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Nonlinear Nutation Control of Spacecraft Using Two Momentum Wheels

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Abstract

In this work, the nutation control of rigid spacecraft with only two momentum wheels is addressed by applying the feedback linearization technique. In this strategy, the primary performance index is to regulate the nutational angle by the momentum control of wheels. The spacecraft attitude equations of motion are transformed to a general linearized form by feedback linearization technique, including a guaranteed control law promising the internal dynamics stability to accomplish the nutation angle small. It is proven that the configuration of inertia properties plays a key role in analyzing spacecraft energy level. The behavior of the momentum wheels is also studied analytically and numerically. Finally, the effectiveness of the proposed nonlinear control law for the momentum transfer is verified by conducting numerical simulations.

Key words: : Nutation control, Under-actuated system, Spacecraft attitude control, Two-wheels

1. Introduction

At least three actuators, either gas jets or momentum wheels, are required necessarily for the 3-axis attitude control of spacecraft so that arbitrary reorientation maneuvers can be accomplished using smooth feedback[1]–[6]. On the other hand, it is also well proven that with less than three momentum wheel actuators the system becomes uncontrollable[7]. As a result, arbitrary reorientation maneuvers are analytically impossible using only two momentum wheel actuators. Nevertheless, the attitude stabilization problem using only two momentum wheel actuators have been performed by several researchers since the wheels can fail to operate in space like FUSE and Hayabusa examples[7–9]. Krishnan *et al*. derived a discontinuous feedback control law that stabilized the spacecraft about any equilibrium attitude[8]. In this strategy, a series of eight maneuvers is required to accomplish the three axis attitude reorientation. Also, Krishnan *et al*. considered stabilizing an under-actuated rigid spacecraft using two momentum wheels with the assumption that the initial velocity vector lies in the same plane as the two momentum wheels[9]. Tsiotras and Longuski have considered stabilizing an axially symmetric spacecraft using only two external pairs of gas jets[10]. However, the initial velocity is restricted to the control input plane. The problem of finding suboptimal spacecraft maneuver control laws for handling under-actuated system, with only two control torques available was addressed by Kim *et al*.[11,12]. One of the drawbacks of this technique is that a series maneuvers based on Euler angle is required.

An adaptive control based on feedback linearization technique with neural networks was addressed for the momentum transfer control of a torque-free gyrostat with an attached spring-mass-dashpot damper. With very restricted resources, it was proven that the stability of the control law by using the Lyapunov stability criterion[13]. Nevertheless, the problem in this research is that it has very undesirable control torque profiles for the nutation angle regulation. In this paper, it is focused on that the attitude control problem from arbitrary reorientation maneuver to the initial attitude

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acquisition maneuver by momentum transfer control with only two wheels, guaranteeing implementable control torque profiles. The nonlinear feedback linearization technique is applied for the primary attitude control of spacecraft, so that the equations of motion of spacecraft installed with two wheels are easily transformed to a general linearized form including internal dynamics with a stability condition by Lyapunov function candidate. In this work, the role of the secondary wheel is analyzed deeply how well contribute on the performance of the attitude control. Furthermore, misalignment of the momentum wheels is considered as well.

This paper is organized as follows. First, the dynamic equations of motion of spacecraft model installed with two wheels resulting in equations in terms of angular velocity and momentum are formulated. From the generalized equations of motion with the restriction that two wheels are located in the same plane, it is simplified by placing two wheels on each spacecraft principal axis. Next, a control law using feedback linearization technique is presented and the stability of the control law is proven by using Lyapunov stability theorem. Finally, simulation results are presented to verify the effectiveness of the proposed control law.

2. Equations of Motion

Consider a rigid spacecraft installed with two momentum wheel actuators as shown in Fig. 1. The system consists of a rigid body Bs, containing rigid two axisymmetric wheels R_1 and R_2 spinning about the axes defined by the unit vectors and $\hat{\mathbf{a}}_1$ fixed in $\hat{\mathbf{a}}_2$. We assume that the wheel spin axes, $\hat{\mathbf{a}}_1$ and $\hat{\mathbf{a}}_2$, lie on the principal axes $\mathbf{b}_1$ - $\mathbf{b}_2$ plane. The total angular momentum of the spacecraft is given by

$$\mathbf{h} = \mathbf{h}_b + \mathbf{h}_{w_1} + \mathbf{h}_{w_2} \tag{1}$$

where $\mathbf{h}_b$, $\mathbf{h}_{w_1}$ and $\mathbf{h}_{w_2}$ denote the angular momentum vector of the spacecraft, R_1 and R_2, respectively. At first, the angular momentum of the platform, $\mathbf{h}_b$, can be expressed as

