 24, 25, 30]. Despite soft set theory and rough set theory are different tools to deal with uncertainty, some researchers [1,8] have shown that there is some kind of linkage between these two different theories. Feng et al. [7] provided a framework to combine fuzzy sets, rough sets and soft sets all together, which gives rise to several interesting new concepts such as rough soft sets, soft rough sets and soft rough fuzzy sets. Shabir et al. [26] presented the notion of modified soft rough set to improve some difficulties in definition of Feng's soft rough set. Meng et al. [16] developed some important approximation operators for soft rough fuzzy set which are the improved version of Feng's model. According to these approximation operators a very strong condition was implemented, that is soft set as approximation space should be a full soft set. If the approximation space is not full soft set then there will be shortcomings (undefinable set will not always have upper or lower approximation). The purpose of this paper aims at improving the basic structure of the approximations to overcome these shortcomings by defining *modified soft rough fuzzy sets*. Rest of this paper is arranged in the following manner. In Section 2, some basic notions are given to understand our proposal. In Section 3, modified soft rough fuzzy sets and its approximation operators are developed. In Section 4, conclusion of the paper is given. This study presents a preliminary, but potentially interesting research direction.

2. Preliminaries

First we review some basic concepts, necessary to understand our proposal.

Let U be a crisp universe of generic elements, a *fuzzy set* $\mu_{\mathcal{A}}$ in the universe U is a mapping from U to $[0, 1]$. For any $u \in U$, the value $\mu_{\mathcal{A}}(u)$ is called the *degree of membership of* u in $\mu_{\mathcal{A}}$. If membership value of the elements is 0 or 1 then that fuzzy set is also called as crisp set. So the membership value of all the elements in universal set U is 1 and the membership value of all the elements in empty set is 0. Universal set U in the form of fuzzy set is denoted by μ_U and $\mu_U(u) = 1$ for all $u \in U$. Similarly, empty set $\emptyset$ in the form of fuzzy set is denoted by $\mu_{\emptyset}(u) = 0$ for all $u \in U$. The family of all subsets of U is denoted by $P(U)$ and family of all fuzzy sets in U is denoted by $FS(U)$. With the min-max system proposed by Zadeh, fuzzy set intersection, union and complement are defined component wise as follow:

$(\mu_{\mathcal{A}} \cap \mu_{\mathcal{B}})(u) = \mu_{\mathcal{A}}(u) \wedge \mu_{\mathcal{B}}(u)$,
$(\mu_{\mathcal{A}} \cup \mu_{\mathcal{B}})(u) = \mu_{\mathcal{A}}(u) \vee \mu_{\mathcal{B}}(u)$,
$\mu^c_{\mathcal{A}}(u) = 1 - \mu_{\mathcal{A}}(u)$,

where $\mu_{\mathcal{A}}, \mu_{\mathcal{B}}$ are fuzzy sets and $u \in U$. By $\mu_{\mathcal{A}} \subseteq \mu_{\mathcal{B}}$, we mean that $\mu_{\mathcal{A}}(u) \leq \mu_{\mathcal{B}}(u)$ for all $u \in U$. Clearly, $\mu_{\mathcal{A}} = \mu_{\mathcal{B}}$ if $\mu_{\mathcal{A}}(u) = \mu_{\mathcal{B}}(u)$ for all $u \in U$.

Definition 2.1. [32] α-level set of $\mu_{\mathcal{A}}$ is defined as $(\mu_{\mathcal{A}})_\alpha = \{u \in U; \mu_{\mathcal{A}}(u) > \alpha\}$.

In 1999, Molodtsov [17] introduced the concept of soft sets. Let U be the universe set and E the set of all possible parameters under consideration with respect to U. Usually, parameters are attributes, characteristics, or properties of objects in U. Molodtsov [17] defined the notion of a soft set in the following way:

Definition 2.2. [17] A pair (F, A) is called a soft set over U, where $A \subseteq E$ and F is a mapping given by $F : A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of U. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A).For $u \in U$, $F(e)u = 1$ if $u \in F(e)$ and $F(e)u = 0$ if $u \notin F(e)$.

Definition 2.3. [8] Let $S = (F, A)$ be a soft set over U. Then the pair $SAS = (U, S)$ is called a soft approximation space. Based on SAS, following two operations are defined:

$\underline{sra}_{SAS}(X) = \{u \in U : \exists a \in A[u \in F(a) \subseteq X]\}$
$\overline{sra}_{SAS}(X) = \{u \in U : \exists a \in A[u \in F(a), F(a) \cap X \neq \emptyset]\}$

for any subset X of U. Two subsets $\underline{sra}_{SAS}(X)$ and $\overline{sra}_{SAS}(X)$ called the lower and upper soft rough approximations of X in SAS, respectively are obtained. If $\underline{sra}_{SAS}(X) = \overline{sra}_{SAS}(X)$, X is said to be soft definable; otherwise X is called a soft rough set.

