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A RELATION OF GENERALIZED q-w-EULER NUMBERS AND POLYNOMIALS

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Abstract. In this paper, we study the generalizations of Euler numbers and polynomials by using the q-extension with p-adic integral on $\mathbb{Z}_p$. We call these: the generalized q-w-Euler numbers $E^{(\alpha)}_{n,q,w}(a)$ and polynomials $E^{(\alpha)}_{n,q,w}(x;a)$. We investigate some elementary properties and relations for $E^{(\alpha)}_{n,q,w}(a)$ and $E^{(\alpha)}_{n,q,w}(x;a)$.

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1. Introduction

Many mathematicians are interested in the Euler numbers and polynomials because they possess many interesting properties and arise in many areas of mathematics and physics. Due to these reasons, recently various analogues for Euler numbers and polynomials have been studied. C. S. Ryoo and H. Y. Lee studied the generalized w-Euler numbers and polynomials and they observed that the distribution of roots for $E_{n,q,w}(x;a)$ is different from the Euler polynomials $E_n(x) = 0$ (see [4]).

Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$, $\mathbb{C}_p$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ will be denote the ring of p-adic integers, the field of p-adic rational numbers, the completion of algebraic closure of $\mathbb{Q}_p$, the set of natural numbers, the ring of rational integers, the field of rational numbers, the set of complex numbers respectively and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$.

Let ν_p be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{-\nu_p(p)} = p^{-1}$. When one talks of q-extension, q is considered in many ways such as an indeterminate, a complex number $q \in \mathbb{C}$, or p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$ one normally assume that $|q| < 1$. If $q \in \mathbb{C}_p$, then we normally assume that

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$|q-1|_p < p^{-\frac{1}{p-1}}$ so that $q^x = \exp(x \log q)$ for $|x|_p \leq 1$. Throughout this paper, we use the notation:

$$[x]_q = \frac{1-q^x}{1-q}, \quad [x]_{-q} = \frac{1-(-q)^x}{1+q} \text{ (see [1-2, 4-9])} .$$

Hence $\lim_{q\to 1}[x]_q = x$ for any x with $|x|_p \leq 1$ in the present p-adic case. For

$$g \in UD(\mathbb{Z}_p) = \{g | g : \mathbb{Z}_p \to \mathbb{C}_p \text{ is a uniformly differentiable function}\},$$

the fermionic p-adic q-integral on $\mathbb{Z}_p$ is defined by T. Kim as below:

$$I_{-1}(g) = \int_{\mathbb{Z}_p} g(x) d\mu_{-1}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-1}} \sum_{x=0}^{p^N-1} g(x)(-1)^x. \tag{1}$$

Let

$$T_p = \cup_{m\geq 1} C_{p^m} = \lim_{m\to\infty} C_{p^m}$$

where $C_{p^m} = \{w | w^{p^m} = 1\}$ is the cyclic group of order p^m. For $w \in T_p$, we denote by $\phi_w : \mathbb{Z}_p \to \mathbb{C}_p$ the locally constant function given by $x \longmapsto w^x$.

If we take $g_1(x) = g(x+1)$ in (1), then we easily see that

$$I_{-1}(g_1) + I_{-1}(g) = 2g(0). \tag{2}$$

From (2), we obtain

$$I_{-1}(g_n) + (-1)^{n-1} I_{-q}(g) = 2 \sum_{l=0}^{n-1} (-1)^{n-1-l} g(l), \tag{3}$$

where $g_n(x) = g(x+n)$ (see [1-2, 4-9]).

As well-known definition, the Euler polynomials are defined by

$$F(t) = \frac{2}{e^t+1} = e^{Et} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}$$

and

$$F(t,x) = \frac{2}{e^t+1} e^{xt} = e^{E(x)t} = \sum_{n=0}^{\infty} E_n(x) \frac{t^n}{n!}$$

with the usual convention of replacing $E^n(x)$ by $E_n(x)$. In the special case, $x = 0$, $E_n(0) = E_n$ are called the n-th Euler numbers (see [1-2, 4-9]).

