Stochastic Programming for the Optimization of Transportation-Inventory Strategy

Mou Deyi, Zhang Xiaoqian*
Institute of Mathematics for Applications, Civil Aviation University of China, Tianjin, China

(Received: March 07, 2016 / Revised: June 17, 2016; September 29, 2016 / Accepted: October 6, 2016)

ABSTRACT

In today’s competitive environment, supply chain management is a major concern for a company. Two of the key issues in supply chain management are transportation and inventory management. To achieve significant savings, companies should integrate these two issues instead of treating them separately. In this paper we develop a framework for modeling stochastic programming in a supply chain that is subject to demand uncertainty. With reasonable assumptions, two stochastic programming models are presented, respectively, including a single-period and a multi-period situations. Our assumptions allow us to capture the stochastic nature of the problem and translate it into a deterministic model. And then, based on the genetic algorithm and stochastic simulation, a solution method is developed to solve the model. Finally, the computational results are provided to demonstrate the effectiveness of our model and algorithm.

Keywords: Transportation-Inventory, Joint Optimization, Random Demand, Stochastic Dynamic Programming, Hybrid Intelligent Algorithm

* Corresponding Author, E-mail: bay122328@hotmail.com

1. INTRODUCTION

Supply chain is defined as an integrated process where different business entities such as suppliers, manufacturers, distributors, and retailers work together to deliver products to customers. This chain is concerned with two distinct flows: a forward flow of materials and a backward flow of information (Karakitsiou and Migdalas, 2008). The competition forces companies to look at their operations from a supply-chain perspective and to seek improvements from better coordination and communication across the supply chain.

In this paper, we consider the problem of selecting the appropriate distribution strategy for delivering a family of products from a set of suppliers to a set of retailers so that the total transportation and inventory costs are minimized. There were some research works about this issue. Harris (1990) and Burns (1985) have presented a model to deal with these problems in an integrated manner. Viswanathan (1997) have considered a two-stage model for multi-product capacity planning of supply chain networks. Ye and Du (2003) formulated the minimization of total cost as the objective function, and established the optimal model of the supplier’s maximum inventory and distribution periods. However, these models do not take any uncertain factors into account.

Bertazzi and Cherubini (2013) researched an inventory-transportation system with stochastic demand. They studied a logistic system in which a supplier has to deliver a set of products to a set of retailers to face a random demand over a given time horizon. Agrawal and Smith (2013) optimized inventory management for a retail chain with diverse store demands. Accounting for these variations appropriately in inventory management can significantly improve retailers’ profits. Pyke (1993) have developed a stochastic programming model that can be used to
determine the optimal replenishment policies as well as the optimal dispatch strategy simultaneously. Shu (2005) dealt with the problem of minimizing the sum of transportation and inventory costs for shipping several products on a common link where the demand is a random variable. Zhang (2005) presented a model to design simple inventory policies and transportation strategies to satisfy uncertainty demands over a finite horizon, while minimizing system wide cost by taking advantage of quantity discount in the transportation cost structures. Hu et al. (2014) studied on the influence of supply chain joint decision from the perspective of supply chain finance. Based on the research for Xi’an equipment manufacturing and biomedicine industry enterprises, structural equation model is established. We know that, by improving the use frequency of supply chain finance and enhancing trust, communication, cooperation, small and medium-sized enterprises have a significant impact. Fu (2007) discussed an analytical model for coordinating inventory and transportation decisions in a vendor-managed inventory system and constructed the adaptive genetic algorithm to solve the model. Liu (2009) presented a model to consolidate inbound freight at transshipment point from multiple sources to a single destination to minimize the overall transportation and inventory costs, which is subject to demand uncertainty, at the same time, heuristic algorithm was also used to solve the two sub-problems of inventory and transportation. As the new research area in supply chain management, financial supply chain focuses that cash flow should correspond with information and logistic. Such as Liu (2016) and Lambert (2016), through communication, information resource sharing, efficient and low cost supply chain formation, enterprises can better cope with the fierce market competition.

From the latest literatures, we can know that the research of supply chain still has certain significance and value. However, these references only consider single-period constraint model, didn’t put the periodic into account. Similarly, we also focus on minimizing the transportation and inventory costs in a supply chain where the demand is a random variable. But, further, we extend the model to the multi-period with random variables, and then, translate the stochastic programming (Panos et al., 2015) into a deterministic model. Based on the genetic algorithm and stochastic simulation, a hybrid intelligent algorithm (HIA) having been explored can help us to find a good system solution.