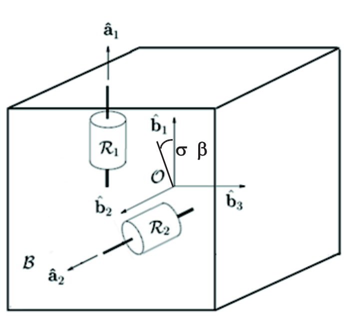

Fig. 1. A rigid spacecraft using two momentum wheel actuators

$$\mathbf{h}_b = \mathbf{J}_b \omega \tag{2}$$

where $\mathbf{J}_b$ is the inertia matrix containing spacecraft inertia terms and ω is the angular velocity of the spacecraft with respect to the body frame. Moreover, the angular momentum of the two wheels, $\mathbf{h}_{w_1}$ and $\mathbf{h}_{w_2}$, can be expressed as

$$\mathbf{h}_{w_i} = \mathbf{J}_{w_i} \omega + J_{w_i} \Omega_{w_i} \hat{\mathbf{a}}_i, \quad i = 1,2 \tag{3}$$

where

$$\mathbf{J}_{w_i} = J_{t_i} \mathbf{1} + (J_{w_i} - J_{t_i}) \hat{\mathbf{a}}_i \hat{\mathbf{a}}_i^T, \quad i = 1,2 \tag{4}$$

is the inertia tensor of each wheel. Ω_{w_i}, $\mathbf{1}$, J_{w_i} and J_{t_i} are the relative angular velocity of each wheel about its spin axis $\hat{\mathbf{a}}_1$, unit matrix, axial and transverse moments of inertia of each axisymmetric momentum wheel, respectively. Therefore, the total angular momentum is the sum of three components which becomes

$$\mathbf{h} = \mathbf{J}\omega + h_{w_1} \hat{\mathbf{a}}_1 + h_{w_2} \hat{\mathbf{a}}_2 \tag{5}$$

where

$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_{w_1} + \mathbf{J}_{w_2}$$

$$h_{w_i} = J_{w_i} \Omega_{w_i}, \quad i = 1,2$$

By assuming that the product of inertia of the spacecraft is zero, we denote the system inertia matrix as **I** rather than **J**. The total angular momentum of the spacecraft in the body frame is as follows

$$\mathbf{h} = \begin{bmatrix} I_1\omega_1 \\ I_2\omega_2 \\ I_3\omega_3 \end{bmatrix} + \begin{bmatrix} h_{w_1} \mathrm{c}\alpha \\ h_{w_1} \mathrm{s}\alpha \\ 0 \end{bmatrix} + \begin{bmatrix} h_{w_2} \mathrm{c}\beta \\ h_{w_2} \mathrm{s}\beta \\ 0 \end{bmatrix} = \begin{bmatrix} I_1\omega_1 + h_{w_1}\mathrm{c}\alpha + h_{w_2}\mathrm{c}\beta \\ I_2\omega_2 + h_{w_1}\mathrm{s}\alpha + h_{w_2}\mathrm{s}\beta \\ I_3\omega_3 \end{bmatrix} \tag{6}$$

where $\mathrm{c}\alpha \equiv \cos\alpha$, $\mathrm{s}\alpha \equiv \sin\alpha$, $\mathrm{c}\beta \equiv \cos\beta$ and $\mathrm{s}\beta \equiv \sin\beta$. α and β are the clockwise alignment angle of each wheel about $\hat{\mathbf{b}}_1$-axis. Of particular interest is the absolute angular momentum of the wheel about its spin axis, the axis of symmetry and becomes

$$h_{a_i} = J_{w_i} \Omega_{w_i} + J_{w_i} \hat{\mathbf{a}}^T \omega, \quad i = 1,2 \tag{7}$$

Notice that the absolute angular momentum of the wheel includes the relative angular momentum, $h_{w_i} = J_{w_i} \Omega_{w_i}$, as well as a contribution from the system angular velocity, ω. Assuming that external force and torque are zero, the governing equations of motion can be expressed as

$$\begin{aligned} \dot{\mathbf{h}} &= -\omega \times \mathbf{h} \\ \dot{h}_{w_i} &= u_i, \quad i = 1,2 \end{aligned} \tag{8}$$

where × denotes the cross product.