Definition 2.4. [8] Let $S = (F, A)$ be a soft set over U. If $\bigcup\limits_{a \in A} F(a) = U$, then S is called a full soft set.

Definition 2.5. [26] Let (F, A) be a soft set over U, where F is a map $F : A \to P(U)$. Let $\phi : U \to P(A)$ be another map defined as $\phi(x) = \{a : x \in F(a)\}$. Then the pair $MSAS = (U, \phi)$ is called modified soft approximation space and for any $X \subseteq U$, lower modified soft rough approximation is defined as

$$\underline{msra}_{MSAS}(X) : \{x \in X : \phi(x) \neq \phi(y) \text{ for all } \ y \in X^c\},$$

where $X^c = U - X$ and its upper modified soft rough approximation is defined as

$$\overline{msra}_{MSAS}(X) = \{x \in U : \phi(x) = \phi(y) \text{ for all } \ y \in X\}.$$

If $\underline{msra}_{MSAS}(X) \neq \overline{msra}_{MSAS}(X)$, then X is said to be modified soft rough set.

Definition 2.6. [7] Let $S = (F, A)$ be a full soft set over U and $SAS = (U, S)$ be a soft approximation space. For a fuzzy set $\mu_{\mathcal{A}} \in FS(U)$, the lower and upper soft rough approximations of $\mu_{\mathcal{A}}$ with respect to SAS are denoted by $\underline{SRA}_{SAS}(\mu_{\mathcal{A}})$ and $\overline{SRA}_{SAS}(\mu_{\mathcal{A}})$, respectively, which are fuzzy sets in U given by:

$\underline{SRA}_{SAS}(\mu_{\mathcal{A}})(x) = \wedge\{\mu_{\mathcal{A}}(y); \exists a \in A(\{x, y\} \subseteq F(a))\}$
$\overline{SRA}_{SAS}(\mu_{\mathcal{A}})(x) = \vee\{\mu_{\mathcal{A}}(y); \exists a \in A(\{x, y\} \subseteq F(a))\}$
for all $x \in U$. The operators $\underline{SRA}_{SAS}$ and $\overline{SRA}_{SAS}$ are called the lower and upper soft rough approximation operators on fuzzy sets. If $\underline{SRA}_{SAS}(\mu_{\mathcal{A}}) = \overline{SRA}_{SAS}(\mu_{\mathcal{A}})$, $\mu_{\mathcal{A}}$ is said to be soft definable; otherwise μ is called a soft rough fuzzy set.

3. Modified Soft Rough Fuzzy Set (MSRFS)

Shabir et al. [26] highlighted few drawbacks in Feng's soft rough set [7, 8] and gave a new model for soft rough set. Meng et al. [16] showed that the soft rough fuzzy set is an extension of Feng's soft rough set. From these results in this section is proposed the modified soft rough fuzzy set. This extension overcomes the drawbacks of Feng's and Meng's soft rough fuzzy sets.

Definition 3.1. Let (F, A) be a soft set over U, where F is a map $F : A \to P(U)$. Let $\phi : U \to P(A)$ be another map defined as $\phi(x) = \{a : x \in F(a)\}$. Then the pair $MSAS = (U, \phi)$ is called modified soft approximation space. For any fuzzy set $\mu_{\mathcal{A}} \in FS(U)$, the lower and upper modified soft rough approximations of $\mu_{\mathcal{A}}$ with respect to $MSAS$ are denoted by $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$ and $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$, respectively, which are fuzzy sets in U given by:

$$\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = \begin{cases} \mu_{\mathcal{A}}(x) & \text{if } \phi(x) \neq \phi(y) \text{ for all } y \in ((\mu_{\mathcal{A}})_0)^c \\ 0 & \text{if } \phi(x) = \phi(y) \text{ for some } y \in ((\mu_{\mathcal{A}})_0)^c \end{cases}$$

for all $x \in (\mu_{\mathcal{A}})_0$ and

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = \begin{cases} 1 & \text{if } \phi(x) \neq \emptyset \text{ and } \phi(x) = \phi(y) \text{ for some } y \in (\mu_{\mathcal{A}})_0 \\ \mu_{\mathcal{A}}(x) & \text{if } \phi(x) = \emptyset \text{ and } \phi(x) = \phi(y) \text{ for some } y \in (\mu_{\mathcal{A}})_0 \\ 0 & \text{if } \phi(x) \neq \phi(y) \text{ for all } y \in (\mu_{\mathcal{A}})_0 \end{cases}$$

for all $x \in U$. The operators $\underline{MSRA}_{MSAS}$ and $\overline{MSRA}_{MSAS}$ are called the lower and upper modified soft rough approximation operators on fuzzy sets. If $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) = \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$, then $\mu_{\mathcal{A}}$ is called modified soft definable; otherwise $\mu_{\mathcal{A}}$ is a modified soft rough fuzzy set.

