ON THE TOPOLOGY OF THE DUAL SPACE OF CROSSED PRODUCT $C^*$-ALGEBRAS WITH FINITE GROUPS

Firuz Kamalov

Abstract. In this note we extend our previous result about the structure of the dual of a crossed product $C^*$-algebra $A \rtimes_G G$, when $G$ is a finite group. We consider the space $\tilde{\Gamma}$ which consists of pairs of irreducible representations of $A$ and irreducible projective representations of subgroups of $G$. Our goal is to endow $\tilde{\Gamma}$ with a topology so that the orbit space $G \setminus \tilde{\Gamma}$ is homeomorphic to the dual of $A \rtimes G$. In particular, we will show that if $\hat{A}$ is Hausdorff then $G \setminus \tilde{\Gamma}$ is homeomorphic to $\hat{A} \rtimes G$.

1. Introduction

The dual space of a crossed product $A \rtimes_G G$ has a rich and deep structure. Describing this structure in a general setting is a difficult task. To gain any meaningful insight about $\hat{A} \rtimes_G G$ one has had to impose various conditions on $A$ and $G$ [1, 2, 4, 5, 7, 8]. Recently Echterhoff and Williams gave a concrete description of the dual space in the case of a strictly proper action on a continuous trace $C^*$-algebra [3]. In this paper, we investigate the topology of $\hat{A} \rtimes_G G$ when $G$ is finite.

The first step in understanding the structure of $\hat{A} \rtimes_G G$ is to describe it as a set. Let $\Gamma$ be the set of all pairs $(\pi, W)$, where $\pi \in \hat{A}$ and $W$ is an irreducible projective representation of $G$ associated to a certain 2-cocycle $\omega_\pi$. There exists a natural action of $G$ on $\Gamma$. If $G$ is finite, then $\hat{A} \rtimes_G G$ corresponds bijectively, via a certain map $\Phi$, to the orbit space $G \setminus \Gamma$ as a set [4]. The next step is to equip $\Gamma$ with a suitable topology so that $\hat{A} \rtimes_G G$ is homeomorphic to $G \setminus \Gamma$. Indeed, this is the main goal of the paper. We will show that if $\hat{A}$ is Hausdorff, then $G \setminus \tilde{\Gamma}$ is homeomorphic to $\hat{A} \rtimes_G G$.

We define the topology on $G \setminus \Gamma$ based on the approach used in [3]. In Proposition 4, we show that the map $\Phi$ is continuous. In Lemma 5 and Lemma 6, we show that if $\hat{A}$ is a Hausdorff space, then $\Phi$ is a closed map. Our main result is stated in Theorem 8.

Received August 25, 2015; Revised September 21, 2016.
2010 Mathematics Subject Classification. 46L55, 46L05.
Key words and phrases. crossed product $C^*$-algebra.
2. Preliminaries

In this section, we give a brief overview of the correspondence between the set \( \Gamma \) and \( \hat{A} \rtimes \sigma G \). We refer the reader to [4] for further details. Let \( G \) be a finite group acting on a C*-algebra \( A \) and let \((A, G, \sigma)\) be the corresponding dynamical system. We will assume throughout this paper that \( A \) is a separable C*-algebra. The action of \( G \) on \( A \) induces an action of \( G \) on \( \hat{A} \) given by \( [\pi] \mapsto [\pi \circ \sigma_s] \) for all \([\pi] \in \hat{A}\) and \( s \in G \). Let \( G_\pi \) denote the stability group at each \([\pi] \in \hat{A}\). Then for each \( s \in G_\pi \) there is a unitary \( V_s \) such that \( V_s \pi V_s^* = \pi \circ \sigma_s \). The map \( s \mapsto V_s \) defines a projective representation of \( G_\pi \). Let \( \omega \) be the multiplier of the projective representation \( V \). Let \( \hat{G}_\pi \) denote the set of all irreducible \( \omega \)-representations of \( G_\pi \). Then for each \( W \in \hat{G}_\pi \) we can construct a corresponding covariant representation of \((A, G_\pi, \sigma)\) (1) \((\pi \otimes 1_m, V \otimes W^*)^G\).

Let \( \tilde{\Gamma} \) be the set of all equivalence classes in \( \Gamma \). Then the map \( \Phi \) factors through from \( \tilde{\Gamma} \) into \( \hat{G}_\pi \). Moreover, \( \Phi \) is surjective.

