Journal of the Korean Society of Marine Environment & Safety
Research Paper
Vol. 23, No. 3, pp. 313-319, May 31, 2017, ISSN 1229-3431(Print) / ISSN 2287-3341(Online) https://doi.org/10.7837/kosomes.2017.23.3.313

Development of Design Formula for Predicting Post-Buckling Behaviour and Ultimate Strength of Cylindrical Shell

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Abstract : *Cylindrical shells are often used in ship structures at deck plating with a camber, side shell plating at fore and aft parts, and bilge structure part. It has been believed that such curved shells can be modelled fundamentally by a part of a cylinder under axial compression. From the estimations with the usage of cylinder models, it is known that, in general, curvature increases the buckling strength of a curved shell subjected to axial compression, and that curvature is also expected to increase the ultimate strength. We conduct series of elasto-plastic large deflection analyses in order to clarify the fundamentals in buckling and plastic collapse behaviour of cylindrical shells under axial compression. From the numerical results, we derive design formula for predicting the ultimate strength of cylindrical shell, based on a series of the nonlinear finite element calculations for all edges, simply supporting plating, varying the slenderness ratio, curvature and aspect ratio, as well as the following design formulae for predicting the ultimate strength of cylindrical shell. From a number of analysis results, fitting curve can be developed to use parameter of slenderness ratio with implementation of the method of least squares. The accuracy of design formulae for evaluating ultimate strength has been confirmed by comparing the calculated results with the FE-analysis results and it has a good agreement to predict their ultimate strength.*

Key words : *Cylindrical shell, Ultimate strength, Elasto-plastic analysis, Deflection, Design formula, Buckling strength*

1. Introduction

Cylindrical shells are often used in ship structures at deck plating with a camber, side shell plating at fore and aft parts, and bilge structure part. It has been believed that such curved plates can be modelled fundamentally by a part of a cylinder under axial compression. The estimations using cylinder models, were conducted and it was derived that in general, curvature increases the buckling strength of a curved plate subjected to axial compression, and that curvature is also expected to increase the ultimate strength. We conduct series of elasto-plastic large deflection analysis in order to clarify the fundamentals in buckling and plastic collapse behaviour of cylindrical shells under axial compression. According to numerous FE-results, we derived design formula for predicting the ultimate strength of cylindrical shell, based on a series of the nonlinear finite element calculations for all edges simply supporting plating varying the slenderness ratio, curvature and aspect ratio, as well as the following design formulae to predict the ultimate strength of cylindrical shell.

A brief review is made in previous research works related to buckling and ultimate strength behaviour of flat plates and stiffened plates. Maeno et al. (2004) performed a series of elasto-plastic large deflection analyses to investigate buckling and plastic collapse behaviour of ship's bilge strakes, which are unstiffened curved plates under axial compression. On the basis of the calculated results, a simple formula was derived to calculate buckling and ultimate strength and to simulate stress-average strain relationship of the bilge structure under axial compression. It was found that the bilge structure with a conventional shape and size reaches the ultimate strength by yielding before buckling.

Park et al. (2009) developed a method for predicting the buckling/post-buckling behaviour including secondary buckling and ultimate strength of a cylindrical shell subjected to axial compression. The elastic large deflection as well as elasto-plastic analysis of curved plates with initial imperfection, simply supporting four edges, is performed with application of FEM. Analytical formulation is made based on the theory of the elastic large deflection analysis. It is shown that Faulkner's ultimate strength formula developed for a flat plate can allow a reasonable estimate of ultimate strength of curved plate with relatively small flank angle, when the newly defined slenderness parameter considering the increase of the elastic buckling strength due to

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curvature is applied.

Tran and Davaine (2011) developed a method for predicting the ultimate strength of a cylindrical shell subjected to uniform axial compression. The methodology is based on the formal procedure adopted by Eurocode 3, for all types of stability verification. A series of numerical simulations were carried out in order to clarify and examine the fundamental buckling behaviour of curved panels. On the basis of those results, the formulas for elastic buckling and ultimate strength were derived.