Remark 1. For any fuzzy set $\mu_{\mathcal{A}}$, it is easy to see that
$\mu_{\emptyset} \subseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \mu_U$ and $\mu_{\emptyset} \subseteq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \mu_U$.

Theorem 3.2. *Let (F, A) be a soft set over U, $MSAS = (U, \phi)$ be a modified soft approximation space and $\mu_{\mathcal{A}} \in FS(U)$. Then we have*

1. $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \mu_{\mathcal{A}} \subseteq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$,
2. $\underline{MSRA}_{MSAS}(\mu_U) = \mu_U = \overline{MSRA}_{MSAS}(\mu_U)$,
3. $\underline{MSRA}_{MSAS}(\mu_{\emptyset}) = \mu_{\emptyset} = \overline{MSRA}_{MSAS}(\mu_{\emptyset})$.

Proof. Point wise proof is;

1. There are two cases for $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x)$.

Case i. If $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = \mu_{\mathcal{A}}(x)$ then we can write that

$$\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \mu_{\mathcal{A}}(x).$$

Case ii. If $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = 0$ then $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \mu_{\mathcal{A}}(x)$ because we know that $0 \leq \mu_{\mathcal{A}}(x) \leq 1$.

Thus, $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \mu_{\mathcal{A}}(x)$.

Now we want to prove that $\mu_{\mathcal{A}}(x) \leq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x)$.

There are three cases for $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x)$.

Case i. If $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = 1$ then we can write that

$$\mu_{\mathcal{A}}(x) \leq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x).$$

Case ii. If $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = \mu_{\mathcal{A}}(x)$ then $\mu_{\mathcal{A}}(x) \leq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x)$.

Case iii. $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = 0$ when $\phi(x) \neq \phi(y)$ for all $y \in (\mu_{\mathcal{A}})_0$, which further implies that $x \notin (\mu_{\mathcal{A}})_0$. So $\mu_{\mathcal{A}}(x) = 0$.
Thus $\mu_{\mathcal{A}}(x) \leq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x)$.
Hence

$$\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \mu_{\mathcal{A}} \subseteq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}).$$

2. By (1), we can write that $\mu_U \subseteq \overline{MSRA}_{MSAS}(\mu_U)$.
By definition of $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$, we can write $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \mu_U$ for any fuzzy set $\mu_{\mathcal{A}}$. So $\overline{MSRA}_{MSAS}(\mu_U) \subseteq \mu_U$.
Thus, $\mu_U = \overline{MSRA}_{MSAS}(\mu_U)$.
By definition, it is noted that $\mu_U(x) = 1$ for all $x \in U$. So $\phi(x) \neq \phi(y)$ for all $y \in ((\mu_U)_0)^c$ then $\underline{MSRA}_{MSAS}(\mu_U)(x) = \mu_U(x)$.
Thus, $\underline{MSRA}_{MSAS}(\mu_U)(x) = 1$ for all $x \in U$.
Hence

$$\underline{MSRA}_{MSAS}(\mu_U) = \mu_U = \overline{MSRA}_{MSAS}(\mu_U).$$

3. By (1), we can write that $\underline{MSRA}_{MSAS}(\mu_\emptyset) \subseteq \mu_\emptyset$.
By definition of $\underline{MSRA}_{MSAS}(\mu_\emptyset)$, we can write $\mu_\emptyset \subseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$ for any fuzzy set $\mu_{\mathcal{A}}$. So $\mu_\emptyset \subseteq \underline{MSRA}_{MSAS}(\mu_\emptyset)$.
Hence $\underline{MSRA}_{MSAS}(\mu_\emptyset) = \mu_\emptyset$.
It is obvious that $(\mu_\emptyset)_0 = \emptyset$.
So there does not exist any y in $(\mu_\emptyset)_0$.
This implies that $\phi(x) \neq \phi(y)$ for all $y \in (\mu_\emptyset)_0$.
Thus, $\overline{MSRA}_{MSAS}(\mu_\emptyset)(x) = 0$ for all $x \in U$.
Hence

$$\underline{MSRA}_{MSAS}(\mu_\emptyset) = \mu_\emptyset = \overline{MSRA}_{MSAS}(\mu_\emptyset).$$

□

THEOREM 3.3. *Let (F, A) be a soft set over U, $MSAS = (U, \phi)$ be a modified soft approximation space and $\mu_{\mathcal{A}}, \mu_{\mathcal{B}} \in FS(U)$. Then we have*

1. $\mu_{\mathcal{A}} \subseteq \mu_{\mathcal{B}} \Rightarrow \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{B}})$,
2. $\mu_{\mathcal{A}} \subseteq \mu_{\mathcal{B}} \Rightarrow \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})$.