Let \( G \setminus \tilde{\Gamma} \) be the set of orbits in \( \tilde{\Gamma} \) under the group action. Then the map \( \Phi \) defines a bijective correspondence between \( G \setminus \tilde{\Gamma} \) and the dual space \( \hat{A} \rtimes \sigma G \) [4].

3. Topology on \( \tilde{\Gamma} \)

We endow the set \( \tilde{\Gamma} \) with the same topology as in [3, Theorem 4.1]. This topology is defined in terms of convergent sequences.

**Definition 1.** Let \((\pi_n, W_n)\) be a sequence in \( \tilde{\Gamma} \). We say that \((\pi_n, W_n)\) converges to \((\pi_0, W_0)\) in \( \tilde{\Gamma} \) with respect to the topology \( \Omega \) if

(a) \( \pi_n \to \pi_0 \)

(b) there is \( N \in \mathbb{N} \) such that \( G_{\pi_n} \leq G_{\pi_0} \) and \( W_n \leq W_0|_{G_{\pi_n}} \) for all \( n \geq N \).
We will show that the map $\Phi : (\hat{\Gamma}, \Omega) \rightarrow \hat{A} \rtimes_{\sigma} G$ is continuous. Furthermore, we will show that if $\hat{A}$ is Hausdorff, then $\Phi$ is a closed map. First, we need a few of ancillary results.

**Lemma 2.** Let $(A, G, \sigma)$ be a dynamical system where $G$ is finite. Let $Q$ be in $\text{Prim}(A)$. Suppose there is a sequence $P_n \in \text{Prim}(A)$ such that $(\bigcap_{n \in G} sP_n)_n$ converges to $\bigcap_{A} sQ$. Then there exists a subsequence $P_{n_k}$ and $s_0 \in G$ such that $P_{n_k}$ converges to $s_0Q$ for some $s_0 \in G$.

**Proof.** Since $(\bigcap_{n \in G} sP_n)_n$ converges to $\bigcap_{A} sQ$ it follows that

$$\bigcap_{n \in G} sP_n \subseteq \bigcap_{s \in G} sQ.$$ 

Let $J = \bigcap_{n} P_n$. Then $\bigcap_{n \in G} sJ \subseteq Q$. Since $Q$ is a prime ideal, then $s_0J \subseteq Q$ for some $s_0 \in G$. In particular, $\bigcap_{n} s_0P_n \subseteq Q$. Let $I$ be an ideal of $A$ such that $I \not\subseteq Q$ and let $O_I = \{I' \in \text{Prim}(A) : I \not\subseteq I'\}$ denote the corresponding open set in $\text{Prim}(A)$. Suppose, for contradiction, that $s_0P_n \notin O_I$ for all $n$. Then $I \subseteq s_0P_n$ for all $n$ and $I \subseteq Q$. It follows that for every open set $O_I$ containing $Q$ there exists $s_0P_n$ such that $s_0P_n \in O_I$. □

The next tool we need is the Forbenius Reciprocity Theorem for crossed products. The proof of the theorem is similar to the classical proof for the case of groups.

**Theorem 3** (Frobenius Reciprocity). Let $A \rtimes_{\sigma} G$ be a crossed product where $G$ is finite. Let $H$ be a subgroup of $G$. Let $\pi \rtimes_{\sigma} U$ be a representation of $A \rtimes_{\sigma} G$ on a Hilbert space $\mathcal{H}$ and $\delta \rtimes_{\lambda} \lambda$ a representation of $A \rtimes_{\sigma} H$ on $\mathcal{K}$. Then

$$\text{Hom}_{A \rtimes_{\sigma} G}(\mathcal{H}, \mathcal{K}^{G}) = \text{Hom}_{A \rtimes_{\sigma} H}(\mathcal{H}, \mathcal{K}).$$

In this isomorphism the $A \rtimes_{\sigma} G$-module homomorphism $\Theta : \mathcal{H} \rightarrow \mathcal{K}^{G}$ corresponds to the $A \rtimes_{\sigma} H$-module homomorphism $\theta : \mathcal{H} \rightarrow \mathcal{K}$, by the following formulae

$$\theta(\xi) = \Theta(\omega)(1), \quad \Theta(\omega)(g) = \theta(U(g)\omega).$$