Tran et al. (2014) examined the buckling and collapse behaviour of curved stiffened plates under uniform longitudinal compression. They address the linear buckling and the ultimate strength which are both influenced by the coupled effects of curvature and stiffening. They finally proposed design methodology based on stiffened flat plates adopted by European Standards as well as column-like behaviour.

Fig. 1 conceptually compares the flat plate, the circular cylinder and the cylindrical shell for elastic large deflection behaviour under axial loading. One may see there flat plating which buckles in the lowest stress as compared to cylindrical shell structure with curvature from author's engineering experience as well as some classical books (Timoshenko and Woinowsky-Krieger, 1959). Especially, in case of evaluating the buckling behaviour of the circular cylinder, it is necessary to examine carefully their characteristic components.

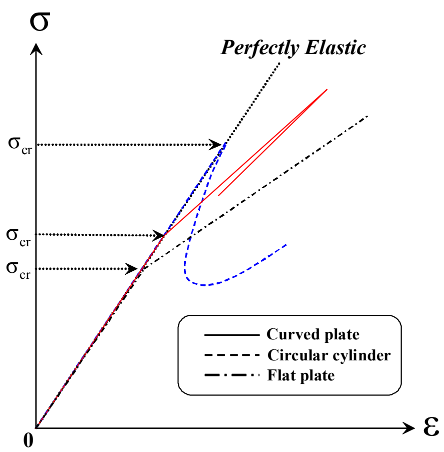


Fig. 1. Schema of the elastic large deflection behaviour considering three kinds of structure model under axial load.

2. Analysis and FEM (Finite Element Method)

A linear material model is satisfactory when only small quantities of the material are exposed to the yield stress or greater. The bilinear kinematic material model is recommended for general small strain used for materials that comply with von Mises yield criteria (it includes most metals) as shown by Fig. 2. Kinematic hardening assumes that the yield surface retains a constant size but moves in stress space with continued yielding.

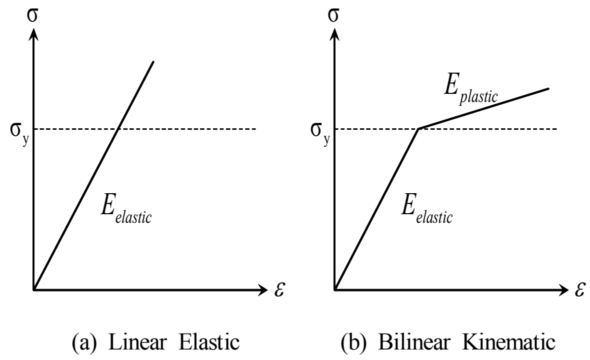


Fig. 2. Material properties of the cylindrical shell.

In the bilinear kinematic material model it is necessary to specify the yield stress and subsequent slope (i.e tangent) for the stress and strain curve beyond initial yield. This is expressed as a percentage of the original Young's elastic modulus. Cylindrical shell geometries were modelled with the ANSYS 14.0 code using standard structural shell element namely SHELL181. This element is suitable for analysing thin to moderately-thick shell structures. It is a 4-node element with six degree of freedom at each node: translation in the x, y, z and rotations about x, y, and z-axes. The target structures of this research are shown in Fig. 3. Curved plates have dimensions of a in length, b in width, t in thickness and in flank angle (theta), where the width b is kept constant 1,000 mm throughout the present study. There is a relation between the width b and the flank angle (theta), where R is the radius as shown in Fig. 3.

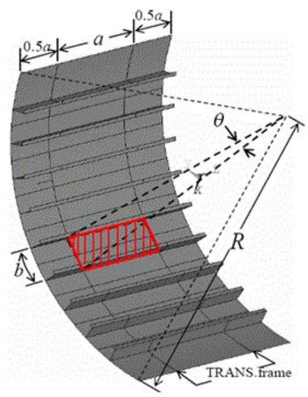


Fig. 3. The definition of cylindrical shell.