The purpose of this paper is to study the generalized q-w-Euler numbers $E^{(\alpha)}_{n,q,w}(a)$ and polynomials $E^{(\alpha)}_{n,q,w}(x; a)$ which are q-analogues of the generalized w-Euler numbers and polynomials, respectively.

2. The generalized q-w-Euler numbers and polynomials

In this section, we study the generalized q-w-Euler numbers $E^{(\alpha)}_{n,q,w}(a)$ and polynomials $E^{(\alpha)}_{n,q,w}(x;a)$. To do this, we define the notions of the generalized q-w-Euler numbers $E^{(\alpha)}_{n,q,w}(a)$ and polynomials $E^{(\alpha)}_{n,q,w}(x;a)$ using fermionic integral.

Definition 2.1. For $a \in \mathbb{R}^+$, $t \in \mathbb{R}$ and $w \in \mathbb{C}$, the generalized q-w-Euler numbers and polynomials is given by

$$\sum_{n=0}^{\infty} E^{(\alpha)}_{n,q,w}(x;a)\frac{t^n}{n!} = \int_{\mathbb{Z}_p} w^{ay} e^{t[ay+x]_{q^\alpha}} d\mu_{-1}(y) \tag{4}$$

and

$$\sum_{n=0}^{\infty} E^{(\alpha)}_{n,q,w}(a)\frac{t^n}{n!} = \int_{\mathbb{Z}_p} w^{ax} e^{t[ax]_{q^\alpha}} d\mu_{-1}(x). \tag{5}$$

From Definition 2.1 and some calculations, we get

$$\begin{aligned}
\sum_{n=0}^{\infty} E^{(\alpha)}_{n,q,w}(x;a)\frac{t^n}{n!} &= \int_{\mathbb{Z}_p} w^{ay} e^{t[ay+x]_{q^\alpha}} d\mu_{-1}(y) \\
&= \lim_{N\to\infty} \frac{1}{[p^N]_{-1}} \sum_{y=0}^{p^N-1} w^{ay} e^{t[ay+x]_{q^\alpha}} (-1)^y \\
&= \lim_{N\to\infty} \sum_{y=0}^{p^N-1} (-w^a)^y \sum_{n=0}^{\infty} [ay+x]^n_{q^\alpha} \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\frac{1}{1-q^\alpha}\right)^n \lim_{N\to\infty} \sum_{y=0}^{p^N-1} (-w^a)^y (1-q^{\alpha(ay+x)})^n \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\frac{1}{1-q^\alpha}\right)^n \lim_{N\to\infty} \sum_{y=0}^{p^N-1} (-w^a)^y \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha a y l} q^{\alpha x l} \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\frac{1}{1-q^\alpha}\right)^n \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha x l} \lim_{N\to\infty} \sum_{y=0}^{p^N-1} (-w^a)^y q^{\alpha a y l} \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\frac{1}{1-q^\alpha}\right)^n \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha x l} \frac{2}{1+w^a q^{\alpha a l}} \frac{t^n}{n!}.
\end{aligned}$$

Then we get

$$E^{(\alpha)}_{n,q,w}(x;a) = 2\left(\frac{1}{1-q^\alpha}\right)^n \sum_{l=0}^{n} \binom{n}{l} (-1)^l q^{\alpha x l} \frac{1}{1+w^a q^{\alpha a l}} \tag{6}$$