The rest of this paper is organized as follows. Section 2 presents a single-period and a multi-period stochastic programming model where the demands are random variables. Section 3 presents HIA to handle the multi-period stochastic programming model. Numerical examples and computational experiments have been reported in Section 4. Section 5 concludes the paper with a summary and direction for future work.

2. MODEL ESTABLISH

In this section, we will give the description of transportation-inventory strategy and how to construct our model.

2.1 Single-Period Model

In this section we present the single-period model that serves as the building block for modeling a multi-period stochastic programming. The model will do the following assumptions:

1. Only consider the same kind of products;
2. The random demand for each product at each retailer form each supplier is any positive number;
3. The transportation and inventory costs are only determined by the quantity, regardless of the weight;
4. The customer demand is a random variable, and the demands of different retailers are independent of each other;
5. The retailers’ inventory supplement take single period \((T, S)\) replenishment policy (According to the predetermined interval \(T\), each retailer need to check the stock, put forward the order, and then, replenish the target inventory stock \(S\));
6. The delivery from each supplier to each retailer can be completed immediately;
7. The transportation costs includes order costs.

Secondly, we introduce the following notation to present the mathematical formulation throughout the remainder of this paper.

Indices, Sets, and Parameters.

\[ I := \text{set of suppliers, } I = \{1, 2, ..., m\}. \]
\[ J := \text{set of retailers, } J = \{1, 2, ..., n\}. \]
\[ i := \text{index for set of suppliers } I. \]
\[ j := \text{index for set of retailers } J. \]
\[ a_i := \text{output of supplier } i. \]
\[ c_{ij} := \text{transportation cost of shipping one unit of product from supplier } i \text{ to retailer } j. \]
\[ c_j := \text{inventory cost of one unit of product for retailer } j. \]
\[ x_{ij} := \text{decision variable, transport volume of products from supplier } i \text{ to retailer } j. \]
\[ y_j := \text{random variable, demand of products of retailer } j. \]
\[ \mu_j := \text{the expected demand volume of retailer } j. \]
\[ \sigma_j := \text{the demand standard deviation of retailer } j. \]
\[ s_j := \text{retailer } j \text{ inventory volume of products at the end of this period.} \]
\[ s_{j_0} := \text{retailer } j \text{ inventory volume of products at the beginning of this period.} \]
\[ \alpha_j := \text{confidence level of retailer } j. \]

Notice: Amount of product units: ten thousand. The cost unit: $
we consider the following objective function as a chance constraint of our model. In addition, the objective function is to minimize the inventory and transportation costs. Then, we take available resources into account as other constraints to construct the stochastic programming for transportation inventory model as follows:

\[
\text{Min } Z = \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}x_{ij} + E\sum_{j=1}^{J} \left[ \max\{s_{ji} + \sum_{i=1}^{I} x_{ij} - y_{j}, 0\} \right] 
\]

subject to

\[
\sum_{j=1}^{J} x_{ij} \leq a_{i}, \quad (\forall i \in I) \tag{1}
\]

\[
P\left\{ s_{ji} + \sum_{i=1}^{I} x_{ij} \geq y_{j} \right\} \geq \alpha_{j}, \quad (\forall j \in J) \tag{2}
\]

\[
x_{ij} \geq 0, \quad (\forall i \in I, \ j \in J) \tag{3}
\]

In the model, (1) is the objective function of minimizing the inventory-transportation costs. Constraints (2) is the significance that the transport volume of the supplier \(i\) to all the number of retailers can not exceed the total output of this supplier. Constraints (3) is a chance constraint, which indicates that the total storage capacity of retailer \(j\) at the periodic initial state is greater than or equal to the probability of the retailer’s current demand that is not less than the confidence level of retailer \(j\). Constraints (4) meets the condition of decision variable is non-negative.

