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Optimal Operation for Reverse Supply Chain Incorporating Inspection Policy into Remanufacturing of Used Products

Shin Yamaguchi, Etsuko Kusukawa*
Graduate School of Electrical and Electronic Systems, College of Engineering,
Osaka Prefecture University, Sakai, Osaka, Japan

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ABSTRACT

This paper discusses a reverse supply chain (RSC) which consists of the process flows from procurement of used products collected from a market, through remanufacturing products from the used products, to sales of the products in a market. In general, it is conceivable for the RSC to face the uncertainty in quality of used products collected from a market. Inspection is one of efficient methods to deal with the problem regarding quality of used products. However, there is a trade-off between inspection cost and inspection accuracy. This paper focuses on the following five types of inspection: (1) 100% inspection, (2) sampling inspection, (3) sampling inspection with screening of rejected lots, (4) sampling inspection with screening of acceptable lots, and (5) no inspection, and determines the optimal operation consisting of the optimal number of procured used products and the optimal inspection policy. Numerical analysis clarifies not only how changes of conditions of the RSC affect the manufacturer's optimal operation but also features of each inspection type. In addition, from the results of numerical analysis, this paper shows the usability to add the proposed inspection in this paper, the sampling inspection with screening of acceptable lots, to choices of inspection type.

Keywords: Reverse Supply Chain, Uncertainty in Quality of Used Products, Sampling Inspection, Inspection Policy, Optimal Operation

* Corresponding Author, E-mail: kusukawa@eis.osakafu-u.ac.jp

1. INTRODUCTION

In recent years, serious environmental problems such as waste of used natural resources and depletion of natural resources have been occurring worldwide (Pochampally *et al.*, 2009). Under above situations, environmental protection, resource saving and sustainable development have become increasingly important. For the purpose of solving above problems about environment protection and resource saving, several evaluation measures and policies have been promoted in order to establish a new supply chain management which incorporates reverse chains/logistics into traditional forward chains/logistics. The traditional forward chains/logistics are composed of the flows from procurement of new materials through production of new products to selling them. The reverse chains/logistics consists of the flows from collection of used products through recycling parts from the used products to reuse the recycled parts. Theoretical analyses and the marginal insights obtained from numerical examples on the reverse chains/logistics are discussed in the following previous papers: Aras *et al*. (2004), Behret and

Korugan (2009), Ferguson *et al*. (2009), Fleischman *et al*. (1997), Guide (2000), Guide and Wassenhove (2001), Inderfurth (2005), Konstantaras *et al*. (2010), Kusukawa and Akizawa (2015), Nenes *et al*. (2010), Pochampally *et al.* (2009), Pokharel and Liang (2012), Savaskan and Van Wassenhove (2006), Tang and Teunter (2006), Teunter and Flapper (2011), Watanabe and Kusukawa (2014), Watanabe *et al*. (2013), Wei *et al*. (2011), Wu (2012), Zhang *et al*. (2014).

Also, a supply chain which organizes the forward chains and the reverse chains has been called a closed-supply chain or reverse supply chain, which is defined in the following previous papers: Fleischman *et al*. (1997), Guide *et al*. (2003), Inderfurth (2005), Shi *et al.* (2010, 2011), Tagaras and Zikopoulos (2008), Thierry *et al*. (1995), Van Wassenhove and Zikopoulos (2010), Wei *et al.* (2012), Yan and Sun (2012), Zhang *et al*. (2014), Zikopoulos and Tagaras (2007, 2008).

In this study, a supply chain with the forward chains and the reverse chains is referred to as a reverse supply chain (RSC). The concept and management of RSC has been evolving to assist practitioners, academic researchers and real-world policymakers regarding production planning in a RSC which try to take some measures and policies in order to promote 3R activities (Reduce-Reuse-Recycle) in the RSC.

In general, it is conceivable for the RSC to face the uncertainty in quality of used products collected from a market. It is necessary for practitioners, academic researchers and real-world policymakers to consider the optimal operations for RSC, and the uncertainty in quality of used products regarding the remanufacturing. Several previous papers have dealt with the optimal operations for RSC, and the uncertainty in quality in used products regarding the remanufacturing in a RSC has been attracting more attention in recent papers. Aras *et al.* (2004) investigated the effect of the stochastic nature of product returns on the optimal production planning and verified conditions under which quality-based categorization was most cost effective. Zikopoulos and Tagaras (2007) investigated how uncertainty regarding the quality of returned products in two collection sites affected the profitability of reuse activities and derived the unique optimal solution (procurement and production quantities). In Guide and Van Wassenhove (2001) and Ferguson *et al.* (2009), returned products were assumed to have N quality categories, and the procurement prices and the remanufacturing costs were different based on the corresponding quality level. Behret and Korugan (2009) discussed a remanufacturing stage with uncertainties in the quality of remanufacturing products, return rates and return times of returned products. After returned products were classified by considering the uncertainty in quality of the returned product, remanufacturing processing times, material recovery rates, the remanufacturing costs and disposal costs were determined by using the ARENA simulation program.

Inspection is one of efficient methods to deal with the uncertainty in quality of used products regarding the remanufacturing. Theoretical analyses and the marginal insights obtained from numerical examples regarding the effects of inspection and sorting of used products on the optimal tactical production planning in RSC are discussed in the following previous papers: Aras *et al.* (2004), Behret and Korugan (2009), Ferguson *et al.* (2009), Guide and Van Wassenhove (2001), Guide *et al.* (2003), Konstantaras *et al.* (2010), Nenes *et al.* (2010), Tagaras and Zikopoulos (2008), Zikopoulos and Tagaras, (2007, 2008). Zikopoulos and Tagaras (2007, 2008) and Van Wassenhove and Zikopoulos (2010) incorporated some quality classification errors into a RSC. Here, quality classification errors were dealt as an uncertain value. Zikopoulos and Tagaras (2008) handled quality classification errors as Type I error and Type II error based on a concept of quality control. However, in their previous studies, possible errors in inspection and sorting of used products were considered as a given Type I error and a given Type II error, and those errors were incorporated into the RSC. Besides, in the above previous studies, a concept of the optimal decision-making for the lower limit of quality to recycle used products was not considered for a RSC.

However, above previous papers did not discussed a trade-off between inspection cost and inspection accuracy. When inspection processes are incorporated into operations in a RSC, it is necessary for firms related to operation of the RSC to consider a trade-off between inspection cost and inspection accuracy. In addition, it isn't always possible to develop and apply an adequately accurate inspection technology at low cost. In the cases, a sampling inspection may be a realistic alternative to mitigate the inspection cost (Lee and Rosenblatt. 1985, Panagiotidou *et al.*, 2013). Panagiotidou *et al.* (2013) discussed the optimal decision for a procurement quantity of used products and a sampling size. A sampling inspection for a batch of used products was conducted before the procurement. The procurement quantity and the sample size were determined according to the estimated quality condition from results of the sampling inspection. Establishing a sampling inspection might produce the potential economic benefits. However, this previous paper didn't sufficiently consider how disposal of un-remanufactured used products affected the expected profit of a remanufacturer. Here, a sampling inspection has two essential demerits: (I) remanufacturable used products are disposed and (II) unremanufacturable used products are conveyed to a remanufacturing process.

This paper focuses on the optimal operation for a RSC to remanufacture a single type of used products such as consumer electronics (mobile phone, personal computer), semiconductor and electronic component. (Guide,

2000; Guide and Jayaraman, 2000; Guide and Van Wassenhove, 2001; Guide *et al*., 2003; Ferguson *et al*., 2009; Van Wassenhove and Zikopoulos, 2010). When the RSC is operated to remanufacture parts extracted from used products, it is necessary for firms to discuss how the quality of used products collected from a market and a trade-off between inspection cost and inspection accuracy affects the optimal operation for a RSC. Also, firms may need to determine the optimal operation when used products are classified into a small number N of quality categories according to a quality classification list provided by a firm which remanufacturers parts/products (Guide and Van Wassenhove, 2001; Guide *et al*., 2003; Ferguson *et al*., 2009; Van Wassenhove and Zikopoulos, 2010).

The motivation of this paper is to find solutions for the following problems in the remanufacturing of used products: (i) a trade-off between inspection cost and inspection accuracy, (ii) the problem that remanufacture able used products are disposed, (iii) the problem that unremanufactur able used products are conveyed to a remanufacturing process, (iv) the problem that used products are classified into a small number N of quality categories. This paper tries to provide the optimal operation for a RSC incorporating inspection policy into remanufacturing of used products to practitioners, academic researchers, and real-world policymakers who make plan for the optimal operation in a RSC, considering the above problems in the remanufacturing of used products in a RSC.

Concretely, the inspection process is incorporated into a RSC with a single manufacturer who remanufactures products from procured used products collected from a market. As the optimal operation for the RSC, the number of procured lots of used products and the inspection policy are discussed. This paper focuses on the following five inspection types (ITs): (IT-1) 100% inspection, (IT-2) sampling inspection, (IT-3) sampling inspection with screening of rejected lots, (IT-4) sampling inspection with screening of acceptable lots, and (IT-5) no inspection. IT-2 corresponds to a solution for the problem (i). IT-3 can resolve the demerit (I) and corresponds to a solution for the problem (ii). IT-4 can resolve the demerit (II) and corresponds to a solution for the problem (iii). This paper determines the optimal operation for the RSC, consisting of the optimal number of procured lots of used products and the optimal inspection policy, so as to maximize the manufacturer's expected total profit.

The numerical analysis investigates how changes of quality distribution of used products, inspection cost of used products, disposal cost of used products, manufacturing cost of new products, and the maximal number of procured lots of used products, affect the maximal expected total profit of a manufacturer and the optimal operation in a RSC.

Also, from the results of numerical analysis, this paper shows features of each inspection type and the usability to add inspection types (4), which is the proposed inspection in this study, to choices of inspection type for a RSC.

Contribution of this paper is to consider the following topics regarding a RSC.

- The 5 ITs: (IT-1) 100% inspection, (IT-2) sampling inspection, (IT-3) sampling inspection with screening of rejected lots, (IT-4) sampling inspection with screening of acceptable lots (the proposed inspection type in this paper), and (IT-5) no inspection, are applied to the inspection process in the RSC.
- Used products are classified into the discrete quality level in IT-2, IT-3, and IT-4, so as to make application of model analysis in this paper easier for a decision-maker of the RSC.
- The effect of a variety of quality in used products is considered on the probability that each part extracted from used product is remanufacture able and the remanufacturing cost depending on the quality level of each used product in the model analysis of the RSC.
- Influences of the difference in disposal costs depending on the disposing stage on the maximal expected total profit of a manufacturer in the RSC are clarified.
- This paper guarantees that no situation where used products taken as sample are inspected again occurs in each inspection type by defining the flows of used products and conducting the model analysis according to the clarified flows.
- The optimal decision procedure for each inspection type is proposed to determine the optimal number of procured lots of used products and the optimal sampling inspection scheme so as to maximize the expected total profit of a manufacturer. The optimal operation, which consists of the optimal number of procured lots of used products and the optimal inspection policy, are finally determined by comparing the maximal expected total profits among five inspection types. As a result, not only the optimal operation for the RSC but also features of each inspection type can be clarified.

The rest of this paper is organized as follows: in Section 2, notation used in considered model is defined. In Section 3, operational flows of a RSC considered in this paper and model assumptions of the RSC are described. In Section 4, the manufacturer's expected total profit is formulated for each inspection type which is incorporated into the RSC. Section 5 describes the optimal decision procedures for the optimal number of procured lots of used products and the optimal sampling inspection scheme in each inspection type. In Section 6, numerical analysis shows the optimal operation, which consists of the optimal number of procured lots of used products and the optimal inspection policy, in a RSC and investigates how changes of conditions of the RSC affect the optimal operation and the maximal expected total profit of a manufacturer in the RSC. In Section 7, conclusions, mana-

gerial insights and future researches for this paper are summarized.