Now, the preceding equations of motion can be rewritten in the body frame as

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$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \\ \dot{\omega}_3 \\ \dot{h}_{w_1} \\ \dot{h}_{w_2} \end{bmatrix} = \begin{bmatrix} I_1 & 0 & 0 & \mathrm{c}\alpha & \mathrm{c}\beta \\ 0 & I_2 & 0 & \mathrm{s}\alpha & \mathrm{s}\beta \\ 0 & 0 & I_3 & 0 & 0 \\ J_{w_1}\mathrm{c}\alpha & J_{w_1}\mathrm{s}\alpha & 0 & 1 & 0 \\ J_{w_2}\mathrm{c}\beta & J_{w_2}\mathrm{s}\beta & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} (I_2 - I_3)\omega_2\omega_3 + \omega_3 h_{w_1}\mathrm{s}\alpha + \omega_3 h_{w_2}\mathrm{s}\beta \\ (I_3 - I_1)\omega_3\omega_1 - \omega_3 h_{w_1}\mathrm{c}\alpha - \omega_3 h_{w_2}\mathrm{c}\beta \\ (I_1 - I_2)\omega_1\omega_2 - (\omega_1\mathrm{s}\alpha - \omega_2\mathrm{c}\alpha)h_{w_1} - (\omega_1\mathrm{s}\beta - \omega_2\mathrm{c}\beta)h_{w_2} \\ u_1 \\ u_2 \end{bmatrix} \tag{9}$$

Finally, the equations of motion can be simplified as

$$\begin{aligned} I_1^*\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 - h_{w_2}\omega_3 + u_1 &= 0 \\ I_2^*\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 + h_{w_1}\omega_3 + u_2 &= 0 \\ I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 + h_{w_2}\omega_1 - h_{w_1}\omega_2 &= 0 \\ I_1^*\dot{h}_{w_1} + J_{w_1}(I_2 - I_3)\omega_2\omega_3 + J_{w_1}h_{w_2}\omega_3 - I_1 u_1 &= 0 \\ I_2^*\dot{h}_{w_2} + J_{w_2}(I_3 - I_1)\omega_1\omega_3 - J_{w_2}h_{w_1}\omega_3 - I_2 u_2 &= 0 \end{aligned} \tag{10}$$

where $I_i^* = I_i - J_{w_i}$, i=1, 2, $\omega_i\omega_i$ is the angular rate of spacecraft, u_i denotes the applied control input, and h_w and J_w represent the angular momentum and the moment of inertia of the momentum wheels, respectively. The total kinetic energy of the spacecraft is written as

$$\begin{aligned} E &= \frac{1}{2}\left(I_1^*\omega_1^2 + I_2^*\omega_2^2 + I_3^*\omega_3^2\right) \\ &+ \frac{1}{2}\left(J_{w_1}(\omega_1 + \Omega_{\omega_1})^2 + J_{t_1}\omega_2^2 + J_{t_1}\omega_3^2\right) + \frac{1}{2}\left(J_{t_2}\omega_1^2 + J_{w_2}(\omega_2 + \Omega_{\omega_2})^2 + J_{t_2}\omega_3^2\right) \\ &= \frac{1}{2}\left(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_1^3\right) + \omega_1 h_{w_1} + \omega_2 h_{w_2} + \frac{1}{2J_{w_1}}h_{w_1}^2 + \frac{1}{2J_{w_2}}h_{w_2}^2 \end{aligned} \tag{11}$$

The time derivative of the total kinetic energy is given in the form

$$\dot{E} = \Omega_{\omega_1}u_1 + \Omega_{\omega_2}u_2 \tag{12}$$

Note that the total energy is changed by nonzero torque input independent of inertia properties and spacecraft angular velocity components[14]. The time derivative of the total kinetic energy is related to the torque input and angular speed of each wheel.

As a criterion for the momentum transfer of spacecraft, let us consider the nutation angle of the spacecraft defined as[15]

$$\theta = \left(\frac{I_1\omega_1 + \cos(\alpha)h_{w_1} + \cos(\beta)h_{w_2}}{|\mathbf{h}|}\right) \tag{13}$$

where $|\mathbf{h}|$ denotes the magnitude of the total angular momentum of spacecraft. Note that $|\mathbf{h}|$ is constant by momentum conservation principle. Therefore, the nutation angle decreases from 90 deg, with the initial spin of the spacecraft about $\hat{\mathbf{b}}_2$ or $\hat{\mathbf{b}}_3$-axis to a small value.