The material is assumed to be hardening of elasto-plasticity with a Young's modulus E=205.8 GPa, Poisson's ratio is 0.3 and yield stress is 352.8 MPa. To fabricate the cylindrical shell of the ship structure, welding is normally used and thus the post-weld initial imperfections (initial deflections and residual stresses) develop in the structure. In advanced ship structural design, capacity calculations of ship plating should accommodate post-weld initial imperfections as parameters of influence. The characteristics of the post-weld initial imperfections are uncertain, and the sinusoidal deflection model is used as shown in equation (1). Smith's et al. (1987) suggested the following maximum values of representative initial deflections for plating in merchant vessel structures which may be used as shown in equation (1) and (2).

$$\frac{w_o}{w_{opl}} = \sum_{i=1}^{M} B_{oi} \sin\frac{i\pi x}{a} \sin\frac{\pi x}{b} \tag{1}$$

$$\frac{w_{opl}}{t} = \begin{Bmatrix} 0.025\beta^2 & \text{for slight level} \\ 0.1\beta^2 & \text{for average level} \\ 0.3\beta^2 & \text{for severe level} \end{Bmatrix} \tag{2}$$

where, a is plate length, b is plate width, B_{oi} is welding induced initial deflection amplitude normalized by the maximum initial deflection, w_{0pl}, can be determined based on the initial deflection measurements, $\beta=(b/t)\sqrt{(\sigma_y/E)}$, *slenderness ratio of plate*

3. Elastic Large Deflection Analysis of the circular cylinder

Thin-walled circular cylinders are widely used in numerous applications such as tanks, silos, space launchers. A correct design has to take into account buckling phenomena, which may occur under specific loading conditions, and cause total collapse of the structure. The circular cylinder can also be subject to axial compression (pipes, silos, tanks, off-shore jacket legs, etc.). The simplest case to analysis is the axially compressed circular cylinder, with elastic critical buckling stress as follows in equation (3).

$$\sigma_{cr} = \frac{E}{\sqrt{3(1-\nu^2)}} \times \frac{t}{R} \tag{3}$$

where, R is radius, t is thickness, ν is Poisson's ratio, E is Young's elastic modulus, σ_{cr} is critical buckling strength

This buckling load is derived on the assumption that the pre-buckling increase of the radius due to the Poisson's effect is unrestrained and that the two edges are held against translational movement in the radial and circumferential directions during buckling, but are able to rotate about the local circumferential axis. These edge restraints are usually called "classical boundary conditions".

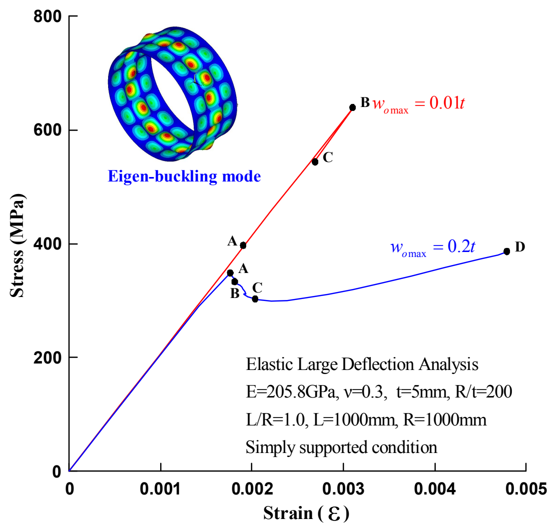


Fig. 4. A comparison of stress and strain curves of circular cylinder varying the magnitude of initial imperfection under axial compression.

Fig. 4 shows relationships of stress and strain for circular cylinder varying the magnitude of initial imperfection under axial compression. In these calculations initial imperfection of 1 % and 20 % of plate thickness are considered respectively. Different elastic large deflection behaviours are observed when the amplitude of initial imperfection increases. Consequently, imperfections can reduce the buckling load of a circular cylinder drastically compared to that of the perfect circular cylinder. The behaviour under displacement control snaps down to a low post-buckling value and increases the wave number of the buckling mode during increasing axial loading.

The computed structural behaviour of circular cylinder under axial loading is presented in Fig. 4 During the pre-buckling state of stress and strain curve is almost of linear behaviour. Buckling takes place at point B and A separately, the outside wall deformations are characterized by one or several localized ellipse-like deflection patterns as shown in Fig. 5 When the axial loading increases, the ellipse-like deflection changes into larger diamond shaped buckles as shown at point D.