Proof. 1. Let $\mu_{\mathcal{A}} \subseteq \mu_{\mathcal{B}}$ which implies that $\mu_{\mathcal{A}}(x) \leq \mu_{\mathcal{B}}(x)$ for all $x \in U$.
If $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = 0$ then it is easy to see that $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \underline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x)$ for all $x \in U$.

If $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = \mu_{\mathcal{A}}(x)$ and $\underline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x) = \mu_{\mathcal{B}}(x)$ then $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \underline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x)$ for all $x \in U$.

But $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = \mu_{\mathcal{A}}(x) \neq 0$ implies that $\phi(x) \neq \phi(y)$ for all $y \in ((\mu_{_A})_0)^c$ and $x \in (\mu_{_A})_0$. So obviously $x \in (\mu_{_B})_0$ as $\mu_{\mathcal{A}} \subseteq \mu_{\mathcal{B}}$. It can be written that $\phi(x) \neq \phi(y)$ for all $y \in ((\mu_B)_0)^c$, so by definition $\underline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x) = 0$

Hence

$$\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{B}}).$$

2. Let $\mu_{\mathcal{A}} \subseteq \mu_{\mathcal{B}}$ which implies that $\mu_{\mathcal{A}}(x) \leq \mu_{\mathcal{B}}(x)$ for all $x \in U$.

We want to show that $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x)$ for all $x \in U$.

If $\overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x) = 1$ then $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x)$.

$\overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x) = \mu_{\mathcal{B}}(x) \neq 1$ and $\mu_{\mathcal{B}}(x) \neq 0$, when $\phi(x) = \emptyset$ and $\phi(x) = \phi(y)$ for some $y \in (\mu_B)_0$. There are two possible cases:

Case i. When $\phi(x) = \emptyset$ and $\phi(x) = \phi(y)$ for some $y \in (\mu_A)_0$ then

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = \mu_{\mathcal{A}}(x).$$

Since $\mu_{\mathcal{A}}(x) \leq \mu_{\mathcal{B}}(x)$ for all $x \in U$.

So

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x).$$

Case ii. When $\phi(x) = \emptyset$ and $\phi(x) \neq \phi(y)$ for all $y \in (\mu_A)_0$ then

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) = 0.$$

Thus $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})(x)$.

Hence

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}}).$$

□

The following theorem can be easly proved by using Theorem 3.4.

THEOREM 3.4. *Let (F, A) be a soft set over U, $MSAS = (U, \phi)$ be a modified soft approximation space and $\mu_{\mathcal{A}}, \mu_{\mathcal{B}} \in FS(U)$. Then we have*

1. $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}} \cup \mu_{\mathcal{B}}) \supseteq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \cup \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})$,
2. $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}} \cap \mu_{\mathcal{B}}) \subseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \cap \underline{MSRA}_{MSAS}(\mu_{\mathcal{B}})$,
3. $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}} \cup \mu_{\mathcal{B}}) \supseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \cup \underline{MSRA}_{MSAS}(\mu_{\mathcal{B}})$,

4. $\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}} \cap \mu_{\mathcal{B}}) \subseteq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \cap \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}})$.

THEOREM 3.5. *Let (F, A) be a soft set over U, $MSAS = (U, \phi)$ be a modified soft approximation space and $\mu_{\mathcal{A}} \in FS(U)$. Then we have*

1. $\overline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_A)) \supseteq \underline{MSRA}_{MSAS}(\mu_A)$,
2. $\overline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_A)) \supseteq \underline{MSRA}_{MSAS}(\mu_A)$,
3. $\underline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_A)) \subseteq \underline{MSRA}_{MSAS}(\mu_A)$,
4. $\underline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_A)) \subseteq \overline{MSRA}_{MSAS}(\mu_A)$,
5. $\overline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_A)) \supseteq \overline{MSRA}_{MSAS}(\mu_A)$.

Proof. Point wise proof is given below.