2. NOTATION

• Decision Variables for a RSC

i : index number of inspection type $(i \in \{1, 2, \cdots, 5\})$

R : the number of procured lots of used products, referred to simply as the procured lots

n : sample size taken from a single lot for sampling inspection $(0 \le n \le Q)$, where Q denotes the number of used products in a single lot

Q_n : acceptance criterion for sampling inspection $(0 \le Q_n \le n)$

• Other Variables and Functions in a RSC

y : quality of used products $(0 \le y \le 1)$

$g(y)$: the probability density function (pdf) of quality y

I : the total number of quality classification

ℓ : quality level of used products $(1 \le \ell \le I)$

$G(\ell)$: the ratio of used products with quality level ℓ in procured used products

$\mathrm{E}[y \mid \ell]$: the conditional expectation of quality y under quality level ℓ

Q : the number of used products in a single lot

R_{Max} : the maximal number of procured lots of used products

a : procurement cost per lot of used products

c_c : classification cost per unit of used products

c_d : disassembly cost per unit of used products

c_I : inspection cost per unit of parts extracted from used products

$c_{D1}(n)$: the unit disposal cost of used products not taken as sample from lots which are judged as rejected lots in sampling inspection for sample size n, referred to simply as the unit first disposal cost

c_{D2} : the unit disposal cost of unremanufacturable (defective) parts in inspected parts extracted from used products including the used products taken as samples, referred to simply as the unit second disposal cost

k : ratio of the minimum of $c_{D1}(n)$ to c_{D2}

c_{D3} : the unit disposal cost of unremanufacturable (defective) parts in used products which are not inspected and sent to the remanufacturing process, referred to simply as the unit third disposal cost

$r(\ell)$: the probability of each of the parts extracted from used products with quality level ℓ to be remanufacturable (conforming), referred to simply as the conforming probability for quality level ℓ

$c_r(\ell)$: the unit remanufacturing cost of conforming parts extracted from used products with quality level ℓ

x : demand of a single product, referred to simply as product demand

$f(x)$: the probability density function (pdf) of product demand x

$F(x)$: the cumulative distribution function (cdf) of products demand x

S_{Max} : the maximal number of products which the manufacturer can supply in a market

v : sales price per unit of products

h_r : the unit inventory holding cost of excess remanufactured products for demand x per unit time

c_M : the unit manufacturing cost of new products from new parts

s : shortage penalty cost per unsatisfied demand

$Q_r^i(R, n, Q_n)$: the expected number of remanufactured products in inspection type i for procured lots R, sample size n, and acceptance criterion Q_n

$P(Q_r)$: the expected profit for number of remanufactured products Q_r, consisting of the expected sales of products, the expected inventory holding cost of excess remanufactured products, the expected manufacturing cost of new products, and the expected shortage penalty cost of unsatisfied product demand

$TP^i(R, n, Q_n)$: the expected total profit (of a single manufacturer) in inspection type i for procured lots R, sample size n, and acceptance criterion Q_n

$R^{i*}(n, Q_n)$: the optimal procured lots of used products in inspection type i for sample size n and acceptance criterion Q_n

n^{i*} : the optimal sample size in inspection type i

Q_n^{i*} : the optimal acceptance criterion in inspection type i

R^{i*} : the optimal procured lots of used products in inspection type i, corresponding to the optimal procured lots $R^{i*}(n^{i*}, Q_n^{i*})$ for the optimal sample size n^{i*} and the optimal acceptance criterion Q_n^{i*} in inspection type i

TP^{i*} : the maximal expected total profit in inspection type i, corresponding to the expected total profit $TP^i(R^{i*}, n^{i*}, Q_n^{i*})$ in inspection type i for the optimal number of procured lots R^{i*}, the optimal sample size n^{i*}, and the optimal acceptance criterion Q_n^{i*}

Q_r^{i*} : the expected number of remanufactured products under optimal situation in inspection type i, corresponding to the expected number of remanufactured products $Q_r^i(R^{i*}, n^{i*}, Q_n^{i*})$ in inspection type i for the optimal number of procured lots R^{i*}, the optimal sample size n^{i*}, and the optimal acceptance criterion Q_n^{i*}

c_r^{i*} : the average of the expected total cost per unit of remanufactured products under optimal situation in inspection type i

3. MODEL DESCRIPTIONS

3.1 Flows of Used Products in a RSC

This paper focuses on the optimal operation for a RSC to remanufacture a single type of used products such as consumer electronics (mobile phone, personal computer), semiconductor and electronic component. This paper considers a RSC with a manufacturer. The manufacturer procures a single type of used products at lot unit. After disassembly of used products in procured lots to a single type of parts and inspection of the parts, the manufacturer remanufactures a single type of products from the remanufacturable (conforming) parts and sells the products in a market. This paper proposes the optimal operation, which consists of the procured lots and inspection policy, so as to maximize the manufacturer's expected total profit in a single period. The flows of used products, parts, and remanufactured products in a RSC addressed in this paper are shown as follows.

(1) A manufacturer procures R lots of used products. The procurement cost per lot is a. A single lot consists of used products Q. The maximal number of procured lots is R_{Max}.
(2) The manufacturer chooses one IT from five ITs, IT-1 is 100% inspection, IT-2 is sampling inspection, IT-3 is sampling inspection with screening of rejected lots, IT-4 is sampling inspection with screening of acceptable lots, and IT-5 is no inspection and applies the IT to all of procured used products. In specified ITs, before the inspection, the manufacturer classifies each of procured used products QR within $1 \le \ell \le I$ of quality levels according to the its quality y at the unit cost c_c. ℓ denotes quality level of used products, and ℓ is a natural number. After that, the manufacturer collects up the used products with the same quality level at lot unit. A single lot has the quantity Q of used products with the same quality level.
(3) Used products which passed any IT are conveyed to the remanufacturing process after they are disassembled to parts at the unit cost c_d. Here, the used products which have already been disassembled to parts for inspection are sent there directly. If unremanufacturable (defective) parts are sent to the remanufacturing process, they are found surely and disposed there. The manufacturer remanufactures all of the conforming parts into the products without considering the aspect of the total profit.
(4) Remanufactured products are sold in a market at the unit price v. The manufacturer incurs the unit inventory holding cost h_r of excess remanufactured products for the product demand x per unit time. In contrast, when the quantity of remanufactured products doesn't satisfy the product demand x, the manufacturer produces new products from new parts at the unit manufacturing cost c_M. c_M includes all the required cost from procuring the new parts to manufacturing the new products. The maximal number of products which the manufacturer can supply in a market is S_{Max}, combining the remanufactured products and the new products. When S_{Max} is doesn't satisfy the products demand x, the manufacturer incurs the shortage penalty cost s per unsatisfied demand.

3.2 Inspection Type (IT) Incorporated into a RSC

This paper incorporates five ITs into a RSC. Here, $i \in \{1, 2, \cdots, 5\}$ is an index number of IT. Figures 1-5 show the flows of system operations when IT-*i* is incorporated into a RSC.

• 100% Inspection (IT-1)

From Figure 1, all of procured used products are disassembled to parts and inspected without classification. The conforming parts are conveyed to the remanufacturing process. The manufacturer remanufactures the products from them. The defective parts are disposed in the inspection process at the unit cost c_{D2}.

• Sampling Inspection (IT-2)

From Figure 2, the manufacturer classifies the procured used products as to quality and collects up the used products with the same quality level at lot unit. After that, the manufacturer takes the used products n from each lot as samples, and the manufacturer disassembles the samples to parts and inspects them. Based on the magnitude relation between the number of conforming parts Q_C in the samples n and the acceptance criterion Q_n, each lot is judged acceptable or rejected. Concretely, if $Q_n \le Q_C \le n$, the lot is judged as 'acceptable lot'. In this case, the used products $(Q-n)$ not taken in the lot are disassembled to parts and conveyed to the remanufacturing process.

The conforming parts in the conveyed parts are remanufactured into the products, while the defective parts are disposed in the remanufacturing process at the unit cost c_{D3}. In contract, if $0 \le Q_C \le Q_n - 1$, the lot is judged as 'rejected lot.' In this case, the used products $(Q-n)$ not taken in the lot are disposed in the inspection process in a bundle at the unit cost $c_{D1}(n)$. Regardless whether a single lot has passed the sampling inspection or not, the conforming parts in samples are conveyed to the remanufacturing process and they are remanufactured into the products, while the defective parts in them are disposed in the inspection process at the unit cost c_{D2}.

• Sampling Inspection with Screening of Rejected Lots (IT-3)

From Figure 3, as with IT-2, after the used products are classified and are collected up at lot unit as to their quality, the samples n taken from each lot are disassembled to parts and inspected. Based on the magnitude relation between Q_C and Q_n, each lot is judged as 'acceptable lot' or 'rejected lot.' The used products $(Q-n)$ not taken in the acceptable lots and the samples n are respectively treated as with IT-2. The used products

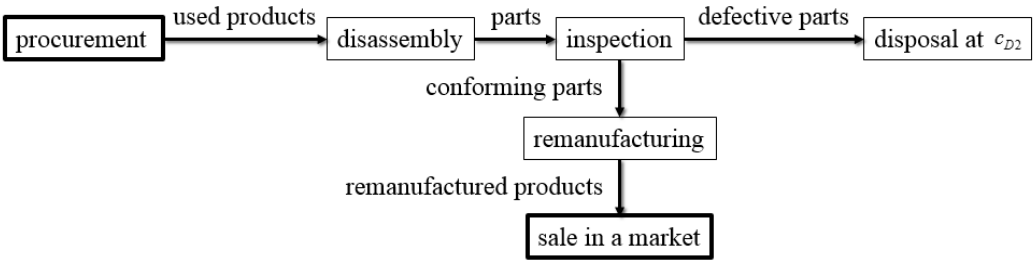

Figure 1. The flows of the operations when IT-1 is incorporated into a RSC.

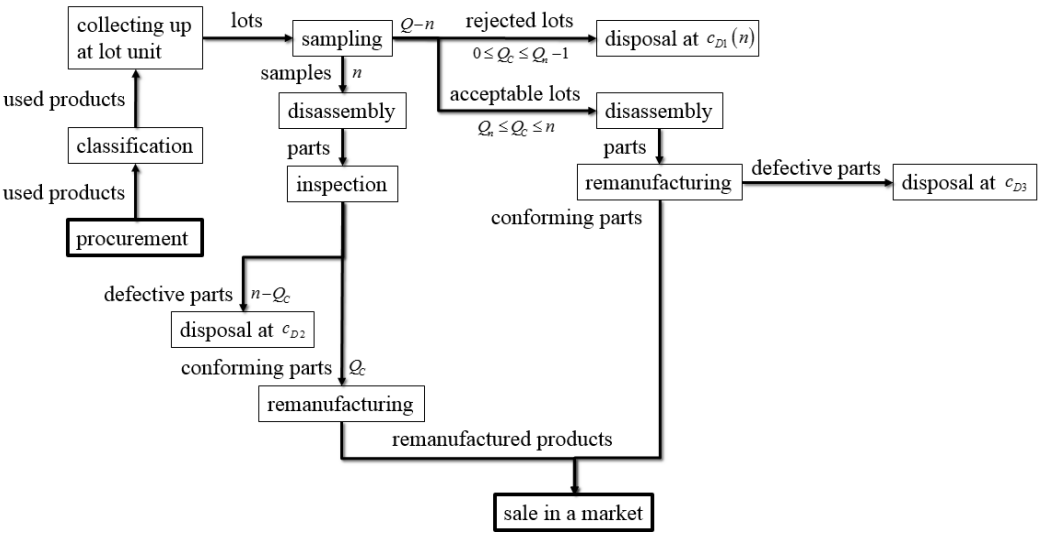

Figure 2. The flows of the operations when IT-2 is incorporated into a RSC.