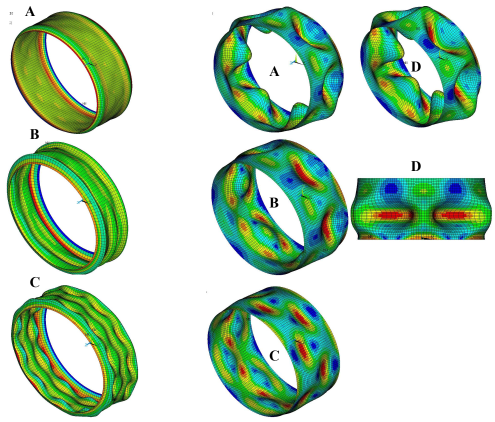


Fig. 5. A change of the deflection varying the magnitude of initial deflection.

3.1 Elasto-Plastic Large Deflection Analysis of the curved plate

A series of elasto-plastic large deflection analyses is performed to clarify the fundamentals in buckling and plastic collapse behaviour of cylindrical shell under axial compression. The thickness of the plate is taken as 10 mm. Maximum magnitude of the initial deflection is 1 % of the thickness.

In each case, calculated average stress strain relationships are summarized, using FEM code (ULSAS and ANSYS) as shown in Fig. 6.

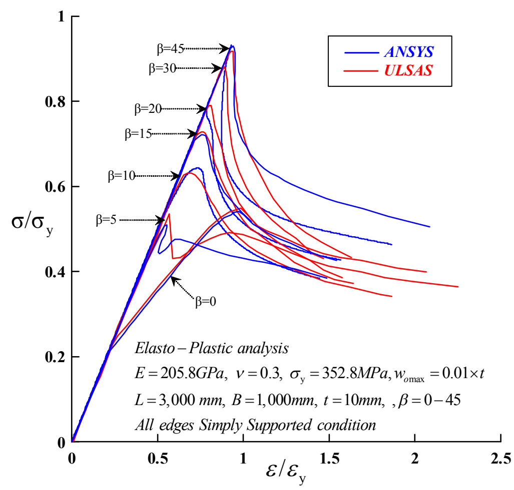
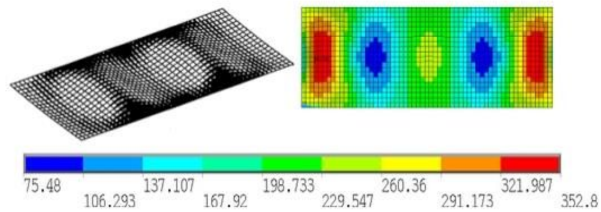
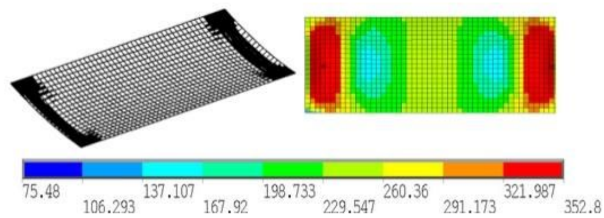


Fig. 6. A comparison of average stress and average strain curves of the cylindrical shell varying the flank angle under axial compression.

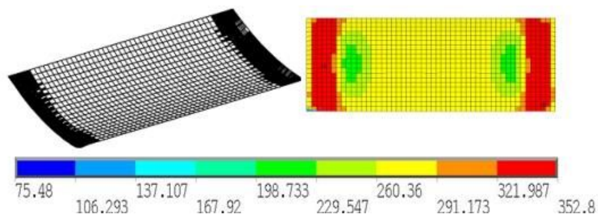
In case of a thin plate (t=10 mm) with a flank angle 5 degrees the secondary buckling takes place accompanied by a snap-through. Except for the case of flank angle being 5 degrees, the ultimate strength increases also with increase in the flank angle. The collapse behaviour simulated with the usage of different FEM codes (ULSAS and ANSYS) shows good agreement in evaluating the ultimate strength. Almost the same local yielding near the loading edge part of the curved plate is observed commonly among the all cases with different flank angles as shown in Fig. 7.