Also from some calculations, we get

$$\begin{aligned}
E^{(\alpha)}_{n,q,w}(x;a) &= 2\left(\frac{1}{1-q^{\alpha}}\right)^{n}\sum_{l=0}^{n}\binom{n}{l}(-1)^{l}q^{\alpha x l}\frac{1}{1+w^{a}q^{\alpha a l}} \\
&= 2\left(\frac{1}{1-q^{\alpha}}\right)^{n}\sum_{l=0}^{n}\binom{n}{l}(-1)^{l}q^{\alpha x l}\sum_{k=0}^{\infty}(-w^{a}q^{\alpha a l})^{k} \\
&= 2\left(\frac{1}{1-q^{\alpha}}\right)^{n}\sum_{k=0}^{\infty}(-1)^{k}w^{ak}\sum_{l=0}^{n}\binom{n}{l}(-1)^{l}q^{\alpha x l}q^{\alpha a l k} \\
&= 2\left(\frac{1}{1-q^{\alpha}}\right)^{n}\sum_{k=0}^{\infty}(-1)^{k}w^{ak}(1-q^{\alpha x+\alpha a k})^{n} \\
&= 2\sum_{k=0}^{\infty}(-1)^{k}w^{ak}\left(\frac{1-q^{\alpha(x+ak)}}{1-q^{\alpha}}\right)^{n} \\
&= 2\sum_{k=0}^{\infty}(-1)^{k}w^{ak}[x+ak]^{n}_{q^{\alpha}} \\
&= \sum_{k=0}^{\infty}2(-1)^{k}w^{ak}[x+ak]^{n}_{q^{\alpha}}.
\end{aligned}$$

Therefore we get

$$E^{(\alpha)}_{n,q,w}(x;a) = 2\sum_{k=0}^{\infty}(-1)^{k}w^{ak}[x+ak]^{n}_{q^{\alpha}}.$$

In the above result, if $x=0$, then we get

$$E^{(\alpha)}_{n,q,w}(a) = 2\sum_{k=0}^{\infty}(-1)^{k}w^{ak}[ak]^{n}_{q^{\alpha}}.$$

Hence we get the theorem as below:

Theorem 2.2. *For $a \in \mathbb{R}^{+}$ and $w \in \mathbb{C}$, we get*

$$E^{(\alpha)}_{n,q,w}(x;a) = 2\sum_{k=0}^{\infty}(-1)^{k}w^{ak}[x+ak]^{n}_{q^{\alpha}} \tag{7}$$

and

$$E^{(\alpha)}_{n,q,w}(a) = 2\sum_{k=0}^{\infty}(-1)^{k}w^{ak}[ak]^{n}_{q^{\alpha}}. \tag{8}$$

From (6) and some calculations, we get

$$E^{(\alpha)}_{n,q^{-1},w^{-1}}(a-x;a) = 2\left(\frac{1}{1-q^{-\alpha}}\right)^{n}\sum_{l=0}^{n}\binom{n}{l}(-1)^{l}q^{-\alpha(a-x)l}\frac{1}{1+w^{-a}q^{-\alpha a l}}$$

$$= 2(-1)^n q^{\alpha n}\left(\frac{1}{1-q^{\alpha}}\right)^n \sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha x l} q^{-\alpha a l}\frac{w^a q^{\alpha a l}}{1+w^a q^{\alpha} a l}$$
$$= 2(-1)^n q^{\alpha n} w^a\left(\frac{1}{1-q^{\alpha}}\right)^n \sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha x l}\frac{1}{1+w^a q^{\alpha} a l}$$
$$= (-1)^n q^{\alpha n} w^a E^{(\alpha)}_{n,q,w}(x;a).$$

Hence, we get the following theorem.

Theorem 2.3. *For $a \in \mathbb{R}^+$, $w \in \mathbb{C}$ and $n \in \mathbb{N}$, we have*

$$E^{(\alpha)}_{n,w^{-1},q^{-1}}(a-x;a) = (-1)^n q^{\alpha n} w^a E^{(\alpha)}_{n,q,w}(x;a).$$