**Proof.** Suppose that $\Theta$ is an $A \rtimes_{\sigma} G$-module homomorphism. We will show that $\theta$ is an $A \rtimes_{\sigma} H$-module homomorphism. Indeed, for each $a \in A, h \in H$ and $\xi \in \mathcal{H}$, we have

$$\theta(\pi(a)U(h)\xi) = \Theta(\pi(a)U(h)\xi)(1) = (\delta^{G}(a)\lambda^{G}(h)\Theta(\xi))(1) = \delta(a)\Theta(\xi)(h) = \delta(a)\lambda(h)\Theta(\xi)(1) = \delta(a)\lambda(h)\theta(\xi).$$

Conversely, suppose that $\theta$ is an $A \rtimes_{\sigma} H$-module homomorphism. Then, for each $a \in A, \xi \in \mathcal{H}$ and $g, s \in G$, we have

$$\Theta(\pi(a)U(g)\xi)(s) = \theta(U(s)\pi(a)U(g)\xi)$$
It follows that $\Phi(\pi, a) \in \mathcal{I}(A \rtimes_\sigma H)$ also since $\Theta(\pi, a) \in \mathcal{I}(A \rtimes_\sigma G)$. 

Therefore, $\tilde{\Phi} : (\pi, W) \rightarrow (\pi, G)$ is a continuous map. 

\[ \tilde{\Phi}(\pi, W) := \delta(\pi, a) \theta(U(sg)) \xi \]

\[ = \delta(\pi, a) \theta(U(sg)) \xi \]

\[ = \delta(\pi, a) \Theta(\xi)(sg) \]

\[ = \delta(\pi, a) (\lambda^G(g) \Theta(\xi)(s)) \]

\[ = (\delta^G(a) \lambda^G(g) \Theta(\xi))(s). \]

\[ \square \]

Induced representations give us a natural map from the set of representations of $A \rtimes_\sigma H$ to that of $A \rtimes_\sigma G$. There exists a corresponding map $\text{Ind}_H^G : \mathcal{I}(A \rtimes_\sigma H) \rightarrow \mathcal{I}(A \rtimes_\sigma G)$ between the ideal spaces. We equip $\mathcal{I}(A \rtimes_\sigma G)$ with the topology with subbasic open sets indexed by $J \in \mathcal{I}(A \rtimes_\sigma G)$ given by

\[ \mathcal{O}_J = \{ I \in \mathcal{I}(A \rtimes_\sigma G) : J \not\subseteq I \}. \]

The map $\text{Ind}_H^G$ is continuous with respect to the above topology [8, §5.3].

**Proposition 4.** Let $(A, G, \sigma)$ be a dynamical system where $G$ is finite. Let $\Phi : (\Gamma, \Omega) \rightarrow A \rtimes_\sigma G$ be as above. Then $\Phi$ is a continuous map.

**Proof.** Let $(\pi_n, W_n)$ be a sequence in $\tilde{\Gamma}$ converging to $(\pi_0, W_0) \in \tilde{\Gamma}$. Denote $(\pi_n, W_n) = (\pi_n \otimes 1, V_n \otimes W_n^\sigma)$ to be the corresponding representations of $(A, G\pi_n, \sigma)$. Since $G$ is finite we can assume $G\pi_n = H \leq G\pi_0$ and $W_n = W \leq W_0|_H$ for all $n$. Then $\pi_n \rtimes_\sigma W_n$ converge to $\pi_0 \rtimes_\sigma W$. In particular, $\ker(\pi_n \rtimes_\sigma W_n) \rightarrow \ker(\pi_0 \rtimes_\sigma W)$ in $\text{Prim}(A \rtimes_\sigma H)$. Since the map $\text{Ind}_H^{G\pi_0}$ is continuous it follows that

\[ \text{Ind}_H^{G\pi_0} \ker(\pi_n \rtimes_\sigma W_n) \rightarrow \text{Ind}_H^{G\pi_0} \ker(\pi_0 \rtimes_\sigma W). \]

Also since $\pi_0 \rtimes_\sigma W \leq (\pi_0 \rtimes_\sigma W_0)|_{A \rtimes_\sigma H}$, then by the Frobenius Theorem $\pi_0 \rtimes_\sigma W_0 \leq \text{Ind}_H^{G\pi_0} (\pi_0 \rtimes_\sigma W_0)$. Then

\[ \text{Ind}_H^{G\pi_0} \ker(\pi_n \rtimes_\sigma W_n) \rightarrow \ker(\pi_0 \rtimes_\sigma W_0). \]