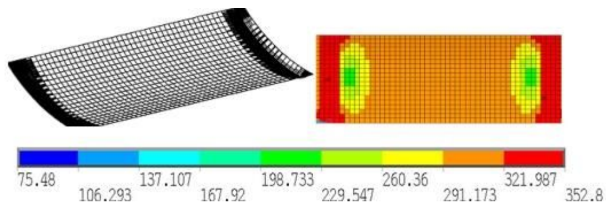
(a) flank angle 5 degrees



(b) flank angle 15 degrees



(c) flank angle 30 degrees



(d) flank angle 45 degrees

Fig. 7. Distribution of yielding stress of the cylindrical shell (left side : ULSAS results, right side : ANSYS results).

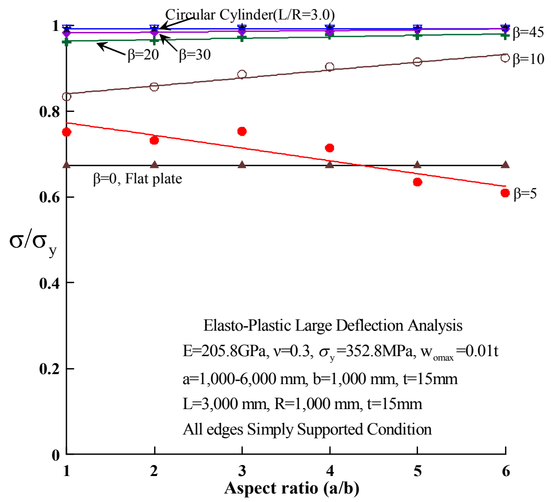


Fig. 8. A comparison of ultimate strengths of cylindrical shell with various aspect ratios under axial compression.

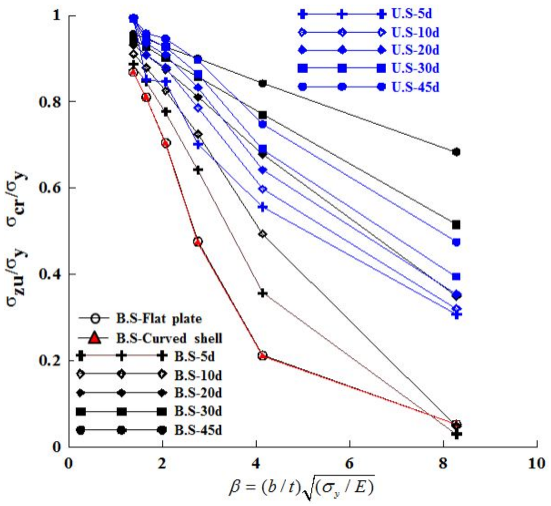


Fig. 9. Comparison of buckling and ultimate strengths of cylindrical shell with various slenderness ratio under axial compression.

Fig. 8 summarizes the calculated results showing the effect of aspect ratio on the ultimate strength for the cylindrical shell with various flank angles and circular cylinders subjected to axial load. In case of flank being 5 degrees, the ultimate strength is sensitively affected due to the changing of aspect ratio. However, when the flank angle is more than 20 degrees, the effect of aspect ratio can be ignored. Also, when the flank angle is more than 30 degrees, the curves almost overlap those of the cylinders irrespective of the aspect ratio. Thus, the effect of aspect ratio can be neglected, when the flank angle is over 20 degrees with the same radius, and thickness ratio can be predicted for ultimate strength. The estimations for cylinders on buckling/ultimate strength can be used in place of the estimation of the buckling/ultimate strength for the cylindrical shell with the flank angles over 30 degrees. If the increase of the magnitude of initial imperfection is getting bigger, the influence of aspect ratio can be neglected except for flank angle being 5 degrees.

Fig. 9 shows relationship of the average stress and slenderness ratio ranging from 1.0 to 8.2 of the cylindrical shell under axial loading, aspect ratio is fixed 3.0, plate thickness is taken 15 mm, magnitude of initial imperfection uses Smith's formula (slight level). The triangle symbol shows results under DnV class guideline (2013), symbols of black colour are calculated under elastic buckling stress with plastic correction, symbols of blue colour represents results under ultimate strength.