1. By definition we know that $\mu_{\mathcal{A}}(x) \leq \overline{MSRA}_{MSAS}(\mu_A)(x)$ for any fuzzy set $\mu_{\mathcal{A}}$. Now replace $\mu_{\mathcal{A}}$ by $\underline{MSRA}_{MSAS}(\mu_A)$ then we get $\underline{MSRA}_{MSAS}(\mu_A)(x) \leq \overline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_A))(x)$.
   Hence
   $$\underline{MSRA}_{MSAS}(\mu_A)(x) \subseteq \overline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_A))(x).$$
2. By Theorem 3.4(1) we know that $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \subseteq \mu_{\mathcal{A}}$ and by using Theorem 3.4(2) it can be written that $\underline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) \subseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$.
3. By definition we know that $\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})(x) \leq \mu_{\mathcal{A}}(x)$ for any fuzzy set $\mu_{\mathcal{A}}$. Now replace $\mu_{\mathcal{A}}$ by $\overline{MSRA}_{MSAS}(\mu_A)$ then we get
   $$\underline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_A))(x) \leq \overline{MSRA}_{MSAS}(\mu_A)(x).$$
   Hence
   $$\underline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_A)) \subseteq \overline{MSRA}_{MSAS}(\mu_A).$$
4. By Theorem 3.4(1) we know that $\mu_{\mathcal{A}} \supseteq \overline{MSRA}_{MSAS}(\mu_A)$ and by using Theorem 3.4(2) it can be written that $\overline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_A)) \supseteq \overline{MSRA}_{MSAS}(\mu_A)$.

□

EXAMPLE 3.6. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be the set of six utility stores (universe set) and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$, where e_1 represents empowerment of sales, e_2 represents perceived quality of products, e_3 represents high traffic location, e_4 represents covered area. The soft set (F, A) is representing this data in Table 1.
$F : A \to P(U)$
$\phi : U \to P(A)$

Table 1. Soft set (F, A)

	u_1	u_2	u_3	u_4	u_5	u_6
e_1	1	1	1	1	1	1
e_2	1	0	1	1	0	1
e_3	1	0	0	1	0	1
e_4	0	1	0	0	1	0

Then the MSAS (U, ϕ) will be $\phi(u_1) = \{e_1, e_2, e_3\}$, $\phi(u_2) = \{e_1, e_4\}$, $\phi(u_3) = \{e_1, e_2\}$, $\phi(u_4) = \{e_1, e_2, e_3\}$, $\phi(u_5) = \{e_1, e_4\} = \phi(u_2)$, $\phi(u_6) = \{e_1, e_2, e_3\} = \phi(u_4) = \phi(u_1)$.

$\mu_{\mathcal{A}} = \{(u_1, 0.3), (u_2, 0.4), (u_3, 0), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) = \{(u_1, 0), (u_2, 0), (u_3, 0), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) = \{(u_1, 1), (u_2, 1), (u_3, 0), (u_4, 1), (u_5, 1), (u_6, 1)\}$
$\mu_{\mathcal{B}} = \{(u_1, 0), (u_2, 0.4), (u_3, 0), (u_4, 0.7), (u_5, 0), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{B}}) = \{(u_1, 0), (u_2, 0), (u_3, 0), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\overline{MSRA}_{MSAS}(\mu_{\mathcal{B}}) = \{(u_1, 1), (u_2, 1), (u_3, 0), (u_4, 1), (u_5, 1), (u_6, 1)\}$
$\mu_{\mathcal{A}} \cup \mu_{\mathcal{B}} = \{(u_1, 0.3), (u_2, 0.4), (u_3, 0), (u_4, 0.7), (u_5, 0), (u_6, 0)\}$
$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}} \cup \mu_{\mathcal{B}}) = \{(u_1, 1), (u_2, 1), (u_3, 0), (u_4, 1), (u_5, 1), (u_6, 1)\}$
$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \cup \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}}) = \{(u_1, 1), (u_2, 1), (u_3, 0), (u_4, 1), (u_5, 1), (u_6, 1)\}$

Note that

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \cup \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}}) = \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}} \cup \mu_{\mathcal{B}}).$$

$\mu_{\mathcal{A}} \cap \mu_{\mathcal{B}} = \{(u_1, 0), (u_2, 0.4), (u_3, 0), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}} \cap \mu_{\mathcal{B}}) = \{(u_1, 0), (u_2, 1), (u_3, 0), (u_4, 0), (u_5, 1), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{B}} \cap \mu_{\mathcal{B}}) = \{(u_1, 0), (u_2, 0), (u_3, 0), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{B}}) \cap \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) = \underline{MSRA}_{MSAS}(\mu_{\mathcal{B}} \cap \mu_{\mathcal{B}})$.
$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \cap \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}}) = \{(u_1, 1), (u_2, 1), (u_3, 0), (u_4, 1), (u_5, 1), (u_6, 1)\}$

Where

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \cap \overline{MSRA}_{MSAS}(\mu_{\mathcal{B}}) \nsubseteq \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}} \cap \mu_{\mathcal{B}}).$$