Obviously, our model above is a stochastic chance-constrained programming model (SCCPM). We assume that customer demands is subject to normal distribution, that customer demands is subject to normal distribution, and the demand standard deviation of retailer \(j\) is \(\sigma_{j}\). Density function and distribution function are, respectively, \(f(y)\) and \(F(y)\). After the equivalent transformation, the above model will be equivalent to the following deterministic model:

\[
\text{Min } Z = \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}x_{ij} + \sum_{j=1}^{J} \int_{x_{ij}}^{\infty} F_{j}(y) \left( s_{ji} + \sum_{i=1}^{I} x_{ij} - y \right) f_{j}(y) d(y) \tag{5}
\]

subject to

\[
\sum_{j=1}^{J} x_{ij} \leq a_{i}, \quad (\forall i \in I) \tag{6}
\]

\[
\sum_{i=1}^{I} x_{ij} \geq \Phi_{j}^{-1}(\alpha_{j})\sigma_{j} + \mu_{j} - s_{ji}, \quad (\forall j \in J) \tag{7}
\]

\[
x_{ij} \geq 0, \quad (\forall i \in I, \ j \in J) \tag{8}
\]

This is a deterministic nonlinear programming model, and we can call directly the MATLAB optimizer to solve.

### 2.2 Multi-Period Model

In real strategy decision-making process, the multi-period situation is more common and in this situation the decision-maker can revise his production plan when more information about the customer demand becomes available. Many latest papers have addressed such situation, for example, Masoumeh et al. (2009) proposed a multi-product, multi-period planning problem under quality of raw materials. And then, we will construct the multi-period model which is an extension of the single-period stochastic programming in Section 2.1. At first, we add the following assumptions:

1. The customer demand is a random variable, and the demands of different retailers and the different periods of the same retailers are independent of each other.
2. The retailers’ inventory supplement take multiple period replenishment policy and each replenishment period must satisfy the constraint conditions.
3. It is assumed that the moment of replenishment in each period is the periodic initial state.

Secondly, we introduce the following notation to present the mathematical formulation throughout the remainder of this paper.

Indices, Sets, and Parameters.

- \(I\): set of suppliers, \(I = \{1, 2, ..., m\}\).
- \(J\): set of retailers, \(J = \{1, 2, ..., n\}\).
- \(T\): set of periods, \(T = \{1, 2, ..., p\}\).
- \(i\): index for set of suppliers \(I\).
- \(j\): index for set of retailers \(J\).
- \(t\): index for set of periods \(T\).
- \(s_{ji}\): output of supplier \(i\) in period \(t\).
- \(c_{ij}\): transportation cost of shipping one unit of product from supplier \(i\) to retailer \(j\) in period \(t\).
- \(x_{ij}\): decision variable, transport volume of products from supplier \(i\) to retailer \(j\) in period \(t\).
- \(y_{j}\): random variable, demand of products of retailer \(j\) in period \(t\).
- \(\mu_{j}\): the expected demand volume of retailer \(j\) in period \(t\).
- \(\sigma_{j}\): the demand standard deviation of retailer \(j\) in period \(t\).
- \(s_{ji}\): retailer \(j\) inventory volume of products at the end of the period \(t\).
- \(s_{ji}\): retailer \(j\) inventory volume of products at the beginning of the period.
- \(\alpha_{j}\): confidence level of retailer \(j\) in period \(t\).
Notice: Amount of product units: ten thousand.

The cost unit: $

Suppose that it is a multi-period delivering system from $m$ suppliers to $n$ retailers. Output of supplier $i$ in period $t$ is $a_i$. Suppliers delivery products at the periodic initial state for retailers, and each period’s total transport volume is $\sum x_{ijt}$ from all suppliers to the retailer $j$. Set up the first period of the retailer $j$’s inventory is $s_{ij0} = 0$. Obviously, the inventory at the end of the last period is the initial inventory of the next period. We consider the following objective function as a chance constraint of our model. In addition, the objective function is to minimize the cost of transportation-inventory. Then, we take available resources into account as other constraints to construct the model as follows:

$$\text{Min } Z = \sum_i \sum_j \sum_t c_{ijt} x_{ijt} + P\left(\max(\sum_j x_{ijt} - y_j, 0)\right)$$

subject to

$$\sum_{j \in J} x_{ijt} \leq a_i, (\forall i \in I, \forall t \in T)$$

$$P\left(\sum_{j \in J} x_{ijt} \geq y_j, (\forall j \in J, \forall t \in T)\right) \geq \alpha_p,$$

$$x_{ijt} \geq 0,$$

$$s_{ij0} = s_{ijt} = 0, (\forall j \in J)$$

The objective function (9) represents the sum of the total cost of transportation and inventory in all periods, and it needs to be minimized. Constraints (10) are expressed the number goods from supplier $i$ to all retailers in period $t$ can not exceed the total output of the supplier $i$. Constraints (11) is chance constrained conditions, which indicate that the total storage capacity $(s_{ij0} + \sum x_{ijt})$ of retailers $j$, at the periodic initial state of period $t$, is greater than or equal to the probability of the demand of the retailer that’s not less than the confidence level $\alpha_p$. Constraint condition (12) is a non negative condition of decision variable. (13) indicates that the stocks at the beginning and end of the whole period is 0.