From Definition 2.1 and some calculations, we get

$$\begin{aligned}
E^{(\alpha)}_{n,w,q}(x;a) &= \int_{\mathbb{Z}_p} w^{ay}[ay+x]^n_{q^{\alpha}}\, d\mu_{-1}(y) \\
&= \int_{\mathbb{Z}_p} w^{ay}([x]_{q^{\alpha}} + q^{\alpha x}[ay]_{q^{\alpha}})^n\, d\mu_{-1}(y) \\
&= \int_{\mathbb{Z}_p} w^{ay}\sum_{l=0}^{n}\binom{n}{l}[x]^{n-l}_{q^{\alpha}} q^{\alpha x l}[ay]^l_{q^{\alpha}}\, d\mu_{-1}(y) \\
&= \sum_{l=0}^{n}\binom{n}{l}[x]^{n-l}_{q^{\alpha}} q^{\alpha x l}\int_{\mathbb{Z}_p} w^{ay}[ay]^l_{q^{\alpha}}\, d\mu_{-1}(y) \\
&= \sum_{l=0}^{n}\binom{n}{l}[x]^{n-l}_{q^{\alpha}} q^{\alpha x l} E^{(\alpha)}_{l,q,w}(a) \\
&= \left([x]_{q^{\alpha}} + q^{\alpha x}E^{(\alpha)}_{q,w}(a)\right)^n.
\end{aligned}$$

Hence we get an identity as below:

$$E^{(\alpha)}_{n,w,q}(x;a) = \left([x]_{q^{\alpha}} + q^{\alpha x}E^{(\alpha)}_{q,w}(a)\right)^n. \tag{9}$$

Let m be a positive integer. From the equation (6), we get

$$\begin{aligned}
&[m]^n_{q^{\alpha}}\sum_{i=0}^{m-1}(-1)^i w^{ai} E^{(\alpha)}_{n,q^{\alpha m},w^m}\left(\frac{ai+x}{m};a\right) \\
&= [m]^n_{q^{\alpha}}\sum_{i=0}^{m-1}(-1)^i w^{ai}\times 2\left(\frac{1}{1-q^{\alpha m}}\right)^n \sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha m\frac{ai+x}{m}l}\frac{1}{1+w^{m\alpha}q^{\alpha m a l}} \\
&= \left(\frac{1-q^{\alpha m}}{1-q^{\alpha}}\right)^n \times 2 \times \left(\frac{1}{1-q^{\alpha m}}\right)^n \\
&\qquad \times \sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha x l}\frac{1}{1+w^{ma}q^{\alpha m a l}}\sum_{i=0}^{m-1}(-w^a)^i q^{\alpha a i l}
\end{aligned}$$

$$= 2\left(\frac{1}{1-q^\alpha}\right)^n \sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha x l}\frac{1}{1+w^{ma}q^{\alpha mal}}\frac{1+w^{am}q^{\alpha aml}}{1+w^a q^{\alpha al}}$$
$$= 2\left(\frac{1}{1-q^\alpha}\right)^n \sum_{l=0}^{n}\binom{n}{l}(-1)^l q^{\alpha x l}\frac{1}{1+w^a q^{\alpha al}}$$
$$= E_{n,q,w}^{(\alpha)}(x;a).$$

Hence we get the distribution relation as below:

Theorem 2.4. *For $a \in \mathbb{R}^+$, $w \in \mathbb{C}$ and $n, m \in \mathbb{N}$, we have*

$$[m]_{q^\alpha}^n \sum_{i=0}^{m-1}(-1)^i w^{ai} E_{n,q^{\alpha m},w^m}^{(\alpha)}\left(\frac{ai+x}{m};a\right) = E_{n,q,w}^{(\alpha)}(x;a).$$