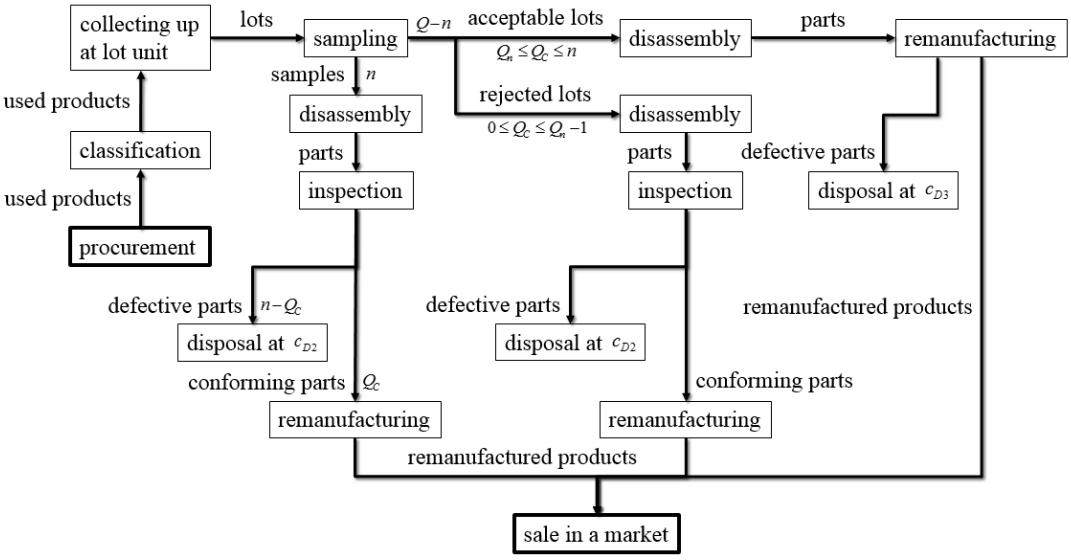

Figure 3. The flows of the operations when IT-3 is incorporated into a RSC.

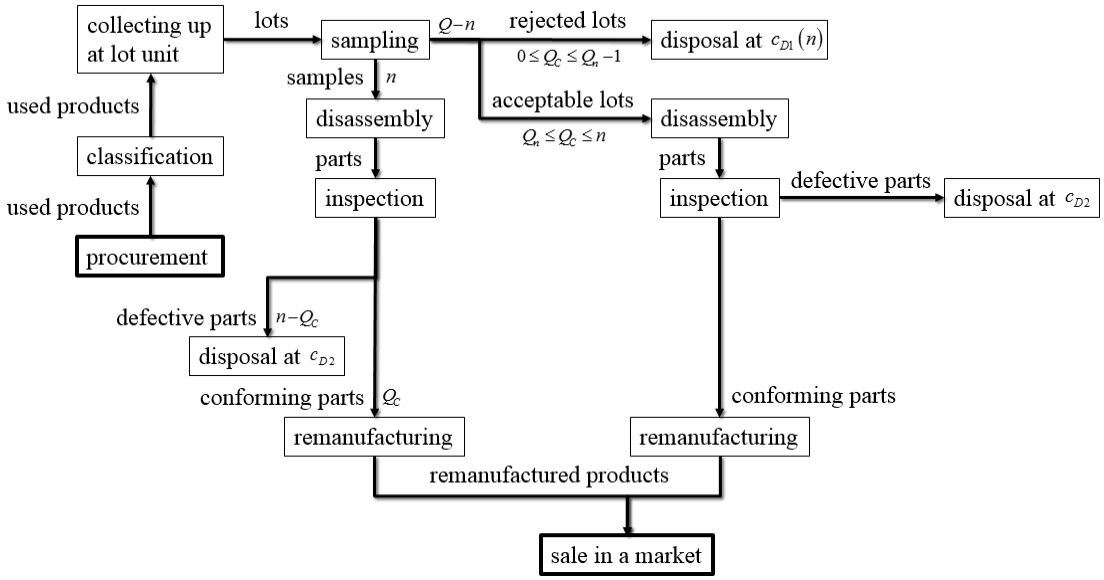

Figure 4. The flows of the operations when IT-4 is incorporated into a RSC.

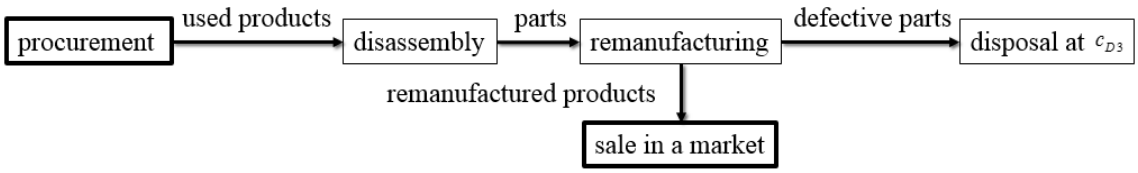

Figure 5. The flows of the operations when IT-5 is incorporated into a RSC.

$(Q-n)$ not taken in the rejected lots are disassembled to parts and inspected. The conforming parts are conveyed to the remanufacturing process and remanufactured into the products, while the defective parts are disposed in the inspection process at the unit cost c_{D2}.

- Sampling Inspection with Screening of Acceptable Lots (IT-4): the Proposed IT

From Figure 4, as with IT-2 and IT-3, after the used products are classified and collected up at lot unit as to their quality, the samples n taken from each lot are disassembled to parts and inspected. Based on the magnitude relation between Q_C and Q_n, each lot is judged as 'acceptable lot' or 'rejected lot'. Differently from IT-3, in IT-4, the used products $(Q-n)$ not taken in the acceptable lots are disassembled to parts and inspected. The conforming parts are conveyed to the remanufacturing process and remanufactured into the products, while the defective parts are disposed in the inspection process at the unit cost c_{D2}. The used products $(Q-n)$ not taken in the rejected lots are disposed in the inspection process in a bundle at the unit cost $c_{D1}(n)$ as with IT-2.

- No Inspection (IT-5)

From Figure 5, after all of procured used products are disassembled to parts without classification as to quality and inspection, they are conveyed to the remanufacturing process. The conforming parts are remanufactured into the products, while the defective parts are disposed in the remanufacturing process at the unit cost c_{D3}.

3.3 Model Assumptions

(1) The manufacturer doesn't know the quality y of each of used products but knows the pdf $g(y)$ of quality y of the whole procured used products. The higher value of y, the higher the quality.
(2) The manufacturer doesn't know the product demand x but knows the pdf $f(x)$ of demand x.
(3) It is necessary to disassemble used products to parts at the unit cost c_d before they are inspected regardless of IT. The inspection is conducted at the unit cost c_I. It enables to discriminate conforming parts from defective parts.
(4) Each IT doesn't conduct the destructive inspection. In this case, the manufacturer can use conforming parts which are inspected in order to remanufacture the products.
(5) As written in Section 2.2, $c_{D1}(n)$ is the unit disposal cost of used products not taken as samples from lots which are judged as rejected lots in sampling inspection for sample size n, c_{D2} is the unit disposal cost of defective parts in inspected parts extracted from used products including the used products taken as samples, and c_{D3} is the unit disposal cost of defec-

tive parts in used products which are not inspected and sent to the remanufacturing process. The values of these disposal costs don't depend on whether the used products have already been disassembled to parts or not, but the cost to dispose the used products or the parts in a bundle is lower than the cost to dispose them one by one. Thus, $c_{D1}(n)$ becomes higher as n increases or the disposal quantity of the used products not taken in a single rejected lot decreases. Also, $c_{D1}(Q-1)=c_{D2}$ and $c_{D1}(n)\le c_{D2}(0\le n\le Q-1)$ are satisfied.

(6) The disposal cost in the remanufacturing process is higher than the sum of the inspection cost and the disposal cost in the inspection process. Therefore, $c_I + c_{D2} < c_{D3}$ is satisfied.

(7) In this case, the number of the used products with quality level ℓ is $QRG(\ell)$, and the number of lots consisting of the used products Q with quality level ℓ is $RG(\ell)$. The level 1 indicates the lowest quality, and the level I indicates the highest quality. $G(\ell)$ is calculated as

$$G(\ell)=\int_{(\ell-1)/I}^{\ell/I} g(y)\,dy\,. \tag{1}$$

(8) Parts extracted from used products with quality level ℓ are referred to as parts with quality level ℓ. They are invariable within $(\ell-1)/I\le y\le \ell/I$ regardless whether the used products are classified or not.

(9) The higher quality level ℓ of used products, the higher the conforming probability $r(\ell)$ and the lower the remanufacturing cost $c_r(\ell)$. Therefore, $r(\ell)$ and $c_r(\ell)$ have the following properties of functions with respect to the quality level ℓ within $0\le \ell\le I$: $dr(\ell)/d\ell\ge 0$, $dc_r(\ell)/d\ell\le 0$.

(10) The manufacturer knows $r(\ell)$, but the numbers of the conforming parts and the defective parts in parts with quality level ℓ in each lot are uncertain. In the situation, it is appropriate to assume that the number of the conforming parts and the defective parts in them follow a binomial distribution with two parameters, the number of parts with quality level ℓ, N and the conforming probability $r(\ell)$. Using the binomial distribution, the probability $p\{N, Q_C, r(\ell)\}$ that the conforming parts Q_C are included in parts with quality level ℓ, N is formulated as

$$p\{N,Q_C,r(\ell)\}=\binom{N}{Q_C}\{r(\ell)\}^{Q_C}\{1-r(\ell)\}^{N-Q_C} \tag{2}$$

In this case, the expected number of the conforming parts Q_C is obtained as

$$\sum_{Q_C=0}^{N}\left[Q_C p\{N,\ Q_C,\ r(\ell)\}\right]$$

$$=\sum_{Q_C=0}^{N}\left[Q_C\binom{N}{Q_C}\{r(\ell)\}^{Q_C}\{1-r(\ell)\}^{N-Q_C}\right]=Nr(\ell) \tag{3}$$

(See Appendix A). Here, to express Eq. (3) simply, the following definition is used as

$$B_N(Q_X, Q_Y, z, \ell)$$
$$=\sum_{Q_C=Q_X}^{Q_Y}\left[z\binom{N}{Q_C}\{r(\ell)\}^{Q_C}\{1-r(\ell)\}^{N-Q_C}\right]. \tag{4}$$

Applying Eq. (4) to Eq. (3), Eq. (3) is rewritten as

$$B_N(0,N,Q_C,\ell)=Nr(\ell)\,. \tag{5}$$

Also, when $z=1$ in Eq. (4), Eq. (4) represents the probability to satisfy the condition $Q_X\le Q_C\le Q_Y$. Accordingly, the probability that a single lot of used products with quality level ℓ is acceptable and the probability that it is rejected are respectively represented as $B_n(Q_n, n, 1, \ell)$ and $B_n(0, Q_n-1, 1, \ell)$. Here, the relation $B_n(0, Q_n-1, 1, \ell)+B_n(Q_n, n, 1, \ell)=1$ is satisfied.