Therefore,

\[ \text{Ind}_H^G \ker(\pi_n \rtimes_\sigma W_n) \rightarrow \text{Ind}_H^{G\pi_0} \ker(\pi_0 \rtimes_\sigma W_0). \]

It follows that $\Phi(\pi_n, W_n)$ converges to $\Phi(\pi_0, W_0)$. \[ \square \]

It remains to show that $\Phi$ is a closed map. Let $V$ be a closed set in $\tilde{\Gamma}$ and let $\rho \in \tilde{\Gamma} \rtimes_\sigma G$ be a limit point of $\Phi(V)$. Let $(\pi_n, W_n) \in V$ be a sequence such that $\Phi(\pi_n, W_n) \rightarrow \rho$. We need to show that there exists $(\pi_0, W_0) \in \tilde{\Gamma}$ such that $\Phi(\pi_0, W_0) = \rho$ and $(\pi_n, W_n) \rightarrow (\pi_0, W_0)$ in $(\tilde{\Gamma}, \Omega)$.

**Lemma 5.** Let $\rho \in \tilde{\Gamma} \rtimes_\sigma G$. Suppose there is a sequence $(\pi_n, W_n) \in \tilde{\Gamma}$ such that $\Phi(\pi_n, W_n) \rightarrow \rho$. Then there exists $(\pi, W) \in \tilde{\Gamma}$ such that $\Phi(\pi, W) = \rho$ and $\pi_n \rightarrow \pi$. 


Proof. Let \((\pi_0, W_0) \in \hat{\Gamma}\) such that \(\Phi(\pi_0, W_0) = \rho\). Then \(\ker (\pi_n \otimes 1) \to \ker (\pi_0 \otimes 1)\) in \(\mathcal{I}(A)\). In particular, \((\bigcap_{s \in G} s(\ker \pi_n))_n \to \bigcap_{s \in G} s(\ker \pi_0)\). Then by Lemma 3, there is a subsequence \(n_k\) and \(s_0 \in G\) such that
\[
\ker \pi_{n_k} \to s_0(\ker \pi_0).
\]
It follows that \(\pi_{n_k}\) converges to \(\pi_0 \circ s_{n_k}\). Since \(\Phi(\pi_0, W_0) = \Phi(\pi_0 \circ s_{n_k}, s_0 \cdot W_0)\), then, after reindexing, we get that \(\pi_n\) converges to \(\pi_0 \circ s_{n_k}\) and \(\Phi(\pi_0 \circ s_{n_k}, s_0 \cdot W_0) = \rho\).

**Lemma 6.** In the context of Lemma 5, suppose there is a sequence \((\pi_n, W_n) \in \hat{\Gamma}\) and a point \((\pi_0, W_0) \in \hat{\Gamma}\) such that \(\Phi(\pi_n, W_n) \to \Phi(\pi_0, W_0)\). If \(\hat{A}\) is Hausdorff, then there exists \(N\) such that \(G_{\pi_n} \leq G_{\pi_0}\) and \(W_n \leq W_0\) for all \(n \geq N\).

Proof. Since \(\Phi(\pi_n, W_n) \to \Phi(\pi_0, W_0)\), then by Lemma 5, \(\pi_n \to \pi_0\). Since \(\hat{A}\) is Hausdorff, then by the continuity of the group action there exists \(N\) such that \(G_{\pi_n} \leq G_{\pi_0}\) for all \(n \geq N\). To prove the second part of the claim, suppose for contradiction that there exists a subsequence \((\pi_{n_k}, W_{n_k})\) such that \(W_{n_k} \not\leq W_0\). Since \(G\) is finite, after passing to a subsequence, we may assume that \(G_{\pi_{n_k}} = H\) for all \(n \in \mathbb{N}\). Further, since \(H^2(H, \mathbb{T})\) is finite as well, we may assume that \(\omega_{\pi_n} = \omega\) and \(W_n = W \not\leq W_0|_H\) are also constant for all \(n \in \mathbb{N}\). Then for each \(\pi_n\) we may choose an \(\omega\)-representation \(V_n\) of \(H\) such that \(\Phi(\pi_n, W) = (\pi_n \otimes 1, V_n \otimes W^*)^G\) for all \(n \in \mathbb{N}\). Let \((\pi_0 \otimes 1, V_0 \otimes W^*)\) and \((\pi_0 \otimes 1, V_0 \otimes W_0^*)\) denote the covariant representations of \((A, H, \sigma)\) and \((A, G_{\pi_{n_k}}, \sigma)\) respectively, as defined in Equation 1.