Actually, in many ship yards, DnV formula is widely used for the design of cylindrical shell but it cannot consider effect of their curvature, and relatively lower-estimate buckling strength and ultimate strength as compared to FE-Analysis. In the case of reducing plate thickness, there is more than enough safety margin, in other words, it is very a conservative design.

In general, bilge circle of the actual container ship adopts slenderness ratio, ranging from 2.07 to 2.76. In case of a plate thickness, 20 mm is close to comparison with DnV results and FEM results of the curved plate, but reduced plate thickness represents a larger difference in results. If plate thickness reduced from 14 mm to 10 mm, existing design formula would have an overflow safety margin. Based on the summarized results, it is known that there's a need to correct the formula considering the effect of curvature for the designed cylindrical shell.

4. Development of design formula of the cylindrical shell under axial compression

Based on a series of the nonlinear finite element calculations for all edges simply supporting plating varying the slenderness ratio (Beta),curvature (R/b) and aspect ratio (a/b), we derived the following design formulae for predicting the ultimate strength of cylindrical shell, taking into consideration Smith's slight level, namely

$$\frac{\sigma_{xu}}{\sigma_y} = \alpha_1\beta^3 + \alpha_2\beta^2 + \alpha_3\beta + 1.000$$

$$where,\ \beta = \frac{b}{t}\sqrt{\frac{\sigma_y}{E}}(Slenderness\ ratio)$$

$$\alpha_1 = C_1(\frac{R}{b})^2 + C_2(\frac{R}{b}) + C_3$$

$$\alpha_2 = C_4(\frac{R}{b})^2 + C_5(\frac{R}{b}) + C_6$$

$$\frac{\sigma_{xu}}{\sigma_y} = \alpha_1\beta^3 + \alpha_2\beta^2 + \alpha_3\beta + 1.000$$

$$where,\ \beta = \frac{b}{t}\sqrt{\frac{\sigma_y}{E}}(Slenderness\ ratio) \tag{4}$$

$$\alpha_1 = C_1(\frac{R}{b})^2 + C_2(\frac{R}{b}) + C_3$$

$$\alpha_2 = C_4(\frac{R}{b})^2 + C_5(\frac{R}{b}) + C_6$$

$$\alpha_3 = C_7(\frac{R}{b})^2 + C_8(\frac{R}{b}) + C_9 \tag{5}$$

$$\begin{aligned}
C1 &= 4.4065E^{-5}(a/b)^2 + 0.00032325(a/b) - 0.00050659\\
C2 &= 0.00060313(a/b)^2 - 0.0042239(a/b) + 0.0087107\\
C3 &= -0.00060264(a/b)^2 + 0.0039679(a/b) - 0.0052798\\
C4 &= 0.00024974(a/b)^2 - 0.0019140(a/b) + 0.0032411\\
C5 &= -0.0035117(a/b)^2 + 0.025142(a/b) - 0.055057\\
C6 &= 0.0023925(a/b)^2 - 0.016227(a/b) + 0.0044221\\
C7 &= -0.00033970(a/b)^2 + 0.0026033(a/b) - 0.0043521\\
C8 &= 0.0050426(a/b)^2 - 0.035884(a/b) + 0.072359\\
C9 &= -0.0023269(a/b)^2 + 0.017797(a/b) - 0.0029754
\end{aligned} \tag{6}$$

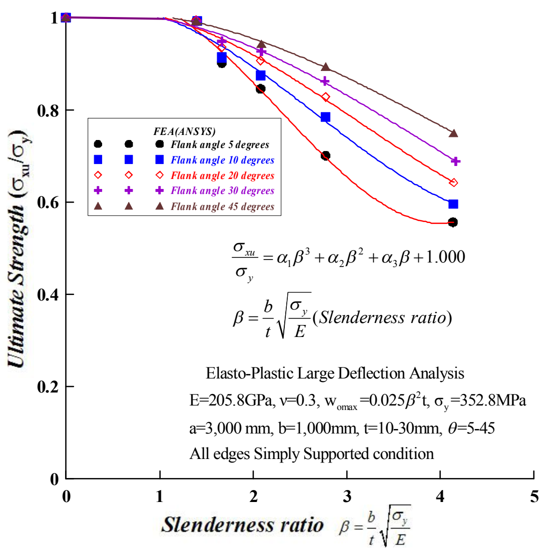


Fig. 10. Comparison of the ultimate strength with the slenderness ratio at the aspect ratio 3.0 considering small initial imperfection.