It is a multi-period dynamic programming model. Hossein (2015) and Nan (2014) have studied the similar problems. In the case of customer demand is a random variable, the number of replenishment for any retailer is not only affected by the amount of pre-inventory, but also depends on the random demand in this period. In view of the characteristics of multi-period model, we can not translate the stochastic programming problem into a deterministic model. We may not solve it by classical optimization methods or intelligent algorithms. The next work is to explore the HIA - based on genetic algorithm and stochastic simulation, so as to handle the model.

### 3 SOLUTION METHOD

The traditional method of solving the chance constrained programming is to convert the chance constraints into their determinate equivalent forms, and then to solve the determinate model (such as single-period model). However, this traditional method isn’t applied to multi-period model above. Because the multi-period model is essentially a dynamic programming model and the variables of different period influence each other. So, in this paper, we explore HIA (Jose et al., 2013) which was mentioned by Liu (2003). This algorithm can solve the general chance-constrained programming. The main thought of this algorithm is to convert the stochastic model to the certain one. First, we adopt stochastic simulation technologies to produce the demands as the input data, and then calculate the output data according to a formulation.

![Figure 1. The flowchart for HIA](image-url)
we have designed. These inputs and outputs are used as the training samples for the back propagation neural network (BP neural network). Second, we train a BP neural network as the fitting function in the next stage. Finally, we use the improved-genetic algorithm to find the best solution. The flowchart for HIA is showed in the following Figure 1:

3.1 Stochastic Simulation Technologies

The SCCPM is different from the traditional model which contains some random variables. The probabilities of random events are taken as constrains. Thus, these probabilities should be obtained first. We use stochastic simulation method to compute them.

3.1.1 The Principle of Monte Carlo Technology Approaching Probability

We consider \( P\{f(x, \xi) \leq b\} \), where \( f(x, \xi) \) is a function of stochastic vector. First of all, \( N \) independent random vectors \( \xi_m, (m = 1, 2, \ldots, N) \) are generated from the probability distribution \( \phi(\xi) \) of random vector \( \xi \).

Denote the number of \( f(\xi) \) which satisfy the condition \( f(\xi) \leq b \), as \( M \). According to the Law of Large Numbers, when \( N \) is sufficiently large, \( P = \frac{M}{N} \) is used as an approximation of \( P\{f(x, \xi) \leq b\} \).

We know that the maximum \( y_\alpha \) which satisfies the inequality:

\[
P\left\{ \sum_{i=1}^{M} x_i \geq y_\alpha \right\} \geq \alpha
\]

should be achieved at the equal sign, that is

\[
P\left\{ \sum_{i=1}^{M} x_i \geq y_\alpha \right\} = \alpha
\]

Here, we denote \( f(\xi) = \sum_{i=1}^{M} x_i \), the sequence \( f(\xi_1), f(\xi_2), \ldots, f(\xi_N) \) is produced after \( N \) simulations.

Define \( h(\xi_i) \),

When \( f(\xi_i) \geq y_\alpha \), \( (k = 1, 2, \ldots, N) \)

\[
h(\xi_i) = 1;
\]

otherwise, \( h(\xi_i) = 0 \)

According to the law of large number, when \( N \) tends to infinity the value of

\[
\frac{\sum_{i=1}^{N} h(\xi_i)}{N}
\]

tend to \( \alpha \), which presents the probability of \( f(\xi) \geq y_\alpha \). On this basis, when the sequence \( f(\xi_1), f(\xi_2), \ldots, f(\xi_N) \) is sorted in ascending order, the \( N \)th element in the ar-

rangement gives the required value, where \( N \) is the integer part of \( \alpha N \).

3.1.2 Monte Carlo Technology to Produce the Simulation Data

Now, note \( \alpha = 0.95 \) and define the following function \( U(x) \).