Let \((V_n \otimes W^*)^{G_{\pi_{n_k}}}\) denote the induced representation of \(G_{\pi_{n_k}}\). Since \(W \not\leq W_0|_H\), then by the Frobenius Reciprocity theorem the representation \((V_n \otimes W^*)^{G_{\pi_{n_k}}}\) is disjoint from the representation \(V_0 \otimes W_0^*\) (see Remark 7). Therefore, for each \(n\), there exists an \(x_n \in C^*(G_{\pi_{n_k}})\) such that \((V_n \otimes W^*)^{G_{\pi_{n_k}}}(x_n) = 0\) and \((V_0 \otimes W_0^*)(x_n) \neq 0\). Since \(G_{\pi_{n_k}}\) is finite, after passing to a subsequence, we may assume that each \((V_n \otimes W^*)^{G_{\pi_{n_k}}}\) decomposes into the same direct sum of irreducible representations up to multiplicity. Furthermore, \((V_n \otimes W^*)^{G_{\pi_{n_k}}}(x_n) = 0\) if and only if \(\rho(x_n) = 0\) for all irreducible subrepresentations \(\rho\) of \((V_n \otimes W^*)^{G_{\pi_{n_k}}}\). It follows that there exists an \(x_0 \in C^*(G_{\pi_0})\) such that \((V_n \otimes W^*)^{G_{\pi_0}}(x_0) = 0\) and \((V_0 \otimes W_0^*)(x_0) = 0\) for all \(n\).

Since \(\hat{A}\) is Hausdorff there exist disjoint open sets \(N\) and \(M\) containing the point \(\pi_0\) and the set \(\{a_{\ker \pi_0}\}_{\pi \in S}\) respectively, where \(S\) is the set of representatives for \(G_{\pi_0}\ \backslash G\) which are not in \(G_{\pi_0}\). We claim that there exists \(a_0 \in A\) such that \(a_0(\pi_0) \neq 0\) and \(a_0(\rho) = 0\) for all \(\rho \in M\). Suppose for contradiction that \(\pi_0(a_0) = 0\) whenever \(\rho(a_0) = 0\) for all \(\rho \in M\). Then \(\bigcap_{\rho \in M} (\ker \rho) \subseteq \ker \pi_0\) and \(\ker \pi_0\) is in the closure of the set \(\{\ker \rho\}_{\rho \in M}\) in the hull-kernel topology. It follows that \(\pi_0\) is in the closure of \(M\) which contradicts our choice of \(N\) and
M. Define \( (a_0 \otimes x_0) : G \to A \) by
\[
(a_0 \otimes x_0)(t) = \begin{cases} 
  a_0x_0(t) & \text{if } t \in G_{\pi_0} \\
  0 & \text{if } t \notin G_{\pi_0}.
\end{cases}
\]
Recall that by induction in stages
\[
\Phi(\pi_n, W) = (\pi_n \otimes 1, V_n \otimes W^*)^G = \left((\pi_n \otimes 1, V_n \otimes W^*)^{G_{\pi_0}}\right)^G.
\]
For each \( n \), let \( \mathcal{H}_n \) denote the Hilbert space corresponding to the representation \( (\pi_n \otimes 1, V_n \otimes W^*)^{G_{\pi_0}} \). Then the representation \( \Phi(\pi_n, W) \) can be viewed as acting on the direct sum \( \bigoplus_r \mathcal{H}_n \), where \( \{ r_i \} \) is a set of representatives for \( G_{\pi_0} \). In addition, \( \Phi(\pi_n) \) is the diagonal operator \( \bigoplus_r r_i(\pi_n \otimes 1)^{G_{\pi_0}} \) and \( \Phi(V_n \otimes W^*) \) is a generalized permutation matrix with the 1 \( \times 1 \) entry given by \((V_n \otimes W^*)^{G_{\pi_0}} \) (see [1]). Note that \((\pi_n \otimes 1)^{G_{\pi_0}} = \bigoplus_r (\pi_n \otimes 1)\), where the direct sum is taken over set of representatives for \( H \setminus G_{\pi_0} \). Since \( \pi_n \to \pi_0 \), then \( r_i(\pi_n \otimes 1)^{G_{\pi_0}} \to r_i(\bigoplus_r l(\pi_n \otimes 1)) = r_i|_{\bigoplus_r l(\pi_n \otimes 1)} \). Let \( N \) and \( M \) be disjoint open sets containing the point \( \pi_0 \) and the set \( \{ r_i(\pi_0) \}_{r_i \in S} \) respectively and \( a_0 \in A \) such that \( \pi_0(a_0) \neq 0 \) and \( \rho(a_0) = 0 \) for all \( r_i \in M \). Since \( r_i(\pi_n \otimes 1)^{G_{\pi_0}} \to \bigoplus_r r_i(\pi_0 \otimes 1) \), then, for each \( r_i \in S \), eventually \( r_i(\pi_n \otimes 1)^{G_{\pi_0}}(a_0) = 0 \). It follows that \( \Phi(\pi_n)(a_0) \to (\pi_0 \otimes 1)(a_0) \bigoplus 0 \). Then we get that \( \Phi(\pi_n, W)(a_0 \otimes x_0) \to (\pi_0 \otimes 1)(a_0)(V_0 \otimes W^*)^{G_{\pi_0}}(x_0) = 0 \). Similarly, let \( \mathcal{H}_0 \) denote the Hilbert space corresponding to the representation \( (\pi_0 \otimes 1, V_0 \otimes W_0^*) \). Then the representation \( \Phi(\pi_0, W) \) can be viewed as acting on the direct sum \( \bigoplus_r \mathcal{H}_0 \), where \( \{ r_i \} \) is a set of representatives for \( G_{\pi_0} \). Likewise, \( \Phi(\pi_0) \) is the diagonal operator \( \bigoplus_r r_i(\pi_0 \otimes 1) \) and \( \Phi(V_0 \otimes W_0^*) \) is a generalized permutation matrix. Since \( \Phi(\pi_0)(a_0) = (\pi_0 \otimes 1)(a_0) \bigoplus 0 \), then \( \Phi(\pi_0, W)(a_0 \otimes x_0) = (\pi_0 \otimes 1)(a_0)(V_0 \otimes W_0^*)(x_0) \neq 0 \). It follows that \( \Phi(\pi_0, W) \) does not converge to \( \Phi(\pi_0, W) \) which contradicts the hypothesis of the lemma.