$\mu_{\mathcal{C}} = \{(u_1, 0), (u_2, 0.4), (u_3, 0.6), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{C}}) = \{(u_1, 0), (u_2, 0), (u_3, 0.6), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\overline{MSRA}_{MSAS}(\mu_{\mathcal{C}}) = \{(u_1, 0), (u_2, 1), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0)\}$
$\mu_{\mathcal{D}} = \{(u_1, 0), (u_2, 0), (u_3, 0.3), (u_4, 0), (u_5, 0.7), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{D}}) = \{(u_1, 0), (u_2, 0), (u_3, 0.3), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\overline{MSRA}_{MSAS}(\mu_{\mathcal{D}}) = \{(u_1, 0), (u_2, 1), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0)\}$

$\mu_{\mathcal{C}} \cup \mu_{\mathcal{D}} = \{(u_1, 0), (u_2, 0.4), (u_3, 0.6), (u_4, 0), (u_5, 0.7), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{C}} \cup \mu_{\mathcal{D}}) = \{(u_1, 0), (u_2, 0.4), (u_3, 0.6), (u_4, 0), (u_5, 0.7), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{C}}) \cup \underline{MSRA}_{MSAS}(\mu_{\mathcal{D}}) = \{(u_1, 0), (u_2, 0), (u_3, 0.6), (u_4, 0), (u_5, 0), (u_6, 0)\}$
Note that

$$\underline{MSRA}_{MSAS}(\mu_{\mathcal{C}} \cup \mu_{\mathcal{D}}) \nsubseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{C}}) \cup \underline{MSRA}_{MSAS}(\mu_{\mathcal{D}}).$$

$\overline{MSRA}_{MSAS}(\mu_{\mathcal{C}} \cup \mu_{\mathcal{D}}) = \{(u_1, 0), (u_2, 1), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0)\}$
$\mu_{\mathcal{C}} \cap \mu_{\mathcal{D}} = \{(u_1, 0), (u_2, 0), (u_3, 0.3), (u_4, 0), (u_5, 0), (u_6, 0)\}$
$\underline{MSRA}_{MSAS}(\mu_{\mathcal{C}} \cap \mu_{\mathcal{D}}) = \{(u_1, 0), (u_2, 0), (u_3, 0.3), (u_4, 0), (u_5, 0), (u_6, 0)\}$
Note that

$$\underline{MSRA}_{MSAS}(\mu_{\mathcal{C}} \cap \mu_{\mathcal{D}}) = \underline{MSRA}_{MSAS}(\mu_{\mathcal{C}}) \cap \underline{MSRA}_{MSAS}(\mu_{\mathcal{D}}).$$

Remark 2. In general

$$\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}) \nsubseteq (\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c$$

$$\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}) \nsupseteq (\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c,$$

$$(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c \nsubseteq \underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}})$$

and

$$(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c \nsupseteq \underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}).$$

Example 3.7. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}$ be the set of nine utility stores (universe set) and $A = \{e_1, e_2, e_3, e_4\} \subseteq E$, where e_1 represents empowerment of sales, e_2 represents perceived quality of products, e_3 represents high traffic location, e_4 represents covered area. The soft set (F, A) is representing this data in Table 2.
$F : A \to P(U)$
$\phi : U \to P(A)$

Table 2. Soft set (F, A)

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9
e_1	1	1	1	1	1	1	0	0	0
e_2	1	0	1	1	0	1	0	0	0
e_3	1	0	0	1	0	1	0	0	0
e_4	0	1	0	0	1	0	0	0	0

Then the MSAS (U, ϕ) will be $\phi(u_1) = \{e_1, e_2, e_3\}$, $\phi(u_2) = \{e_1, e_4\}$, $\phi(u_3) = \{e_1, e_2\}$, $\phi(u_4) = \{e_1, e_2, e_3\}$, $\phi(u_5) = \{e_1, e_4\} = \phi(u_2)$, $\phi(u_6) = \{e_1, e_2, e_3\} = \phi(u_4) = \phi(u_1)$, $\phi(u_7) = \emptyset = \phi(u_8) = \phi(u_9)$.

If we take $\mu_{\mathcal{A}} = \{(u_1, 0.3), (u_2, 0.4), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0), (u_7, 0.2), (u_8, 0), (u_9, 1)\}$, then $\mu^c_{\mathcal{A}} = \{(u_1, 0.7), (u_2, 0.6), (u_3, 0), (u_4, 1), (u_5, 0), (u_6, 1), (u_7, 0.8), (u_8, 1), (u_9, 0)\}$.

Next we calculate some approximations.