Let $g(x) = w^{ax}e^{t[ax]_{q^\alpha}}$. From properties of the fermionic p-adic integral (2), we get

$$I_{-1}(g_1) + I_{-1}(g) = \int_{\mathbb{Z}_p} w^{a(x+1)}e^{t[a(x+1)]_{q^\alpha}}\,d\mu_{-1}(x) + \int_{\mathbb{Z}_p} w^{ax}e^{t[ax]_{q^\alpha}}\,d\mu_{-1}(x)$$
$$= w^a \sum_{n=0}^{\infty}\int_{\mathbb{Z}_p} w^{ax}[ax+a]_{q^\alpha}^n\,d\mu_{-1}(x)\frac{t^n}{n!} + \sum_{n=0}^{\infty}\int_{\mathbb{Z}_p} w^{ax}[ax]_{q^\alpha}^n\,d\mu_{-1}(x)\frac{t^n}{n!}$$
$$= \sum_{n=0}^{\infty}\left(w^a E_{n,q,w}^{(\alpha)}(a;a) + E_{n,q,w}^{(\alpha)}(a)\right)\frac{t^n}{n!}$$
$$= 2.$$

By comparing the coefficients of $\frac{t^n}{n!}$ in both sides, we get an identity as below:

Theorem 2.5. *For $a \in \mathbb{R}^+$, $w \in \mathbb{C}$ and $n \in \mathbb{Z}_+$, we have*

$$w^a E_{n,q,w}^{(\alpha)}(a;a) + E_{n,q,w}^{(\alpha)}(a) = \begin{cases} 2, & \textit{if } n = 0 \\ 0, & \textit{if } n > 0. \end{cases}$$

By Theorem 2.4 and the equation (9), we have the following corollary.

Corollary 2.6. *For $n \in \mathbb{Z}_+$, we have*

$$w^a([a]_{q^\alpha} + q^{\alpha a}E_{n,q,w}^{(\alpha)}(a))^n + E_{n,q,w}^{(\alpha)}(a) = \begin{cases} 2, & \textit{if } n = 0 \\ 0, & \textit{if } n > 0, \end{cases}$$

with the usual convention of replacing $(E_w(a))^n$ by $E_{n,w}(a)$.

Let $g(x) = w^{ax}e^{[ax]_{q^\alpha}t}$. From $g(x)$ and the left side of the equation (3), we get

$$I_{-1}(g_n) + (-1)^{n-1}I_{-1}(g)$$
$$= \int_{\mathbb{Z}_p} w^{a(x+n)}e^{[a(x+n)]_{q^\alpha}^t}\,d\mu_{-1}(x) + (-1)^{n-1}\int_{\mathbb{Z}_p} w^{ax}e^{[ax]_{q^\alpha}^t}\,d\mu_{-1}(x)$$

$$
\begin{aligned}
&= w^{an}\int_{\mathbb{Z}_p} w^{an} e^{[ax+an]_{q^\alpha}^t} d\mu_{-1}(x) + (-1)^{n-1}\int_{\mathbb{Z}_p} w^{ax} e^{[ax]_{q^\alpha}^t} d\mu_{-1}(x)\\
&= w^{an}\sum_{m=0}^{\infty}\left(\int_{\mathbb{Z}_p} w^{ax}[ax+an]_{q^\alpha}^m d\mu_{-1}(x)\right)\frac{t^m}{m!}\\
&\qquad + (-1)^{n-1}\sum_{m=0}^{\infty}\left(\int_{\mathbb{Z}_p} w^{ax}[ax]_{q^\alpha}^m d\mu_{-1}(x)\right)\frac{t^m}{m!}\\
&= \sum_{m=0}^{\infty}\left(w^{an}E_{n,q,w}^{(\alpha)}(an;a) + (-1)^{n-1}E_{n,q,w}^{(\alpha)}(a)\right)\frac{t^m}{m!}. \qquad (10)
\end{aligned}
$$

Also, from $g(x)$ and the right hand side of equation of (3), we get the following :

$$
2\sum_{l=0}^{n-1}(-1)^{n-1-l}w^{al}e^{[al]_{q^\alpha}^t} = \sum_{m=0}^{\infty}\left(2\sum_{l=0}^{n-1}(-1)^{n-1-l}w^{al}[al]_{q^\alpha}^m\right)\frac{t^m}{m!}. \qquad (11)
$$

Hence by (10) , (11) and comparison of the coefficients of $\frac{t^m}{m!}$ in both hand sides, we have the following theorem