3. Control Law Design by Feedback Linearization

3.1 Initial Stability Analysis

The perturbed states from initial pure $\hat{\mathbf{b}}_2$-axis spin are prescribed as

$$\begin{aligned} &\omega_1 = \omega_1^d, \quad \omega_2 = \omega_{20} + \omega_2^d, \quad \omega_3 = \omega_3^d \\ &h_{w_1} = h_{w_1}^d, \quad h_{w_2} = h_{w_2}^d, \quad u_1 = u_1^d, \quad u_2 = u_2^d \end{aligned} \tag{14}$$

where superscript d represents small perturbation. Then, the linearized equations of motion can be expressed as

$$\begin{aligned} I_1^*\dot{\omega}_1^d + (I_3 - I_2)\omega_{20}\omega_3^d + u_1^d &= 0 \\ I_2^*\dot{\omega}_2^d + u_2^d &= 0 \\ I_3\dot{\omega}_3^d + (I_2 - I_1)\omega_{20}\omega_1^d - \omega_{20}h_{w_1} &= 0 \\ I_1^*\dot{h}_{w_1}^d + J_{w_1}(I_2 - I_3)\omega_{20}\omega_3^d - I_1 u_1^d &= 0 \\ I_2^*\dot{h}_{w_2}^d - I_2 u_2^d &= 0 \end{aligned} \tag{15}$$

Therefore, the numerator of the nutation angle, namely, the angular momentum about $\hat{\mathbf{b}}_1$-axis can be written as

$$I_1\omega_1^d + h_{w_1}^d = \int (I_2 - I_3)\omega_{20}\omega_3^d\, dt \tag{16}$$

It is easy to know that ω_3^d>0 condition is required to decrease the nutation angle with the inertia properties satisfying I_2>I_3>I_1. Now, to identify the sign of ω_3^d, combination of linearized equations of motion yields

$$I_3\dot{\omega}_3^d = \left[(I_1 - I_2 - J_{w_1})\omega_1^d + \int u_1^d\, dt\right]\omega_{20} \tag{17}$$

A positive value of u_1^d satisfies ω_3^d>0 condition regardless of the sign of ω_1^d since the integral of u_1^d in time can guarantee positive sign of the right-hand side of Eq. (16). On the contrary, the nutation angle initially decreases to the negative torque value of ud1 about the unstable 5 moment of inertia properties satisfying I_1>I_3>I_2. Moreover, u_2^d is not related to the initial stability condition to reduce nutation angle from Eq. (16). To verify this relationship exactly, the initial angular momentum about $\hat{\mathbf{b}}_2$ axis can be rewritten as

$$I_2\omega_2^d + h_{w_2}^d = \text{constant} \tag{18}$$

It means that u_2^d does not change the angular momentum about $\hat{\mathbf{b}}_2$-axis in the initial state since the spacecraft initially spins about $\hat{\mathbf{b}}_2$-axis.

3.2 Control Law Design by Feedback Linearization

The feedback linearization technique is used for the momentum transfer control of a rigid spacecraft with two momentum wheel actuators. Candidate output and reference functions for feedback linearization based on initial stability analysis are chosen as

$$\begin{aligned} &y_1 = I_1\omega_1 + h_{w_1}, \quad y_2 = I_2\omega_2 + h_{w_2} \\ &y_{1r} = h_T(1 - e^{-t/\tau_1}), \quad y_{2r} = h_T\,\mathrm{sech}(t/\tau_2) \end{aligned} \tag{19}$$

Successive derivatives of output functions until they explicitly contain control input to determine the relative degree are as follows

$$\dot{y}_1 = (I_2 - I_3)\omega_2\omega_3 + h_{w_2}\omega_3, \quad \ddot{y}_1 = P_1(\mathbf{x}) + Q_1(\mathbf{x})u_2$$
$$\dot{y}_2 = (I_3 - I_1)\omega_3\omega_1 - h_{w_1}\omega_3, \quad \ddot{y}_2 = P_2(\mathbf{x}) + Q_2(\mathbf{x})u_1 \tag{20}$$