$\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) = \{(u_1, 0), (u_2, 0.4), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0), (u_7, 0), (u_8, 0), (u_9, 0)\}$

$(\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c = \{(u_1, 1), (u_2, 0.6), (u_3, 0), (u_4, 1), (u_5, 0), (u_6, 1), (u_7, 1), (u_8, 1), (u_9, 1)\}$

$\underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}) = \{(u_1, 0.7), (u_2, 0), (u_3, 0), (u_4, 1), (u_5, 0), (u_6, 1), (u_7, 0), (u_8, 0), (u_9, 0)\}$

$(\underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}))^c = \{(u_1, 0.3), (u_2, 1), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0), (u_7, 1), (u_8, 1), (u_9, 1)\}$

$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) = \{(u_1, 1), (u_2, 1), (u_3, 1), (u_4, 1), (u_5, 1), (u_6, 1), (u_7, 0.2), (u_8, 0), (u_9, 1)\}$

$(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c = \{(u_1, 0), (u_2, 0), (u_3, 0), (u_4, 0), (u_5, 0), (u_6, 0), (u_7, 0.8), (u_8, 1), (u_9, 0)\}$

$\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}) = \{(u_1, 1), (u_2, 1), (u_3, 0), (u_4, 1), (u_5, 1), (u_6, 1), (u_7, 0.8), (u_8, 1), (u_9, 0)\}$

$(\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}))^c = \{(u_1, 0), (u_2, 0), (u_3, 1), (u_4, 0), (u_5, 0), (u_6, 0), (u_7, 0.2), (u_8, 0), (u_9, 1)\}$

Note that

$$(\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c(u_2) < \overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}})(u_2)$$

and

$$\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}})(u_7) < (\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c(u_7).$$

Thus

$$\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}) \nsubseteq (\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c$$

and

$$\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}) \nsupseteq (\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c.$$

Note that

$$\underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}})(u_1) > (\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c(u_1)$$

and

$$(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c(u_7) > \underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}})(u_7).$$

So

$$(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c \nsubseteq \underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}})$$

and

$$(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}))^c \nsupseteq \underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}).$$

It is easy to note that

$$(\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}))^c \nsubseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$$

and

$$(\overline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}))^c \nsupseteq \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}).$$

Therefore

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \nsubseteq (\underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}))^c$$

and

$$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) \nsupseteq (\underline{MSRA}_{MSAS}(\mu^c_{\mathcal{A}}))^c.$$

$\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) = \{(u_1, 0), (u_2, 0.4), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0), (u_7, 0), (u_8, 0), (u_9, 0)\}$
$\underline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) = \{(u_1, 0), (u_2, 0.4), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0), (u_7, 0), (u_8, 0), (u_9, 0)\}$
$\overline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) = \{(u_1, 0), (u_2, 1), (u_3, 1), (u_4, 0), (u_5, 1), (u_6, 0), (u_7, 0), (u_8, 0), (u_9, 0)\}$
Note that

$$\underline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) = \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$$

and

$$\overline{MSRA}_{MSAS}(\underline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) \supset \underline{MSRA}_{MSAS}(\mu_{\mathcal{A}}).$$

$\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}) = \{(u_1, 1), (u_2, 1), (u_3, 1), (u_4, 1), (u_5, 1), (u_6, 1), (u_7, 0.2), (u_8, 0), (u_9, 1)\}$
$\underline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) = \{(u_1, 1), (u_2, 1), (u_3, 1), (u_4, 1), (u_5, 1), (u_6, 1), (u_7, 0), (u_8, 0), (u_9, 0)\}$
$\overline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) = \{(u_1, 1), (u_2, 1), (u_3, 1), (u_4, 1), (u_5, 1), (u_6, 1), (u_7, 0.2), (u_8, 0), (u_9, 1)\}$
Thus

$$\underline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) \subset \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})$$

and

$$\overline{MSRA}_{MSAS}(\overline{MSRA}_{MSAS}(\mu_{\mathcal{A}})) = \overline{MSRA}_{MSAS}(\mu_{\mathcal{A}}).$$

4. Conclusions

Fuzzy set theory has been successfully used to handle vagueness and imprecision in information, meanwhile the soft rough set theory has been remarkably applied to the approximation of undefinable sets. Approximations of undefinable set are important and needed. In the existing literature these approximations do not satisfy the basic properties of approximation measure. Therefore, MSRFSs have been introduced to overcome such deficiencies. It has been also shown that MSRFS provide better approximations of undefinable sets. Our study present preliminary results and has significant potential for new research directions in future.

References

[1] H. Aktas and N. Cagman, *Soft sets and soft groups*, Information Sciences **177** (2007), 2726–2735.

[2] M. I. Ali, *A note on soft sets, rough sets and fuzzy soft sets*, Applied Soft Computing **11** (2011), 3329–3332.

[3] I. Beg and S. Ashraf, *Fuzzy relational calculus*, Bulletin of the Malaysian Mathematical Sciences Society (2) **37** (1) (2014), 203–237.

[4] I. Beg and T. Rashid, *TOPSIS for hesitant fuzzy linguistic term sets*, International Journal of Intelligent Systems **28** (2013), 1162-1171.

[5] R. E. Bellman and L. A. Zadeh, *Decision making in a fuzzy environment*, Management Science **17** (4) (1970), 141–164.