Theorem 2.7. *For a positive integer n, we have*

$$
w^{an}E_{m,q,w}^{(\alpha)}(an;a) + (-1)^{n-1}E_{m,q,w}^{(\alpha)}(a) = 2\sum_{l=0}^{n-1}(-1)^{n-1-l}w^{al}[al]_{q^\alpha}^m.
$$

3. The analogue of the generalized q-w-Euler zeta function

The Euler zeta function is defined by $\zeta_E(s) = \sum_{n=0}^{\infty}\frac{(-1)^{n-1}}{n^s}$ (see [3]). By using the generalized q-w-Euler numbers and polynomials, the generalized Hurwitz q-w-Euler zeta functions and the generalized q-w-Euler zeta functions are defined. These functions interpolate the generalized q-w-Euler numbers and q-w-Euler polynomials, respectively. In this section, let ω be the p^N-th root of unity.

Let $F_{w,q,a}^{(\alpha)}(t;x) = \sum_{n=0}^{\infty}E_{n,q,w}^{(\alpha)}(x;a)\frac{t^n}{n!}$. Then by the k-th differentiation, we get

$$
\left.\frac{d^k}{dt^k}F_{w,q,a}^{(\alpha)}(t;x)\right|_{t=0} = E_{k,q,w}^{(\alpha)}(x;a). \qquad (12)
$$

By using equation (12), we are now ready to define the concept of generalized w-Euler zeta functions.

Definition 3.1. For $s \in \mathbb{C}$, we define

$$
\zeta_{q,w,a}^{(\alpha)}(s;x) = 2\sum_{n=0}^{\infty}\frac{(-1)^n w^{an}}{[x+an]_{q^\alpha}^s}.
$$

Note that $\zeta^{(\alpha)}_{q,w,a}(s;x)$ is a meromorphic function on $\mathbb{C}$. Note that if $w \to 1$, $q \to 1$ and $a = 1$, then $\zeta^{(\alpha)}_{q,w,a}(s;x) = \zeta(x;s)$, which is the Hurwitz Euler zeta function. The relation between $\zeta^{(\alpha)}_{q,w,a}(s;x)$ and $E^{(\alpha)}_{n,q,w}(x;a)$ is given in the following theorem.

Theorem 3.2. *For $k \in \mathbb{N}$, we have*

$$\zeta^{(\alpha)}_{q,w,a}(-s;x) = E^{(\alpha)}_{s,q,w}(x;a).$$

Observe that the function $\zeta^{(\alpha)}_{q,w,a}(s;x)$ interpolates $E^{(\alpha)}_{n,q,w}(x;a)$ at non-negative integers. By using (12), we note that

$$\frac{d^k}{dt^k} F^{(\alpha)}_{w,q,a}(t;0)\bigg|_{t=0} = E^{(\alpha)}_{k,q,w}(0;a) = E^{(\alpha)}_{k,q,w}(a). \tag{13}$$

By using the above equation, we are now ready to define the notion of twisted Hurwitz q-w-Euler zeta functions.

Definition 3.3. For $s \in \mathbb{C}$, we define

$$\zeta^{(\alpha)}_{q,w,a}(s) = 2\sum_{n=1}^{\infty} \frac{(-1)^n w^{an}}{[an]^s_{q^\alpha}}.$$

Note that $\zeta^{(\alpha)}_{q,w,a}(s)$ is a meromorphic function on $\mathbb{C}$. Obserse that if $w \to 1$ and $q \to 1$, then $\zeta^{(\alpha)}_{q,w,a}(s) = \zeta(s)$ is the Hurwitz Euler zeta function. The relation between $\zeta^{(\alpha)}_{q,w,a}(s)$ and $E^{(\alpha)}_{s,q,w}$ is given in the following theorem.

Theorem 3.4. *For $k \in \mathbb{N}$, we have*

$$\zeta^{(\alpha)}_{q,w,a}(-s) = E^{(\alpha)}_{s,q,w}(a).$$

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