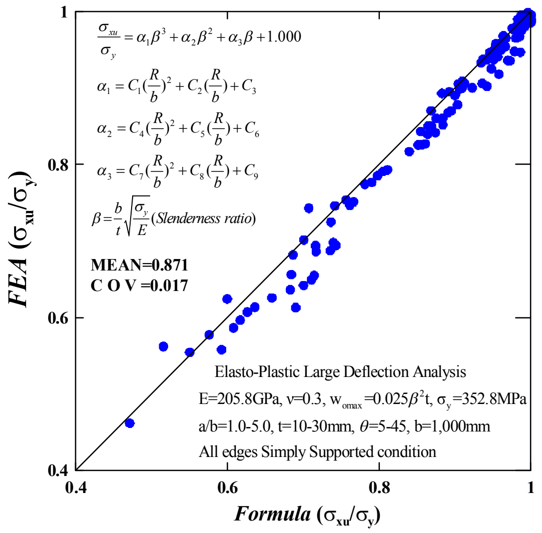


Fig. 11. Comparison of the present formula with the numerical results considering small initial imperfection.

The thickness of the plate varies from 10 mm to 40 mm, flank angle degree varies from 5 to 45 and aspect ratio varies from 1.0 to 5.0 with the magnitude of initial deflection considering Smith's slight level. Fig. 10 shows the relationships between ultimate strength and slenderness ratio with varying flank angles. From the series analysis results, fitting curve can be developed to use parameter of slenderness ratio with implementation of the method of least squares. Fig. 11 illustrates the comparative results between FE-analysis and design formula as obtained from equation (4), (5) and (6). The developed design formula has a good agreement to predict the ultimate strength of cylindrical shell under axial compression and the difference ratio is approximately 1.7 %.

The behaviour of cylindrical shell normally depends on a variety of influential factors, namely, magnitude of initial imperfections, slenderness ratio, curvature and also aspect ratio.

To achieve the advanced buckling and ultimate strength design of cylindrical shell, we would need more sophisticated methods than existing simplified approaches to evaluate buckling and ultimate strength. The aim of the present study is to develop more advanced design formulae to predict their ultimate strength for cylindrical shell.

5. Conclusions

The present paper focuses on the following two studied areas as follows:

1. To examine and clarify the buckling and ultimate strength behaviour with consideration of the effect of several kinds of parameters of the cylindrical shell under axial compression.

2. To develop the design formulae to evaluate ultimate strength of the cylindrical shell.

The validity of the ultimate strength formulations developed in this study has been, to some extent, verified through comparison with nonlinear numerical solutions. In order to raise the reputation of this research, we are going to set up the experimental tests some representative models.

(1) The ultimate strength of a cylindrical shell under axial compression is sensitive to geometrical initial imperfection.

(2) The ultimate strength of cylindrical shell with a flank angle more than 30 degrees can be estimated by that of circular cylinders with the same radius to thickness ratio (R/t).

(3) The effect of aspect ratio has little influence on the ultimate strength of cylindrical shell with the flank angle over 20 degrees.

(4) In case of cylindrical shell with a flank of 10 degrees or less, the ultimate strength differs much, depending on the aspect ratio of the plates, and cannot be estimated by that of an equivalent circular cylinder.

(5) The accuracy of design formulae for evaluating ultimate strength has been confirmed by comparing the calculated results with the FE-analysis results.

Acknowledgment

This paper was supported by the Regional Demand Customized R&D Project (2016-0066) and grant was funded by the Jeollanam-do provincial government.

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Received : 2017. 04. 07.
Revised : 2017. 05. 19. (1st)
: 2017. 05. 27. (2nd)
Accepted : 2017. 05. 29.