Remark 7. In the context of Lemma 6, by the Forbenius Reciprocity theorem the representation \((V_0 \otimes W_0^*)^{G_{\pi_0}} \) is disjoint from \( V_0 \otimes W_0^* \) if and only if \( V_0 \otimes W_0^* \) is disjoint from \( (V_0 \otimes W_0^*)|_{H} \). Since \( G \) is finite we have a direct sum decomposition \( V_0 \otimes W_0^* = \bigoplus_i (v_n_i \otimes W^*) \), where each \( v_n_i \) is an irreducible subrepresentation of \( V_n \). Similarly, we can decompose \((V_0 \otimes W_0^*)|_{H} \) into a direct sum of irreducible representations \( \bigoplus_{i,j} (v_{0i} \otimes w_{0j}) \), where each \( v_{0i} \) is an irreducible subrepresentation of \( V_0|_{H} \) and each \( w_{0j} \) is an irreducible subrepresentation of \( W_0^*|_{H} \). If \( V_0 \otimes W_0^* \) is not disjoint from \( (V_0 \otimes W_0^*)|_{H} \), then \( (v_n_i \otimes W^*) \) is equivalent to \((v_{0i} \otimes w_{0j}) \) for some \( i,j,k \). It would follow that \( W_0^* \) is equivalent \( w_{0k} \) for some \( k \).

We summarize our results in the following theorem.

Theorem 8. Let \( G \) be a finite group acting on a separable \( C^* \)-algebra \( A \). Let \( \Phi : A \times_{\sigma} G \to G \setminus \tilde{\Gamma} \) be the canonical bijection. Then the map \( \Phi \) is continuous. Moreover, if \( \tilde{A} \) is Hausdorff, then \( \Phi \) is in fact a homeomorphism.
Acknowledgments. I would like to express my gratitude to the referee for his/her patience, time and effort in providing me with valuable feedback that greatly improved the content of this paper.

References


FIRUZ KAMALOV
MATHEMATICS DEPARTMENT
CANADIAN UNIVERSITY OF DUBAI
DUBAI, UAE

E-mail address: firuz@cud.ac.ae