[6] D. Dubois and H. Prade, *Fundamentals of Fuzzy Sets*, Kluwer Academic Publishers, Dordrecht - 2000.

[7] F. Feng, C. X. Li, B. Davvaz and M. I. Ali, *Soft sets combined with fuzzy sets and rough sets: a tentative approach*, Soft Computing **14** (2010), 899–911.

[8] F. Feng, X. Liu, V. Leoreanu-Fotea and Y. B. Jun, *Soft sets and soft rough sets, Information Sciences* **181** (2011), 1125–1137.

[9] X. Ge, Z. Li and Y. Ge, *Topological spaces and soft sets, Journal of Computational Analysis and Applications* **13** (2011), 881–885.

[10] S. Greco, B. Matarazzo and R. Slowinski, *Rough set theory for multicriteria decision analysis, European Journal of Operational Research* **129** (2001), 1–47.

[11] T. Herawan and M. M. Deris, *A soft set approach for association rules mining*, Knowledge-Based Systems **24** (2011), 186–195.

[12] T. B. Iwinski, *Algebraic approach to rough sets*, Bulletin of the Polish Academy of Sciences, Mathematics **35** (9–10) (1987).

[13] Y. B. Jun, *Roughness of ideals in BCK-algebra*, Scientiae Mathematicae Japonica **57** (1) (2003), 165–169.

[14] S. J. Kalayathankal and G. S. Singh, *A fuzzy soft flood alarm model*, Mathematics and Computers in Simulation **80** (2010), 887–893.

[15] P. K. Maji, R. Biswas and R. Roy, *Soft set theory*, Computers and Mathematics with Applications **45** (2003), 555–562.
[16] D. Meng, X. Zhang and K. Qin, *Soft rough fuzzy sets and soft fuzzy rough sets*, Computers and Mathematics with Applications **62** (2011), 4635–4645.
[17] D. Molodtsov, *Soft set theory - first results*, Computers and Mathematics with Applications **37** (1999), 19–31.
[18] M. M. Musharif, S. Sengupta and A. K. Ray, *Texture classification using a novel, soft set theory based classification algorithm*, Lecture Notes in Computer Science **3851** (2006), 246–254.
[19] Z. Pawlak, Rough sets, *International Journal of Computing and Information Sciences* **11** (1982), 341–356.
[20] Z. Pawlak, *Rough Sets: Theoretical Aspects of Reasoning about Data*, Kluwer Academic Publishers, Boston, 1991.
[21] Z. Pawlak, *Rough set approach to knowledge-based decision support*, European Journal of Operational Research **99** (1997), 48–57.
[22] Z. Pawlak, *Rough set theory and its applications*, Journal of Telecommunications and Information Technology **3** (2002).
[23] Z. Pawlak and A. Skowron, *Rudiments of rough sets*, Information Sciences **177** (2007), 3–27.
[24] H. Qin, X. Ma, J. M. Zain and T. Herawan, *A novel soft set approach in selecting clustering attribute*, Knowledge-Based Systems **36** (2012), 139–145.
[25] A. S. Sezer, *A new view to ring theory via soft union rings*, ideals and bi-ideals, Knowledge-Based Systems **36** (2012), 300–314.
[26] M. Shabir, M. I. Ali and T. Shaheen, *Another approach to soft rough sets*, Knowledge-Based Systems **40** (2013), 72–80.
[27] L. Shanmei and X. Xiaohao, *Vulnerability analysis for airport networks based on fuzzy soft sets: from the structural and functional perspective*, Chinese Journal of Aeronautics **28** (2015) DOI:http://dx.doi.org/10.1016/j.cja.2015.04.002
[28] H. Tang, *A novel fuzzy soft set approach in decision making based on grey relational analysis and Dempster–Shafer theory of evidence*, Applied Soft Computing **31** (2015), 317–325
[29] Z. Tao, H. Chen, X. Song, L. Zhou and J. Liu, *Uncertain linguistic fuzzy soft sets and their applications in group decision making*, Applied Soft Computing **34** (2015), 587–605
[30] Z. Xio, K. Gong and Y. Zou, *A combined forecasting approach based on fuzzy soft sets*, Journal of Computational and Applied Mathematics **228** (2009), 326–333
[31] L. A. Zadeh, *Fuzzy sets*, Information and Control **8** (1965), 338–353.
[32] H. J. Zimmermann, *Fuzzy Set Theory and its Applications*, second edition, Kluwer Academic Publishers, Boston, (1991).

Ismat Beg
Lahore School of Economics
Lahore, Pakistan.
E-mail: ibeg@lahoreschool.edu.pk

Tabasam Rashid
University of Management and Technology
Lahore, Pakistan.
E-mail: tabasam.rashid@gmail.com