(11) The maximal number of the products supplied by the manufacturer, S_{Max} is greater than or equal to the maximal number of remanufactured products QR_{Max}.

4. THE MANUFACTURER'S EXPECTED TOTAL PROFIT IN EACH INSPECTION TYPE (IT)

According to Section 3, the manufacturer's total profit is obtained from the sales of products, the procurement cost of used products, the classification cost of used products, the disassembly cost of used products, the inspection cost of used products, the disposal costs of used products and parts, the remanufacturing cost of products, the inventory holding cost of remanufactured products, the manufacturing cost of new products, and the shortage penalty cost of products.

In order to express simply $TP^i(R, n, Q_n)$, in advance, the expected profit $P(Q_r)$ for the number of remanufactured products Q_r, consisting of the expected sales of products, the expected inventory holding cost of excess remanufactured products, the expected manufacturing cost of new products, and the expected shortage penalty cost of unsatisfied product demand, is calculated. $P(Q_r)$ is obtained as follows:

$$P(Q_r)=v\left\{\int_0^{S_{Max}} xf(x)\,dx+\int_{S_{Max}}^{\infty} S_{Max}f(x)\,dx\right\}$$
$$-h_r\int_0^{Q_r}(Q_r-x)f(x)\,dx$$
$$-c_M\int_{Q_r}^{S_{Max}}(x-Q_r)f(x)\,dx$$

$$
\begin{aligned}
&-s\int_{S_{Max}}^{\infty}(x-S_{Max})f(x)dx \\
&=v\left\{S_{Max}-\int_0^{S_{Max}}F(x)dx\right\}-h_r\int_0^{Q_r}F(x)dx \\
&-c_M\left\{S_{Max}-Q_rF(S_{Max})-\int_{Q_r}^{S_{Max}}F(x)dx\right\} \\
&-s\left\{\mathrm{E}[x]-S_{Max}+\int_0^{S_{Max}}F(x)dx\right\} \qquad (6)
\end{aligned}
$$

(See Appendix B).

4.1 Case of 100% Inspection (IT-1)

From Section 3.2, both of n and Q_n are 0 in IT-1. Therefore, the expected total profit $TP^1(R,0,0)$ and the expected number of remanufactured products $Q_r^1(R,0,0)$ in IT-1 for R are respectively formulated as

$$
\begin{aligned}
TP^1(R,0,0) &= -aR-(c_d+c_I)QR \\
&-c_{D2}\sum_{\ell=1}^{I}\left[Q\{1-r(\ell)\}RG(\ell)\right] \\
&-\sum_{\ell=1}^{I}\left[c_r(\ell)Qr(\ell)RG(\ell)\right] \\
&+P\left\{Q_r^1(R,0,0)\right\}, \qquad (7)
\end{aligned}
$$

$$
Q_r^1(R,0,0)=\sum_{\ell=1}^{I}Qr(\ell)RG(\ell) \qquad (8)
$$

In Eq. (7), the first term is the procurement cost of used products at lot unit. The second term is the disassembly cost and the inspection cost of all of parts extracted from the used products in the procured lots. The third term is the expected second disposal cost of the defective parts judged in the inspection process. The fourth term is the expected remanufacturing cost of products from the conforming parts. The fifth term is calculated from Eq. (6) in IT-1 where n and Q_n are 0. Note that the expected classification cost of used products aren't included in Eq. (7) since the used products aren't classified in IT-1.

4.2 Case of Sampling Inspection (IT-2)

From Section 3.2, the expected total profit $TP^2(R,n,Q_n)$ and the expected number of remanufactured products $Q_r^2(R,n,Q_n)$ in IT-2 for R, n, and Q_n, are respectively formulated as

$$
\begin{aligned}
TP^2(R,n,Q_n) &= -aR-c_cQR-(c_d+c_I)nR \\
&-c_{D2}\sum_{\ell=1}^{I}\left[n\{1-r(\ell)\}RG(\ell)\right] \\
&-\sum_{\ell=1}^{I}\left[c_r(\ell)nr(\ell)RG(\ell)\right] \\
&-c_{D1}(n)\sum_{\ell=1}^{I}\left[(Q-n)B_n(0,Q_n-1,1,\ell)RG(\ell)\right] \\
&-c_d\sum_{\ell=1}^{I}\left[(Q-n)B_n(Q_n,n,1,\ell)RG(\ell)\right] \\
&-c_{D3}\sum_{\ell=1}^{I}\left[(Q-n)\{1-r(\ell)\}B_n(Q_n,n,1,\ell)RG(\ell)\right] \\
&-\sum_{\ell=1}^{I}\left[c_r(\ell)(Q-n)r(\ell)B_n(Q_n,n,1,\ell)RG(\ell)\right] \\
&+P\left\{Q_r^2(R,n,Q_n)\right\}, \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
Q_r^2(R,n,Q_n) &= \sum_{\ell=1}^{I}\left[nr(\ell)RG(\ell)\right] \\
&+\sum_{\ell=1}^{I}\left[(Q-n)r(\ell)B_n(Q_n,n,1,\ell)RG(\ell)\right] \qquad (10)
\end{aligned}
$$

In Eq. (9), the first term is the procurement cost of used products at lot unit. The second term is the classification cost of used products. The third term is the disassembly cost and the inspection cost of parts extracted from the used products taken as samples. The forth term is the expected second disposal cost of defective parts in samples taken from the procured lots. The fifth term is the expected remanufacturing cost of products from conforming parts in used products as sample. The sixth term is the expected first disposal cost of used products not taken as sample in rejected lots. The seventh term is the expected disassembly cost of used products not taken as sample in acceptable lots. The eighth term is the expected third disposal cost of defective parts not taken as sample in the acceptable lots. The ninth term is the expected remanufacturing cost of products from conforming parts not taken as sample in acceptable lots. The tenth term is calculated from Eq. (6) in IT-2.

4.3 Case of Sampling Inspection with Screening of Rejected Lots (IT-3)

From Section 3.2, the expected total profit $TP^3(R,n,Q_n)$ and the expected number of remanufactured products $Q_r^3(R,n,Q_n)$ in IT-3 for R, n, and Q_n, are respectively formulated as

$$
\begin{aligned}
TP^3(R,n,Q_n) &= -aR-c_cQR-(c_d+c_I)nR \\
&-c_{D2}\sum_{\ell=1}^{I}\left[n\{1-r(\ell)\}RG(\ell)\right]-\sum_{\ell=1}^{I}\left[c_r(\ell)nr(\ell)RG(\ell)\right] \\
&-(c_d+c_I)\sum_{\ell=1}^{I}\left[(Q-n)B_n(0,Q_n-1,1,\ell)RG(\ell)\right] \\
&-c_{D2}\sum_{\ell=1}^{I}\left[(Q-n)\{1-r(\ell)\}B_n(0,Q_n-1,1,\ell)RG(\ell)\right] \\
&-\sum_{\ell=1}^{I}\left[c_r(\ell)(Q-n)r(\ell)B_n(0,Q_n-1,1,\ell)RG(\ell)\right] \\
&-c_d\sum_{\ell=1}^{I}\left[(Q-n)B_n(Q_n,n,1,\ell)RG(\ell)\right] \\
&-c_{D3}\sum_{\ell=1}^{I}\left[(Q-n)\{1-r(\ell)\}B_n(Q_n,n,1,\ell)RG(\ell)\right]
\end{aligned}
$$

$$-\sum_{\ell=1}^{I}\left[c_r(\ell)(Q-n)r(\ell)B_n(Q_n,n,1,\ell)RG(\ell)\right]$$
$$+P\left\{Q_r^3(R,n,Q_n)\right\}, \quad (11)$$

$$Q_r^3(R,n,Q_n)=\sum_{\ell=1}^{I}Qr(\ell)RG(\ell) \quad (12)$$

The terms from first to fifth in Eq. (11) are same as those in Eq. (9). The sixth term is the expected disassembly cost and the expected inspection cost of parts not taken as sample in the rejected lots. The seventh term is the expected second disposal cost of defective parts not taken as sample in the rejected lots. The eighth term is the expected remanufacturing cost of products from conforming parts not taken as sample in the rejected lots. The ninth term is the expected disassembly cost of used products not taken as sample in the acceptable lots. The tenth term is the expected third disposal cost of defective parts not taken as sample in the acceptable lots. The eleventh term is the remanufacturing cost of products from conforming parts not taken as sample in the acceptable lots. The twelfth term is calculated from Eq. (6) in IT-3. Note that $Q_r^3(R,n,Q_n)$ doesn't depend on n and Q_n because any conforming part isn't disposed in IT-3.

4.4 Case of Sampling Inspection with Screening of Acceptable Lots (IT-4)

From Section 3.2, the expected total profit $TP^4(R,n,Q_n)$ and the expected number of remanufactured products $Q_r^4(R,n,Q_n)$ in IT-4 for R, n, and Q_n, are respectively formulated as

$$\begin{aligned}TP^4(R,n,Q_n)&=-aR-c_cQR-(c_d+c_I)nR\\&-c_{D2}\sum_{\ell=1}^{I}\left[n\{1-r(\ell)\}RG(\ell)\right]-\sum_{\ell=1}^{I}\left[c_r(\ell)nr(\ell)RG(\ell)\right]\\&-c_{D1}(n)\sum_{\ell=1}^{I}\left[(Q-n)B_n(0,Q_n-1,1,\ell)RG(\ell)\right]\\&-(c_d+c_I)\sum_{\ell=1}^{I}\left[(Q-n)B_n(Q_n,n,1,\ell)RG(\ell)\right]\\&-c_{D2}\sum_{\ell=1}^{I}\left[(Q-n)\{1-r(\ell)\}B_n(Q_n,n,1,\ell)RG(\ell)\right]\\&-\sum_{\ell=1}^{I}\left[c_r(\ell)(Q-n)r(\ell)B_n(Q_n,n,1,\ell)RG(\ell)\right]\\&+P\left\{Q_r^4(R,n,Q_n)\right\}\end{aligned} \quad (13)$$

$$\begin{aligned}Q_r^4(R,n,Q_n)&=\sum_{\ell=1}^{I}\left[nr(\ell)RG(\ell)\right]\\&+\sum_{\ell=1}^{I}\left[(Q-n)r(\ell)B_n(Q_n,n,1,\ell)RG(\ell)\right].\end{aligned} \quad (14)$$

The terms from first to sixth in Eq. (13) are same as those in Eq. (9). In Eq. (13), the seventh term is the expected disassembly cost and the expected inspection cost of parts extracted from the used products not taken as sample in acceptable lots. The eighth term is the expected second disposal cost of defective parts not taken as sample in the acceptable lots. The ninth term is the expected remanufacturing cost of products from conforming parts not taken as sample in the rejected lots. The tenth term is calculated from Eq. (6) in IT-4.

4.5 Case of No Inspection (IT-5)

From Section 3.2, both of n and Q_n are 0 in IT-5. Therefore, the expected total profit $TP^5(R,n,Q_n)$ and the expected number of remanufactured products $Q_r^5(R,n,Q_n)$ in IT-5 for R are respectively formulated as

$$\begin{aligned}TP^5(R,0,0)&=-aR-c_dQR-c_{D3}\sum_{\ell=1}^{I}\left[Q\{1-r(\ell)\}RG(\ell)\right]\\&-\sum_{\ell=1}^{I}\left[c_r(\ell)Qr(\ell)RG(\ell)\right]+P\left\{Q_r^5(R,0,0)\right\}\end{aligned} \quad (15)$$

$$Q_r^5(R,0,0)=\sum_{\ell=1}^{I}Qr(\ell)RG(\ell). \quad (16)$$

In eq. (15), the first term is the procurement cost of used products at lot unit. The second term is the disassembly cost of parts extracted from the used products. The third term is the expected third disposal cost of defective parts. The forth term is the expected remanufacturing cost of products from conforming parts. The fifth term is calculated from Eq. (6) in IT-5. Note that the expected classification cost of used products aren't included in Eq. (15) since the used products aren't classified in IT-5 as with IT-1.