where

$$P_1(\mathbf{x}) = \frac{I_2 - I_3}{I_3}\left[(I_1 - I_2)\omega_1 + h_{w_1}\right]\omega_2^2 + \frac{I_3 - I_2^*}{I_2^*}\left[(I_1 - I_3)\omega_1 + h_{w_1}\right]\omega_3^2$$
$$+\frac{1}{I_3}\left[(I_1 - 2I_2 + I_3)\omega_2 - h_{w_2}\right]h_{w_2}\omega_1 + \frac{1}{I_3}h_{w_1}h_{w_2}\omega_2$$
$$P_2(\mathbf{x}) = \frac{I_3 - I_1}{I_3}\left[(I_1 - I_2)\omega_2 - h_{w_2}\right]\omega_1^2 + \frac{I_3 - I_1^*}{I_1^*}\left[(I_2 - I_3)\omega_2 + h_{w_2}\right]\omega_3^2$$
$$+\frac{1}{I_3}\left[(I_2 - 2I_1 + I_3)\omega_2 - h_{w_1}\right]h_{w_1}\omega_2 + \frac{1}{I_3}h_{w_1}h_{w_2}\omega_1$$
$$Q_1(\mathbf{x}) = \frac{I_3}{I_2^*}\omega_3, \quad Q_2(\mathbf{x}) = -\frac{I_3}{I_1^*}\omega_3\phi(\mathbf{x})$$

To identify the internal dynamics, a variable $\phi(\mathbf{x})$ is chosen such that

$$\phi(0) = 0, \quad \frac{\partial\phi}{\partial\mathbf{x}}G(\mathbf{x}) = 0 \tag{21}$$

Hence, the partial differential equation can be solved by separating variables to obtain

$$\phi(\mathbf{x}) = I_1\omega_1 + h_{w_1} + I_2\omega_2 + h_{w_2} \tag{22}$$

which satisfies $\phi(0)$=0. The internal dynamics is equivalent to the sum of output functions y_1 and y_2 for the feedback linearization. Now, the equations of motion of the spacecraft are 6 easily transformed to a general feedback linearized form by changing the variables such that

$$\dot{\eta} = \xi_2 + \xi_4$$
$$\dot{\xi} = A_c\xi + B_cQ(\mathbf{x})[u - \gamma(\mathbf{x})] \tag{23}$$
$$y = C_c\xi$$

where

$$\xi = [y_1 \quad \dot{y}_1 \quad y_2 \quad \dot{y}_2]^T, \quad \eta = [\phi], \quad y = [y_1 \quad y_2]^T$$
$$\gamma(\mathbf{x}) = -Q(\mathbf{x})^{-1}P(\mathbf{x}), \quad Q(\mathbf{x}) = \begin{bmatrix} 0 & Q_1(\mathbf{x}) \\ Q_2(\mathbf{x}) & 0 \end{bmatrix}, \quad P(\mathbf{x}) = \begin{bmatrix} P_1(\mathbf{x}) \\ P_2(\mathbf{x}) \end{bmatrix}$$
$$A_c = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The error dynamics and nonlinear feedback law are given by

$$\dot{E} = A_cE + B_c\left[Q(\mathbf{x})(u - \gamma(\mathbf{x})) - S\right] = AE$$
$$u = Q(\mathbf{x})^{-1}(-P(\mathbf{x}) + S + \upsilon) \tag{24}$$

where

$$A^TP + PA = -Q$$

Therefore, A can be Hurwitz by choosing a proper gain matrix K. In other words, there exists a positive definite matrix Q satisfying

$$A^TP + PA = -Q \tag{25}$$

For a Lyapunov function candidate $V = \frac{1}{2}x^TPx > 0$ for $x \neq 0$. The Lyapunov equation guarantees a unique positive definite solution. Based on the Eq. (21), the internal dynamics is not globally asymptotically stable. However, since the Lyapunov stability condition of Eq. (24) guarantees that output functions track reference trajectories eventually, the internal dynamics is stable.

With the restriction that the misalignment angles of α and β are small from orthogonal configuration, output functions for feedback linearization in Eq. (18) can be modified as

$$y_1 = I_1\omega_1 + h_{w_1}\cos\alpha + h_{w_2}\cos\beta$$
$$y_2 = I_2\omega_2 + h_{w_1}\sin\alpha + h_{w_2}\sin\beta \tag{26}$$

It is motivated by the idea that the momenta of two wheels about $\hat{\mathbf{b}}_1$ and $\hat{\mathbf{b}}_2$-axes depend on the alignment angles α and β. The rotor misalignment causes desired nominal spin to 7 deviate from a pure $\hat{\mathbf{b}}_1$-axis spin, since the angular momentum about $\hat{\mathbf{b}}_2$-axis is produced by alignment angle α. However, the added wheel aligned with $\hat{\mathbf{b}}_2$-axis can remove this term, even though the added wheel also has a misalignment angle about $\hat{\mathbf{b}}_2$-axis.