Where

\[
U(x) = \min\{y_\alpha | P\left( s_{j-1} + \sum_{i=1}^{M} x_i \geq y_\alpha \right) \geq 0.95 \}
\]

The steps for simulating \( U(x) \) are as follows:

Step 1. Initialize the parameters: number of simulation \( N \), confidential level \( \alpha = 0.95 \).

Step 2. Generate \((i, j, t)\) randomly, and then, determine \( x_{ijt} \).

Step 3. Generate \( \xi \), randomly. Note that the value generated by the random function should be treated, so as to ensure non-negative.

Step 4. Calculate the value of \( s_{j-1} + \sum_{i=1}^{M} x_{ijt} \).

Step 5. Repeat steps 2-4 \( N \) times and then get \( N \) values of \( s_{j-1} + \sum_{i=1}^{M} x_{ijt} \). Sort these values in ascending order, the \( N \)th is the maximum \( y_\alpha \) needed.

Thus, give an \( x \), there is a \( U(x) \) (the maximum \( y_\alpha \)) through \( N \) times’ simulation. Repeat \( M \) times like this, we obtain \( M \) training samples. By these samples, train a neural network which is used to approximate the uncertain function \( U \).

3.2 Training for Neural Network

Train a neural network (the number of input neuron is \( m \), that of hidden layer neurons is \( y \), and the number of output neurons is 2) by the input and output data to approximate the uncertain function. The uncertain function is used as the evaluation function in the next layer algorithms. Where \( y \) is determined by the formula,

\[
y < \sqrt{p + q + a}
\]

Where \( p \), \( q \) represent the number of input neurons and output neurons respectively, \( a \) is a constant, \( 1 < a < 10 \) (Li Xiaoyan, 2009).

3.3 Genetic Algorithm to Solve the Model

The general steps of genetic algorithm:

Step 1. Initialize the chromosomes, compute the fitness of chromosomes by the trained neural network.
Step 2. Cross, mutate, select, and compute the fitness of the offspring chromosomes by the trained neural network.

Step 3. Pick up chromosomes by roulette wheel selection.

Step 4. Repeat steps until the completion of the given number of cycles.

Step 5. Obtain the best chromosome, adopt this chromosome as the optimal solution.

In the following the given example, assuming that algorithm parameters as follows:

Population size is 30;

Using two-point crossover and crossover probability $P_c = 0.3$;

Mutation probability $P_m = 0.2$;

The parameter in the evaluation function based on the sequence is $\alpha = 0.05$.

4. ILLUSTRATIVE EXAMPLE

In order to test the model of transportation and inventory joint optimization and the solution algorithms applied in the actual situation, we perform numerical tests based on the domestic operation department with reasonable assumptions.

4.1 Illustrative Example of Single-Period Model

We consider two suppliers, $I = \{1, 2\}$, noting $A_1, A_2$, and three retailers, $J = \{1, 2, 3\}$, notes $B_1, B_2, B_3$. This is a two level supply chain system that needs deal with the single-period transportation and inventory optimization problem under the situation of demand is a random variable. Confidence level is $\alpha = 0.95$, so, $\Phi^{-1}(\alpha) = 1.65$. $c_i$ is the transportation cost of shipping one unit of product from supplier $i$ to retailer $j$; $c_i$ is the inventory cost of one unit of product for retailer $j$; $a_i$ is the total output of supplier $i$. The random demand of each retailer is subject to the normal distribution $N(\mu_j, \sigma_j^2)$, and the density function is $f(y_j) = \frac{\exp\left(-\frac{(y_j - \mu_j)^2}{2\sigma_j^2}\right)}{\sqrt{2\pi\sigma_j}}$. The original schedule is Table 1 and Table 2 as follows:

$$
\min Z = \sum_i \sum_j c_i x_{ij} + 
\int_{-\infty}^{+\infty} \sum_j \left[ s_{ij} + \sum x_j - y_j \right] f_j(y) d_j, 
$$

subject to

$$
\sum x_j \leq a_i, \quad (i = 1, 2) 
$$

$$
\sum x_j \geq 1.65\sigma_j + \mu_j - s_{ij}, \quad (j = 1, 2, 3) 
$$

$$
x_j \geq 0, \quad (i = 1, 2, \quad j = 1, 2, 3) 
$$

As shown above, the single-period model is obtained according to the known data. Calling directly MATLAB optimizer to solve, then, we can get the optimal solution making the total cost of transportation inventory joint optimization model least (Minimum value is $17679$) that has been showed in the following Table 3:

4.2 Numerical Experiment of Multi-Period Model

In order to verify the correctness of the model and demonstrate how to use the proposed algorithm to solve the multi-period model, we assume that the product $P$ will be delivered from two suppliers $A_1, A_2$ to three retailers $B_1, B_2, B_3$, and the period $T$ which is made up of three periods $t_1, t_2, t_3$. Table 4 and Table 5 give the value of the parameters and others.