4.6 Features and Relationships of Each IT

In IT-2 and IT-4, all of used products $(Q-n)$ not taken as samples in the rejected lots are disposed at the unit cost $c_{D1}(n)$ even if they have the conforming parts. Meanwhile, in IT-1, IT-3, and IT-5, all of conforming parts in the procured used products are remanufactured into the products. Except for the process to classify the used products and that to collect up the used products with the same quality level at lot unit, when $n=0$, the flows of used products in IT-2 and IT-3 are same as the flow in IT-5, and when $n=Q$, they are same as that in IT-1. When $n=0$ and Q, the flow of used products in IT-4 is same as that in IT-1 except for the process to classify the used products and that to collect up the used products with the same quality level at lot unit.

5. A PROCEDURE TO DETERMINE OPTIMAL OPERATION

The decision procedures for the optimal operation to maximize the manufacturer's expected total profit are discussed.

Here, in this paper, sampling inspection scheme (n, Q_n) consists of sample size n and acceptance criterion Q_n, inspection policy (i, n, Q_n) consists of inspection type i and sampling inspection scheme (n, Q_n), and operation (R, i, n, Q_n) consists of procured lots R and inspection policy (i, n, Q_n). Note that the optimal operation $(R^{i*}, n^{i*}, Q_n^{i*})$ in IT-i, which maximize the expected total profit in IT-i, consists of the optimal procured lots R^{i*} and the optimal sampling inspection scheme (n^{i*}, Q_n^{i*}) in IT-i.

In IT-1 and IT-5, R^{i*} is determined. In IT-2, IT-3, and IT-4, R^{i*} and (n^{i*}, Q_n^{i*}) are determined.

First, the optimal decision for the procured lots R is discussed. It is investigated if the expected total profits in Eqs. (7), (9), (11), (13), and (15) are concave function with respect to R under n and Q_n. Here, $t^i(n, Q_n)$ and $u^i(n, Q_n)$ are defined as functions of two variables (n, Q_n). Using these functions, the expected total profit $TP^i(R, n, Q_n)$ and the expected number of remanufactured products, $Q_r^i(R, n, Q_n)$, in IT-i can be respectively expressed as

$$TP^i(R, n, Q_n) = Rt^i(n, Q_n) + P\{Q_r^i(R, n, Q_n)\}, \quad (17)$$

$$Q_r^i(R, n, Q_n) = Ru^i(n, Q_n) \quad (18)$$

The first- and second-order partial derivatives of $TP^i(R, n, Q_n)$ with respect to R are respectively derived as

$$\frac{\partial}{\partial R}TP^i(R, n, Q_n) = t^i(n, Q_n) + c_M F(S_{Max})u^i(n, Q_n) - (h_r + c_M)F\{Ru^i(n, Q_n)\}u^i(n, Q_n), \quad (19)$$

$$\frac{\partial^2}{\partial R^2}TP^i(R, n, Q_n) = -(h_r + c_M)f\{Ru^i(n, Q_n)\}\{u^i(n, Q_n)\}^2 < 0 \quad (20)$$

(See Appendix C). Therefore, it is proved that $TP^i(R, n, Q_n)$ is a concave function with respect to R under n and Q_n. In this case, the optimal procured lots $R^{i*}(n, Q_n)$ under sample size n and acceptance criterion Q_n in IT-i can be determined as

$$R^{i*}(n, Q_n) = \frac{1}{u^i(n, Q_n)}F^{-1}\left\{\frac{t^i(n, Q_n)}{(h_r + c_M)u^i(n, Q_n)} + \frac{c_M F(S_{Max})}{h_r + c_M}\right\} \quad (21)$$

so as to satisfy

$$\frac{\partial}{\partial R}TP^i(R, n, Q_n)\Big|_{R=R^{i*}(n,Q_n)} = 0 \quad (22)$$

(See Appendix D). Note that if the calculation result satisfies $R^{i*}(n, Q_n) > R_{Max}$, R_{Max} is substituted into $R^{i*}(n, Q_n)$ due to the property that $TP^i(R, n, Q_n)$ is a concave function with respect to R under n and Q_n.

Next, the optimal decisions for the sampling inspection scheme (n, Q_n) are discussed. It is hard to determine analytically the optimal sampling inspection scheme (n^{i*}, Q_n^{i*}) in IT-i.

This paper proposes the following decision procedure for the optimal operation (R^*, i^*, n^*, Q_n^*), which maximizes the expected total profit.

[Step 1] Calculate the provisional $R^{i*}(n, Q_n)$ from Eq. (21) under all feasible combinations of (n, Q_n) and obtain the expected total profit $TP^i(R^{i*}(n, Q_n), n, Q_n)$ by substituting $R^{i*}(n, Q_n)$ and (n, Q_n) into Eqs. (7), (9), (11), (13), or (15), in each IT.

[Step 2] Find the combination (n, Q_n) which maximizes $TP^i(R^{i*}(n, Q_n), n, Q_n)$ by the numerical search. Accordingly, the maximal expected total profit $TP^i(R^{i*}, n^{i*}, Q_n^{i*})$ in each IT is calculated.

[Step 3] By comparing the maximal expected total profit among five ITs, determine finally the optimal operation (R^*, i^*, n^*, Q_n^*) consisting of the optimal procured lots R^* and the optimal inspection policy (i^*, n^*, Q_n^*).

6. NUMERICAL ANALYSIS

This section shows the optimal operation in a RSC, which maximizes the manufacturer's expected total profit, by providing the numerical examples. In addition, sensitivity analyses clarify how each change of conditions of the RSC affects the optimal operation, the expected total profit, and the expected number of remanufactured products, in each IT.

6.1 A Numerical Example

The product demand x follows the normal distribution with mean μ and variance σ^2.

This paper assumes that the quality $\ell(0 \le \ell \le 1)$ of used products has some shapes. In this paper, the quality y of used products is modeled as the beta distribution with parameters (m_a, m_b). This is the reason why we use the beta distribution is not only because it's possible to express various shapes, but more important, it's widely used to measure relative parameters like level $\ell(0 \le \ell \le 1)$, or anything that is between 0–1. Concretely, the beta distribution can express various shapes of distribution of

parts extracted from used products such as the uniform distribution-type shape, the normal distribution-type shape, the exponential distribution-type shape, the left-biased distribution shape, the right-biased distribution shape, by using the following probability density function of quality, $g(y)$ with parameters (m_a, m_b):

$$g(y) = \frac{\Gamma(m_a + m_b)}{\Gamma(m_a)\Gamma(m_b)} y^{m_a} (1-y)^{m_b} \quad (0 \le y \le 1) \tag{23}$$

where $\Gamma(\cdot)$ denotes the gamma function.

The first disposal cost $c_{D1}(n)$ for sample size n is set as $c_{D1}(n) = (1-k)c_{D2}n/(Q-1) + kc_{D2}$, where $0 \le n \le Q-1$, satisfying the conditions: $c_{D1}(n) \le c_{D2}$ and $c_{D1}(Q-1) = c_{D2}$ in Section 3.3 (5). Here, k is a constant within $0 \le k \le 1$, and k represents a ratio of the minimum of $c_{D1}(n)$ to c_{D2} according to $c_{D1}(0) = kc_{D2}$.

The conforming probability and the remanufacturing cost of a part with quality level ℓ are respectively set as $r(\ell) = 0.5 - 0.4\cos(\mathrm{E}[y \mid \ell]\pi)$ and $c_r(\ell) = -20\,\mathrm{E}[y \mid \ell] + 40$, satisfying the properties of functions for quality level ℓ in Section 2.3 (9). Here, $\mathrm{E}[y \mid \ell]$ denotes the conditional expectation of quality level y under quality level ℓ. $\mathrm{E}[y \mid \ell]$ is calculated as

$$\mathrm{E}[y \mid \ell] = \int_{(\ell-1)/I}^{\ell/I} \frac{yg(y)}{G(\ell)} dy \tag{24}$$

The following numerical example are used as system parameters in a RSC: $Q = 100$, $I = 20$, $a = 1000$, $c_c = 3$, $c_d = 5$, $c_I = 15$, $k = 1/4$, $c_{D2} = 30$, $c_{D3} = 65$, $v = 180$, $h_r = 10$, $c_M = 160$, $s = 10$, $R_{Max} = 200$, $S_{Max} = 20000$, $\mu = 7000$, $\sigma = 500$, $m_a = 2$, $m_b = 2$.

6.2 Results of the Numerical Analysis

Table 1 shows the results of the optimal operation in each IT obtained from the numerical example.

Here, c_r^{i*} shown in Table 1 indicates the average of the expected total cost per unit of remanufactured products under the optimal situation in IT-*i*. The expected total cost related to remanufactured products is calculated from the sum of the following expected values of the procurement cost, the classification cost, the disassembly cost, the inspection cost, the first disposal cost of used products, the second disposal cost and the third disposal cost of parts, the remanufacturing cost of products, and the inventory holding cost of remanufactured products.

Table 1. Optimal operation in each IT

i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}	c_r^{i*}
1	133	0	0	407,777	3rd	6,650	118.0
2	161	12	5	425,340	1st	6,689	115.4
3	133	13	8	392,154	4th	6,650	120.4
4	153	11	4	409,179	2nd	6,644	117.8
5	132	0	0	374,600	5th	6,650	123.0

The maximum expected total profit is high in the order of IT-2, IT-4, IT-1, IT-3, and IT-5. Accordingly, the optimal operation in a RSC is finally obtained as $(R^*, i^*, n^*, Q_n^*) = (161, 2, 12, 5)$. In addition, the expected number of remanufactured products in IT-2, Q_r^{2*} is the largest among five ITs. Therefore, IT-2 enables to promote the recycling activity of used products and to improve the maximum expected total profit. The maximum expected total profit in sampling inspection with screening of acceptable lots IT-4, which is the proposed IT in this paper, is higher than that in IT-1. The expected number of remanufactured products in IT-4 is about 99.9% of that in IT-1.

6.2.1 Effect of Quality Distribution of Used Products on Optimal Operation in a RSC

By changing parameters (m_a, m_b) in pdf $g(y)$ of the beta distribution in Eq. (23), the following six cases of the quality distribution of used products are provided as:

Case 1: $(m_a, m_b) = (1, 1)$: a situation where the quality level of used products is distributed uniformly,

Case 2: $(m_a, m_b) = (2, 2)$: a situation where there are many used products with the middle quality level,

Case 3: $(m_a, m_b) = (2, 3)$: a situation where there are many used products with the relatively low quality level,

Case 4: $(m_a, m_b) = (3, 5)$: a situation where there are many used products with the low quality level,

Case 5: $(m_a, m_b) = (3, 2)$: a situation where there are many used products with the relatively high quality level,

Case 6: $(m_a, m_b) = (5, 3)$: a situation where there are many used products with the high quality level.

The probability density function $g(y)$ of quality

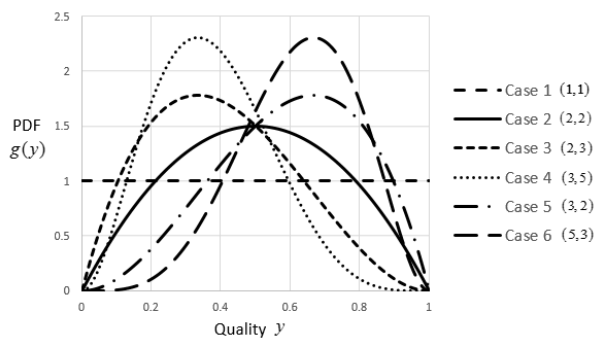

Figure 6. The probability density function $g(y)$ of quality y in each case.

y in each case is shown in Figure 6.

Table 2 shows the effect of the quality distribution on the optimal operation in each IT.

From Table 2, the following results can be seen:

- Under case 1 of the quality distribution, the maximum expected total profit of a manufacturer is high in the order of IT-2, IT-4, IT-1, IT-3, and IT-5 as with the results obtained from numerical examples in Table 1 where the quality distribution is Case 2.
- Under Case 3 of the quality distribution, inspection type 1 is the optimal IT. The manufacturer's expected total profit in proposed inspection type, IT-4, is the second highest. Also, in IT-5, the optimal procured lots $R^{5*} = 0$. In this case, the manufacturer doesn't remanufacture products from conforming parts and manufactures new products from new parts to satisfy the product demand.
- Under Case 4 of the quality distribution, the manufacturer doesn't remanufacture products from used products in all ITs.
- Under Case 5 of the quality distribution, IT-2 is the optimal IT. In addition, the optimal number of procured lots R^{i*} in all ITs is smaller than that under Case 2 provided as the numerical example. This is because mean of the conforming probability of used products is so higher that the required number of used products to satisfy the product demand is smaller.
- Under Case 6 of the quality distribution, R^{i*} is further smaller than that under Case 5 of that. Then, IT-5 is the optimal IT.
- IT-4 is the worst rank of the manufacturer's maximal expected profit under Cases 5 and 6 of the quality distribution.

Therefore, sampling inspection with screening acceptable lots, which is IT-4 (the proposed inspection type), is effective under either situations where the quality level of used products is uniformly distributed or there are many used products with the middle or low quality level.

Table 2. Effect of quality distribution of used products on optimal operation in each IT

$g(y)$	i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}
Case 1	1	133	0	0	415,393	3rd	6,650
	2	170	8	4	494,482	1st	6,762
	3	133	10	6	410,739	4th	6,650
	4	160	7	3	456,541	2nd	6,713
	5	132	0	0	382,158	5th	6,600
Case 2	1	133	0	0	407,777	3rd	6,650
	2	161	12	5	425,340	1st	6,689
	3	133	13	8	392,154	4th	6,650
	4	153	11	4	409,179	2nd	6,644
	5	132	0	0	374,600	5th	6,600
Case 3	1	156	0	0	188,802	1st	6,161
	2	188	15	5	161,785	3rd	6,001
	3	149	19	12	153,506	4th	5,884
	4	181	11	3	184,313	2nd	6,154
	5	0	0	0	140,000	5th	0
Case 4	1	0	0	0	140,000	1st	0
	2	0	0	0	140,000	1st	0
	3	0	0	0	140,000	1st	0
	4	0	0	0	140,000	1st	0
	5	0	0	0	140,000	1st	0
Case 5	1	113	0	0	557,250	4th	6,837
	2	129	11	5	575,675	1st	6,876
	3	113	10	6	557,460	3rd	6,837
	4	124	10	4	541,629	5th	6,841
	5	113	0	0	570,560	2nd	6,837
Case 6	1	108	0	0	591,118	4th	6,878
	2	119	11	5	597,140	2nd	6,864
	3	108	9	5	592,100	3rd	6,878
	4	112	14	5	564,932	5th	6,835
	5	108	0	0	615,850	1st	6,878

6.2.2 Effect of Unit Inspection Cost on Optimal Operation in a RSC

Table 3 shows effect of c_I on optimal operation in each IT. Note that the optimal operation in IT-5 is constant for c_I due to no inspected parts.

From Table 3, the following results can be seen.

- The higher the unit inspection cost c_I is, the smaller the optimal sample size n^{i*} and the optimal acceptance

Table 3. Effect of the unit inspection cost c_I on optimal operation in each IT

c_I	i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}
10	1	135	0	0	471,828	1st	6,750
	2	163	14	6	435,627	4th	6,716
	3	134	21	16	440,975	3rd	6,700
	4	151	12	4	466,007	2nd	6,716
	5	132	0	0	374,600	5th	6,600
15	1	133	0	0	407,777	3rd	6,650
	2	161	12	5	425,340	1st	6,689
	3	133	13	8	392,154	4th	6,650
	4	153	11	4	409,179	2nd	6,644
	5	132	0	0	374,600	5th	6,600
20	1	131	0	0	341,676	5th	6,550
	2	165	9	4	417,582	1st	6,654
	3	132	8	4	355,600	3rd	6,600
	4	156	10	4	353,430	4th	6,565
	5	132	0	0	374,600	2nd	6,600

criterion Q_n^{i*} tend to be in IT-2, IT-3, and IT-4.

- As c_I increases, the optimal procured lots R^{i*} in IT-3 and IT-5 decreases and that in IT-4 increases. Here, in IT-4, a manufacturer incurs the inspection cost of parts $(Q-n)$ not take as samples in the acceptable lots. Thus, in IT-4 with a strict inspection, the more conforming parts are disposed, and the more used products are necessary to remanufacture products.
- IT-1 and IT-4 are largely affected with the change of the unit inspection cost c_I than IT-2 since they need to conduct an inspection for more used products.
- The rank order in IT-4 falls from the second to the forth while c_I changes from 10 to 20. The rank order in IT-1 falls from the first to the worst then. It is indicated that IT-4 is less affected with c_I than IT-1 in Case 2.
- When c_I is high $(c_I = 20)$, $Q_r^{1*} < Q_r^{i*}$ $(i = 2,3,4)$ is satisfied. It is verified that the effect of increment in c_I on the maximal expected total profit and the expected number of remanufactured products can be relieved by introducing IT-2, IT-3, and IT-4.,

Table 4. Effect of the constant k in the first disposal cost $c_{D1}(n)$ of used products on optimal operation in each IT

k	i	R^{i*}	n^{i*}	Q_n^{i*}	$c_{D1}(n^*)$	TP^{i*}	Rank	Q_r^{i*}
0	1	133	0	0	*	407,777	3rd	6,650
	2	178	8	4	2.42	469,170	1st	6,759
	3	133	13	8	*	392,154	4th	6,650
	4	166	9	4	2.73	443,584	2nd	6,694
	5	132	0	0	*	374,600	5th	6,600
$\frac{1}{4}$	1	133	0	0	*	407,777	3rd	6,650
	2	161	12	5	10.23	425,340	1st	6,689
	3	133	13	8	*	392,154	4th	6,650
	4	153	11	4	10	409,179	2nd	6,644
	5	132	0	0	*	374,600	5th	6,600
$\frac{1}{2}$	1	133	0	0	*	407,777	1st	6,650
	2	152	14	5	17.12	395,212	2nd	6,659
	3	133	13	8	*	392,154	3rd	6,650
	4	144	14	4	17.12	387,305	4th	6,624
	5	132	0	0	*	374,600	5th	6,600
$\frac{3}{4}$	1	133	0	0	*	407,777	1st	6,650
	2	144	23	7	24.24	375,395	3rd	6,614
	3	133	13	8	*	392,154	2nd	6,650
	4	137	20	4	24.02	374,775	4th	6,610
	5	132	0	0	*	374,600	5th	6,600
1	1	133	0	0	*	407,777	1st	6,650
	2	132	100	0	30	368,000	5th	6,600
	3	133	13	8	*	392,154	2nd	6,650
	4	134	44	7	30	369,504	4th	6,612
	5	132	0	0	*	374,600	3rd	6,600

6.2.3 Effect of Unit First Disposal Cost on Optimal Operation in a RSC

By varying the constant k in the unit first disposal cost $c_{D1}(n)$, a sensitivity analysis with respect to $c_{D1}(n)$ is conducted.

Table 4 shows effect of k on optimal operation in each IT. Note that the optimal operation in IT-1, IT-3, and IT-5, is unaffected with k according to Section 3.2.

From Table 4, the following results can be seen.

- In IT-2 and IT-4, as k becomes larger, the maximal expected total profit decreases.
- While k changes from 0 to 1, the rank order in IT-4 falls from the second to the forth. The rank order in IT-2 falls from the first to the worst then. It is indicated that IT-4 is less affected with k than IT-2 in Case 2.
- Differently from IT-4, IT-2 has the potential to convey defective parts to the remanufacturing process. It indicates that IT-2 poses a risk that a manufacturer incurs

Table 5. Effect of the second disposal cost c_{D2} of defective parts on optimal operation in each IT

c_{D2}	i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}
20	1	135	0	0	474,818	1st	6750
	2	169	15	7	455,550	3rd	6713
	3	134	17	12	448,840	4th	6700
	4	155	11	4	469,217	2nd	6731
	5	132	0	0	374,600	5th	6600
25	1	134	0	0	441,189	3rd	6700
	2	168	11	5	439,596	1st	6715
	3	134	15	10	419,655	4th	6700
	4	154	11	4	439,108	2nd	6688
	5	132	0	0	374,600	5th	6600
30	1	133	0	0	407,777	3rd	6650
	2	161	12	5	425,340	1st	6689
	3	133	13	8	392,154	4th	6650
	4	153	11	4	409,179	2nd	6644
	5	132	0	0	374,600	5th	6600
35	1	132	0	0	374,600	3rd	6600
	2	158	10	4	412,541	1st	6649
	3	132	11	6	367,019	5th	6600
	4	152	11	4	379,441	2nd	6601
	5	132	0	0	374,600	3rd	6600
40	1	131	0	0	341,676	5th	6550
	2	158	10	4	400,839	1st	6649
	3	131	7	3	345,718	4th	6550
	4	151	11	4	349,904	3rd	6557
	5	132	0	0	374,600	2nd	6600

the third disposal cost, which is the highest of three types of disposal cost, when defective parts are detected in the remanufacturing process. When the first disposal cost is higher due to the increases of k, the optimal sample size n^{i*} becomes larger to avoid the increase of disposal cost. However, as a result, the inspection cost increases as the optimal sample size n^{i*} becomes larger.

6.2.4 Effect of Unit Second Disposal Cost on Optimal Operation in a RSC

Table 5 shows effect of the unit second disposal cost c_{D2} on optimal operation in each IT. Note that the optimal operation in IT-5 is unaffected with c_{D2} according to Section 3.2.

From Table 5, the following results can be seen.

- In IT-1, IT-2, IT-3, and IT-4, as c_{D2} becomes larger, the maximal expected total profit decreases.
- As with the sensitivity analysis with respect to c_I, the rank order in IT-4 falls from the second to the third while c_{D2} changes from 20 to 40. However, IT-4 is less affected with c_{D2} than IT-1 in Case 2.

6.2.5 Effect of Unit Third Disposal Cost on Optimal Operation in a RSC

Table 6 shows effect of c_{D3} on optimal operation in each IT. Note that the optimal operation in IT-1 and IT-4 is constant for c_{D3} since c_{D3} is the unit disposal cost of defective parts conveyed to the remanufacturing process.