**Table 1.** Transportation unit price and total output

<table>
<thead>
<tr>
<th>$i$</th>
<th>$j$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>Output $a_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>35</td>
<td>43</td>
<td>49</td>
<td>180</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>33</td>
<td>46</td>
<td>47</td>
<td>115</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$i$</th>
<th>$j$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>23.72</td>
<td>95.85</td>
<td>37.03</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>53.48</td>
<td>0.00</td>
<td>61.52</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$i$</th>
<th>$j$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>65</td>
<td>73</td>
<td>79</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>63</td>
<td>76</td>
<td>77</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$i$</th>
<th>$j$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>61</td>
<td>70</td>
<td>76</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>65</td>
<td>68</td>
<td>78</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$i$</th>
<th>$j$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>67</td>
<td>75</td>
<td>82</td>
<td></td>
</tr>
<tr>
<td>$A_2$</td>
<td>70</td>
<td>72</td>
<td>80</td>
<td>130</td>
</tr>
</tbody>
</table>

Table 1. Transportation unit price and total output

Table 2. Unit inventory cost, random demand and initial inventory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{ij}$</td>
<td>15</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>$N(\mu_j, \sigma_j^2)$</td>
<td>$N(70,82)$</td>
<td>$N(86,92)$</td>
<td>$N(93,72)$</td>
</tr>
<tr>
<td>$s_{ij}$</td>
<td>6</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>
The confidence levels are $\alpha_j = 0.95$, $(j = 1, 2, 3, t = 1, 2, 3)$, and $\Phi_j^{-1}(0.95) = 1.65$. So, we have the following model about this problem:

$$
\text{Min} \quad Z = \sum \sum \sum c_{ijt} x_{ijt} + \\
\sum \sum \sum c_{ijt} \left[ \max \left( s_{ijt} + \sum x_{ijt} - y_{ijt} \right), 0 \right] \tag{18}
$$

subject to

$$
\sum x_{ijt} \leq a_{ijt} \quad (\forall i \in I, \forall t \in T) \tag{19}
$$

$$
\sum x_{ijt} \geq 1.65 \sigma_{ijt} + \mu_{ijt} - s_{ijt} \quad (\forall j \in J, \forall t \in T) \tag{20}
$$

$$
x_{ijt} \geq 0 \quad (\forall i \in I, \forall j \in J, \forall t \in T) \tag{21}
$$

$$
s_{ijt} = s_{ijt} = 0 \quad (j \in J) \tag{22}
$$

According to the specific algorithm parameters and process in section 3.3, we can make use of MATLAB to program and calculate the procedure. The results can be seen in Table 6. The minimum cost is $484,833.$

We consider a small problem consisting of two suppliers and three retailers whose computational time is only a few seconds. Even if we consider a large-scale case, computation time will not be changed. At the same time, due to space limitations, we don’t need to study large-scale problems.

### 5. CONCLUSIONS

This paper considers the single-period and multi-period two-level supply network and establishes the stochastic chance-constrained programming models. In these models, we consider the whole costs of transportation and inventory as the objective function, and the probabilities of random events are taken as constrains. In order to solve models above, we propose HIA based on genetic algorithm and stochastic simulation. In the end of this paper, numerical examples was given and showed the effectiveness of the method.

A major contribution of this paper is that we provide a comprehensive framework for supply chain under uncertain conditions. Much work still needs to be done to improve on the current framework. One obvious direction is that reducing assumptions and restriction conditions, and studying the transportation of multi-products. Thus, it will be more comprehensive to deal with the issue of supply chain.

### ACKNOWLEDGEMENTS

This work has been financed by the Fundamental Research Funds for the Central Universities (grant No. 3122015L009).

### REFERENCES


Harris, F. W. (1990), How many parts to make at once, *Operations Research, 38*(6), 947-950.


Li, X. (2009), *Study on Optimized Grey Neural Network Prediction Models*, Wuhan University of Science and Technology.