Table 6. Effect of the third disposal cost c_{D3} of defective parts on optimal operation in each IT

c_{D3}	i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}
50	1	133	0	0	407,777	5th	6,650
	2	157	8	3	479,675	1st	6,739
	3	134	0	0	434,489	3rd	6,700
	4	153	11	4	409,179	4th	6,644
	5	135	0	0	474,818	2nd	6,750
65	1	133	0	0	407,777	3rd	6,650
	2	161	12	5	425,340	1st	6,689
	3	133	13	8	392,154	4th	6,650
	4	153	11	4	409,179	2nd	6,644
	5	132	0	0	374,600	5th	6,650
75	1	133	0	0	407,777	2nd	6,650
	2	167	13	6	393,745	3rd	6,645
	3	132	17	12	382,137	4th	6,600
	4	153	11	4	409,179	1st	6,644
	5	130	0	0	309,025	5th	6,500
85	1	133	0	0	407,777	2nd	6,650
	2	132	100	0	368,000	4th	6,600
	3	132	22	17	376,413	3rd	6,600
	4	153	11	4	409,179	1st	6,644
	5	127	0	0	244,673	5th	6,350

From Table 6, the following results can be seen.

- When c_{D3} is high $(c_{D3} = 75, 85)$, IT-4, which is the proposed IT, is the optimal IT. It is verified that it is profitable to incorporate IT-4 into a RSC under a situation where the disadvantage is significant when defective parts are conveyed to the remanufacturing process. Therefore, it is worth adding IT-4 to choices of IT.
- When $c_{D3} = 85$, the optimal sample size in IT-2 is $n^{2*} = 100(=Q)$. It indicates that all of parts extracted from procured used products are inspected and only conforming parts are conveyed to the remanufacturing process. The maximum expected total profit in IT-2 with classification cost is lower than that in IT-1 without it.
- When c_{D3} is low $(c_{D3} = 50)$, the optimal sample size in IT-3 is $n^{3*} = 0$. It indicates that all of parts extracted from procured used products are conveyed to the remanufacturing process without inspection. The maximum expected total profit in IT-3 with classification cost is lower than that in IT-5 without it.

6.2.6 Effect of Manufacturing Cost of New Products on Optimal Operation in a RSC

Table 7 shows the effect of c_M on the optimal operation in each IT.

From Table 7, the following results can be seen:

- The rank order of the ITs and the optimal inspection scheme are unaffected with the change of the unit manufacturing cost c_M.
- When c_M is low $(c_M = 115, 120)$, R^{i*} and Q_r^{i*} become 0. This result indicates that the new production from new parts to new products tends to be promoted as c_M decreases.
- The higher c_M is, the larger the optimal procured lots R^{i*} and the expected remanufactured products Q_r^{i*} are. This result indicates that the remanufacturing activity from used products to products tends to be promoted as c_M becomes higher.
- When IT-3 or IT-5 is incorporated into a RSC, the remanufacturing activity from used products tends to be stopped in the earlier stage than other ITs as c_M becomes lower. This means that the total cost related to the unit of remanufactured products is higher than that in the other inspection types. This can be confirmed from the result in Table 1 that c_r^{i*} in IT-3 and IT-5 are higher than those in other ITs.

6.2.7 Effect of Maximal Number of Procured Lots on Optimal Operation in a RSC

In this section, by changing the maximum number of procured lots of used products R_{Max}, situations where the manufacturer can't procure so many used products are discussed.

Table 7. Effect of the unit manufacturing cost c_M of new products on optimal operation in each IT

c_M	i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}
115	1	0	0	0	455,000	1st	0
	2	0	0	0	455,000	1st	0
	3	0	0	0	455,000	1st	0
	4	0	0	0	455,000	1st	0
	5	0	0	0	455,000	1st	0
120	1	119	0	0	432,037	3rd	5,950
	2	147	12	5	447,901	1st	6,107
	3	0	0	0	420,000	4th	0
	4	137	11	4	433,300	2nd	5,949
	5	0	0	0	420,000	4th	0
125	1	124	0	0	427,412	3rd	6,200
	2	151	12	5	443,815	1st	6,273
	3	122	13	8	412,976	4th	6,100
	4	143	11	4	428,724	2nd	6,210
	5	118	0	0	397,023	5th	5,900
155	1	132	0	0	409,900	3rd	6,600
	2	160	12	5	427,389	1st	6,647
	3	132	13	8	394,393	4th	6,600
	4	153	11	4	411,308	2nd	6,644
	5	131	0	0	376,927	5th	6,550
160	1	133	0	0	407,777	3rd	6,650
	2	161	12	5	425,340	1st	6,689
	3	133	13	8	392,154	4th	6,650
	4	153	11	4	409,179	2nd	6,644
	5	132	0	0	374,600	5th	6,600
180	1	136	0	0	400,343	3rd	6,800
	2	164	12	5	418,222	1st	6,813
	3	135	13	8	384,481	4th	6,750
	4	156	11	4	401,822	2nd	6,775
	5	135	0	0	366,590	5th	6,750

Table 8 shows the effect of R_{Max} on the optimal operation in each IT.

From Table 6, it can be seen that as R_{Max} decreases, the maximal expected total profit in IT-2 and IT-4 becomes lower significantly. This is because IT-2 and IT-4 have the following property: many used products are procured and more used products with high quality level at lot unit are conveyed to the remanufacturing process. This can bring more profits to a manufacturer than the other ITs. Therefore, it is clarified that IT-2 and IT-4 are affected more greatly with R_{Max} than other inspection types.

6.2.8 Sensitivity Analyses in Case 3

In Section 6.2.1, it is indicated that the proposed IT, IT-4, is effective when under the situations where the quality level of used products is uniformly distributed or there are many used products with the middle or low quality level. Moreover, in Sections 6.2.2-6.2.4, IT-4 is detected to be less affected with c_I, k, and c_{D2}. However, these sensitivity analyses are performed only under the situation where there are many used products with the middle quality level, Case 2. This section performs the sensitivity analyses with respect to c_I, k, and c_{D2}, in Case 3 as a situation where the quality of many used products is worse.

Table 8. Effect of the maximal number of procured lots of used products R_{Max} on optimal operation in each IT

R_{Max}	i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}
130	1	130	0	0	406,525	1st	6,500
	2	130	16	5	391,915	2nd	5,893
	3	130	13	8	391,255	3rd	6,500
	4	130	15	4	388,814	4th	6,052
	5	130	0	0	374,025	5th	6,500
140	1	133	0	0	407,777	2nd	6,650
	2	140	15	5	408,843	1st	6,250
	3	133	13	8	392,154	4th	6,650
	4	140	14	4	402,412	3rd	6,440
	5	132	0	0	374,600	5th	6,600
200	1	133	0	0	407,777	3rd	6,650
	2	161	12	5	425,340	1st	6,689
	3	133	13	8	392,154	4th	6,650
	4	153	11	4	409,179	2nd	6,644
	5	132	0	0	374,600	5th	6,600

Table 9-Table 11 show the results of the sensitivity analyses with respect to c_I, k, and c_{D2}, in Case 3, respectively.

From Table 9, it can be seen that IT-4 is less affected with c_I than IT-1 in Case 3 as with Case 2. In addition, when $c_I = 17$, IT-4 is the optimal IT. However, when $c_I = 20$, the manufacturer doesn't procure used products except in IT-2.

When $k = 1/2,\ 3/4,\ 1,$ in Table 10, or $c_{D2} = 20,\ 25,$ in Table 11, all of parts extracted from procured used products are inspected in IT-2 (i.e. $n^{2*} = 100$). Under a situation where low quality used products are many, it is more difficult for IT-2, IT-3, and IT-5, to function because defective parts are conveyed to the remanufacturing process in these ITs. Also, IT-4 is no absolute exception, and IT-1 tends to be more effective and less affected with k and c_{D2} than IT-4 under the situation.

Table 9. Sensitivity analysis with respect to c_I in Case 3

c_I	i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}
10	1	163	0	0	268,659	1st	6,437
	2	159	100	0	220,366	4th	6,279
	3	159	30	23	222,363	3rd	6,279
	4	188	11	3	251,624	2nd	6,392
	5	0	0	0	140,000	5th	0
15	1	156	0	0	188,802	1st	6,161
	2	188	15	5	161,785	3rd	6,001
	3	149	19	12	153,506	4th	5,884
	4	181	11	3	184,313	2nd	6,154
	5	0	0	0	140,000	5th	0
17	1	151	0	0	158,031	2nd	5,963
	2	192	14	5	156,304	3rd	5,936
	3	0	0	0	140,000	4th	0
	4	181	10	3	158,660	1st	5,969
	5	0	0	0	140,000	4th	0
18	1	143	0	0	143,247	3rd	5,647
	2	186	12	4	153,757	1st	5,909
	3	0	0	0	140,000	4th	0
	4	175	10	3	146,306	2nd	5,771
	5	0	0	0	140,000	4th	0
20	1	0	0	0	140,000	2nd	0
	2	191	11	4	149,487	1st	5,830
	3	0	0	0	140,000	2nd	0
	4	0	0	0	140,000	2nd	0
	5	0	0	0	140,000	2nd	0

Table 10. Sensitivity analysis with respect to k in Case 3

k	i	R^{i*}	n^{i*}	Q_n^{i*}	$c_{D1}(n^*)$	TP^{i*}	Rank	Q_r^{i*}
0	1	156	0	0	*	188,802	3rd	6,161
	2	200	10	4	3.03	214,448	2nd	5,798
	3	149	19	12	*	153,506	4th	5,884
	4	200	8	3	2.42	229,117	1st	6,012
	5	0	0	0	*	140,000	5th	0
$\frac{1}{4}$	1	156	0	0	*	188,802	1st	6,161
	2	188	15	5	10.91	161,785	3rd	6,001
	3	149	19	12	*	153,506	4th	5,884
	4	181	11	3	10	184,313	2nd	6,154
	5	0	0	0	*	140,000	5th	0
$\frac{1}{2}$	1	156	0	0	*	188,802	1st	6,161
	2	143	100	0	30.15	143,247	4th	5,647
	3	149	19	12	*	153,506	3rd	5,884
	4	164	19	4	17.88	158,290	2nd	5,968
	5	0	0	0	*	140,000	5th	0
$\frac{3}{4}$	1	156	0	0	*	188,802	1st	6,161
	2	143	100	0	30.08	143,247	4th	5,647
	3	149	19	12	*	153,506	2nd	5,884
	4	150	28	4	24.62	146,339	3rd	5,753
	5	0	0	0	*	140,000	5th	0
1	1	156	0	0	*	188,802	1st	6,161
	2	143	100	0	30	143,247	4th	5,647
	3	149	19	12	*	153,506	2nd	5,884
	4	144	61	6	30	143,429	3rd	5,664
	5	0	0	0	*	140,000	5th	0

Table 11. Sensitivity analysis with respect to c_{D2} in Case 3

c_{D2}	i	R^{i*}	n^{i*}	Q_n^{i*}	TP^{i*}	Rank	Q_r^{i*}
20	1	163	0	0	285,788	1st	6,437
	2	161	100	0	237,168	4th	6,358
	3	161	25	18	242,426	3rd	6,358
	4	191	14	4	273,723	2nd	6,430
	5	0	0	0	140,000	5th	0
25	1	161	0	0	236,758	1st	6,358
	2	156	100	0	189,199	4th	6,161
	3	157	22	15	196,629	3rd	6,200
	4	188	14	4	228,441	2nd	6,329
	5	0	0	0	140,000	5th	0
30	1	156	0	0	188,802	1st	6,161
	2	188	15	5	161,785	3rd	6,001
	3	149	19	12	153,506	4th	5,884
	4	181	11	3	184,313	2nd	6,154
	5	0	0	0	140,000	5th	0
35	1	142	0	0	142,884	1st	5,608
	2	177	12	4	142,757	2nd	5,623
	3	0	0	0	140,000	4th	0
	4	169	10	3	142,215	3rd	5,574
	5	0	0	0	140,000	4th	0
40	1	0	0	0	140,000	1st	0
	2	0	0	0	140,000	1st	0
	3	0	0	0	140,000	1st	0
	4	0	0	0	140,000	1st	0
	5	0	0	0	140,000	1st	0

7. CONCLUSION

This paper incorporated the following five inspection type (IT)s into a RSC which consisted of the process flows from procurement of used products collected from a market, through remanufacturing products from the used products, to sales of the products in a market: (IT-1) 100% inspection, (IT-2) sampling inspection, (IT-3) sampling inspection with screening of rejected lots, (IT-4) sampling inspection with screening of acceptable lots(the proposed IT), and (IT-5) no inspection.

This paper formulated the manufacturer's expected total profit in each IT and derived a procedure to determine the optimal operation for a procurement of used products and an inspection policy, which maximized the manufacturer's expected total profit, by theoretical analysis and numerical search.

In addition, this paper considered the following topics:

- Classification of used products into the discrete quality level,
- The probability that each part extracted from used products is remanufacturable and the remanufacturing cost depending on the quality level of each part,

- The difference in disposal costs depending on the disposing stage in a RSC,
- No situation where used products taken as sample are inspected again.

The numerical analysis investigated how changes of quality distribution of used products, inspection cost of used products, dispose cost of used products, manufacturing cost of new products, and the maximal number of procured lots of used products, affected the maximal expected total profit of a manufacturer and the optimal operation in a RSC. Also, from the results of numerical analysis, this paper showed the usability to add IT-4, which was the proposed inspection in this study, to choices of inspection type for the RSC. Furthermore, this paper determined the conclusive optimal operation for a RSC by comparing the maximal expected total profits among five ITs. Thus, the numerical analysis clarified the features of each IT.

Results of numerical analysis in this paper provided the following managerial insights.

- The appropriate IT should be chosen by a manufacturer who inspects and remanufactures the procured used products as to conditions of the RSC.
- Adding the proposed IT in this paper to choices of IT for a RSC could bring improvement in the manufacturer's expected total profit.
- Especially in a situation where there are many used products with middle quality level, the manufacturer's expected total profit in IT-4, the proposed IT, tended to be less affected with changes of conditions of the RSC including the unit inspection cost and the unit disposal cost.

However, this paper did not discuss the optimal inspection policy for lots with each quality level.

Also, the optimal decision for optimal inspection policy was determined so as to maximize the expected profit of a manufacturer, considering only the expectation of manufacturer's profit. When a third-party-collection trader which collects used products from markets/customers and sorts them is incorporated into the RSC as a procurement source, it is necessary for a manufacturer to determine newly not only the optimal procurement quantity, but also the optimal inspection policy in a RSC, considering quality of used products.

Moreover, to apply this model to different type of products from consumer electronics and electronic components, occasionally destructive inspections should be adopted. By considering the destructive inspection and deterioration in quality, this model may be applied to food products, because the primary materials for food products also have the high uncertainty in quality as with used products in RSC.

From the above, as future researches, it will be necessary to incorporate the following topics into the optimal operation policy for a RSC model in this paper:

- The optimal decision for inspection policy for lots with each quality level in used products,
- The optimal decision for optimal inspection policy considering not only the expectation of manufacturer's profit, but also a variance of it,
- The optimal decision for optimal inspection policy when a RSC consists of a third-party-collection trader who collects used products from markets/customers and sorts them and manufacturer who inspects them by five ITs discussed in this paper,
- Consideration for destructive inspection and deterioration in quality.

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APPENDIX A

The elicitation process of Eq. (3) is shown as follows. If $1 \le Q_C$,

$$Q_C\binom{N}{Q_C} = \frac{Q_C N!}{Q_C!(N-Q_C)!} = \frac{N!}{(Q_C-1)!(N-Q_C)!}$$
$$= \frac{N(N-1)!}{(Q_C-1)!\{(N-1)-(Q_C-1)\}!} = N\binom{N-1}{Q_C-1}$$

is satisfied. Accordingly, the left-hand side of Eq. (3) can be rewritten as

$$\sum_{Q_C=0}^{N}\left[Q_C p\{N, Q_C, r(\ell)\}\right]$$
$$= \sum_{Q_C=0}^{N}\left[Q_C\binom{N}{Q_C}\{r(\ell)\}^{Q_C}\{1-r(\ell)\}^{N-Q_C}\right]$$
$$= 0\cdot\binom{N}{0}\{r(\ell)\}^{0}\{1-r(\ell)\}^{N-0}$$
$$+\sum_{Q_C=1}^{N}\left[N\binom{N-1}{Q_C-1}\{r(\ell)\}^{Q_C}\{1-r(\ell)\}^{N-Q_C}\right]$$
$$= Nr(\ell)\sum_{Q_C=1}^{N}\left[\binom{N-1}{Q_C-1}\{r(\ell)\}^{Q_C-1}\{1-r(\ell)\}^{(N-1)-(Q_C-1)}\right]$$
$$= Nr(\ell)\sum_{Q_C=0}^{N-1}\left[\binom{N-1}{Q_C}\{r(\ell)\}^{Q_C}\{1-r(\ell)\}^{(N-1)-Q_C}\right]$$
$$= Nr(\ell)\left[\{r(\ell)\}+\{1-r(\ell)\}\right]^{N-1} = Nr(\ell)\cdot 1^{N-1} = Nr(\ell).$$

Therefore, Eq. (3) has been proved.

APPENDIX B

The elicitation process of Eq. (6) is shown as follows.

$$P(Q_r)$$
$$= v\left\{\int_0^{S_{Max}} xf(x)dx + \int_{S_{Max}}^{\infty} S_{Max}f(x)dx\right\}$$
$$-h_r\int_0^{Q_r}(Q_r-x)f(x)dx$$
$$-c_M\int_{Q_r}^{S_{Max}}(x-Q_r)f(x)dx$$
$$-s\int_{S_{Max}}^{\infty}(x-S_{Max})f(x)dx$$
$$= v\left\{\left[xF(x)\right]_0^{S_{Max}} - \int_0^{S_{Max}}F(x)dx\right.$$
$$\left.+\int_0^{\infty}S_{Max}f(x)dx - \int_0^{S_{Max}}S_{Max}f(x)dx\right\}$$
$$-h_r\left\{Q_r\left[F(x)\right]_0^{Q_r} - \left[xF(x)\right]_0^{Q_r} + \int_0^{Q_r}F(x)dx\right\}$$
$$-c_M\left\{\left[xF(x)\right]_{Q_r}^{S_{Max}} - \int_{Q_r}^{S_{Max}}F(x)dx - Q_r\left[F(x)\right]_{Q_r}^{S_{Max}}\right\}$$
$$-s\left\{\int_0^{\infty}(x-S_{Max})f(x)dx - \int_0^{S_{Max}}(x-S_{Max})f(x)dx\right\}$$
$$= v\left\{S_{Max}F(S_{Max}) - \int_0^{S_{Max}}F(x)dx + S_{Max} - S_{Max}F(S_{Max})\right\}$$
$$-h_r\int_0^{Q_r}F(x)dx$$
$$-c_M\left\{S_{Max}F(S_{Max}) - Q_rF(S_{Max}) - \int_{Q_r}^{S_{Max}}F(x)dx\right\}$$
$$-s\left\{\mathrm{E}[x] - S_{Max} - \left[xF(x)\right]_0^{S_{Max}}\right.$$
$$\left.+\int_0^{S_{Max}}F(x)dx + S_{Max}\left[F(x)\right]_0^{S_{Max}}\right\}$$
$$= v\left\{S_{Max} - \int_0^{S_{Max}}F(x)dx\right\} - h_r\int_0^{Q_r}F(x)dx$$
$$-c_M\left\{S_{Max} - Q_rF(S_{Max}) - \int_{Q_r}^{S_{Max}}F(x)dx\right\}$$
$$-s\left\{\mathrm{E}[x] - S_{Max} + \int_0^{S_{Max}}F(x)dx\right\}$$

Therefore, Eq. (6) has been proved.

APPENDIX C

The elicitation process of Eq. (20) which indicates that $TP^i(R,n,Q_n)$ is a concave function with respect to R under n and Q_n, is shown as follows.

Due to

$$\frac{\partial}{\partial R}\int_0^{Q_r^i(R,n,Q_n)}F(x)dx = F\{Q_r^i(R,n,Q_n)\}\frac{\partial}{\partial R}Q_r^i(R,n,Q_n),$$
$$\frac{\partial}{\partial R}Q_r^i(R,n,Q_n) = u^i(n,Q_n),$$
$$\frac{\partial^2}{\partial R^2}Q_r^i(R,n,Q_n) = 0,$$

the first- and second-order partial derivatives of $TP^i(R,n,Q_n)$ with respect to R are respectively derived as

$$\frac{\partial}{\partial R}TP^i(R,n,Q_n)$$
$$= t^i(n,Q_n) - h_rF\{Q_r^i(R,n,Q_n)\}\frac{\partial}{\partial R}Q_r^i(R,n,Q_n)$$
$$-c_M\left[-F(S_{Max})\frac{\partial}{\partial R}Q_r^i(R,n,Q_n)\right.$$
$$\left.+F\{Q_r^i(R,n,Q_n)\}\frac{\partial}{\partial R}Q_r^i(R,n,Q_n)\right]$$
$$= t^i(n,Q_n) - (h_r+c_M)F\{Q_r^i(R,n,Q_n)\}\frac{\partial}{\partial R}Q_r^i(R,n,Q_n)$$
$$+c_MF(S_{Max})\frac{\partial}{\partial R}Q_r^i(R,n,Q_n)$$
$$= t^i(n,Q_n) - (h_r+c_M)F\{Ru^i(n,Q_n)\}u^i(n,Q_n)$$
$$+c_MF(S_{Max})u^i(n,Q_n),$$

$$\frac{\partial^2}{\partial R^2} TP^i\left(R,n,Q_n\right)$$

$$= -\left(h_r + c_M\right) f\left\{Ru^i\left(n,Q_n\right)\right\}\left\{u^i\left(n,Q_n\right)\right\}^2 < 0 .$$

Therefore, Eq. (20), indicating that $TP^i\left(R,n,Q_n\right)$ is a concave function with respect to R under n and Q_n, is proved.

APPENDIX D

The elicitation process of Eq. (21) is shown as follows. From Eq. (22),

$$\left(h_r + c_M\right) F\left\{R^{i*}\left(n,Q_n\right)u^i\left(n,Q_n\right)\right\}$$

$$= t^i\left(n,Q_n\right) + c_M F\left(S_{Max}\right)u^i\left(n,Q_n\right)$$

$$\Leftrightarrow F\left\{R^{i*}\left(n,Q_n\right)u^i\left(n,Q_n\right)\right\}$$

$$= \frac{t^i\left(n,Q_n\right)}{\left(h_r + c_M\right)u^i\left(n,Q_n\right)} + \frac{c_M F\left(S_{Max}\right)}{h_r + c_M} \left(\because h_r + c_M > 0\right)$$

$$\therefore R^{i*}\left(n,Q_n\right)$$

$$= \frac{1}{u^i\left(n,Q_n\right)} F^{-1}\left\{\frac{t^i\left(n,Q_n\right)}{\left(h_r + c_M\right)u^i\left(n,Q_n\right)} + \frac{c_M F\left(S_{Max}\right)}{h_r + c_M}\right\}$$

is derived. Therefore, Eq. (21) has been proved.