4. Stability Analysis

In this section, the stability analysis is conducted. A perturbed linear form of the nominal motion is established by defining variables as follows:

$$\omega_1 = \omega_p + \omega_1^d, \quad \omega_2 = \omega_2^d, \quad \omega_3 = \omega_3^d$$
$$h_{w_1} = h_p + h_{w_1}^d, \quad h_{w_2} = h_{w_2}^d \tag{27}$$

where h_p denotes the nominal angular momentum of the wheel, and ω_p is the constant which can be considered as the orbital rate or mean motion for a circular orbit in most practical cases. This allows continuous earth-pointing strategy of the platform.

Next, the perturbed equations of motion are simplified into

$$\dot{\omega}_2^d + \lambda_1\omega_3^d = 0$$
$$\dot{\omega}_3^d + \lambda_2\omega_2^d + \delta h_{w_2}^d = 0 \tag{28}$$
$$\dot{h}_{w_2}^d + J_{w_2}\lambda_1\omega_3^d = 0$$

where

$$\lambda_1 = \frac{(I_1 - I_3)\omega_p + h_p}{I_2^*}, \quad \lambda_2 = \frac{(I_2 - I_1)\omega_p - h_p}{I_3},$$

$$\delta = \frac{\omega_p}{I_3}$$

Then the determinant of coefficients by taking the Laplace transform of Eq. (27)

$$\begin{vmatrix} s & \lambda_1 & 0 \\ \lambda_2 & s & \delta \\ 0 & -J_{w_2}\lambda_1 & s \end{vmatrix}$$

leads to a characteristic equation such as

$$\left[s^2 + \lambda_1 (J_{w_2}\delta - \lambda_2) \right] = 0 \tag{29}$$

Note that the coefficient of Eq. (29) to be positive requires two necessary stability conditions to guarantee non divergent solutions for ω_2 and ω_3 as shown in Table 1. It is noted from the solution 1 that the addition of a spinning rotor within the spacecraft could contain enough angular momentum to stabilize a spacecraft's spin axis, even for minor or intermediate axes. Note that this result does not specify any inertia relationships. However, solution 2 shows that the rotation about either the major or minor inertia axes is stable, and is similar to the rigid body stability condition. Therefore, the spacecraft in most practical cases is only stable when spinning about the major axis in this state. These stability conditions are based on open loop analysis with no control inputs by linearization with respect to an equilibrium point in (26), and the closed loop stability by a feedback control law is already verified in the previous section. The closed loop stability analysis guarantees the asymptotic stability by feedback linearization technique, but there is no relationship between stability conditions and spacecraft dynamics. On the contrary, based on Table 1, the above stability conditions contain moment of inertia properties and angular momentum of the spacecraft

Table 1. Stability conditions for spacecraft installed with two momentum wheels

Con 1	$(I_1\omega_p + h_p) - I_3\omega_p > 0$ $(I_1\omega_p + h_p) - I_2^*\omega_p > 0$
Sol 1	$If\ \ \omega_p = 0, then\ \ h_p > 0$
Sol 2	$If\ \ h_p = 0, then\ \ I_1 - I_3 > 0\ and\ I_1 - I_2^* > 0$ I_1 has to be the maximum moment of inertia
Con 2	$(I_1\omega_p + h_p) - I_3\omega_p < 0$ $(I_1\omega_p + h_p) - I_2^*\omega_p < 0$
Sol 1	$If\ \ \omega_p = 0, then\ \ h_p < 0$
Sol 2	$If\ \ h_p = 0, then\ I_1 - I_3 < 0\ and\ I_1 - I_2^* < 0$ I_1 has to be the minimum moment of inertia

5. Simulation

Several numerical simulations are conducted to verify the performance of proposed control law. Time constants τ_1 and τ_2 determining reference trajectory shapes are set to 800 and 1200, respectively. A feedback gain matrix K is designed as α_0=0.1, α_1=0.001, β_0=0.1, β_1=0.001, to assign the eigenvalues of A_c-B_cK at a desired stable location. To prevent unusually excessive control commands, the torque outputs from the control laws are limited such that $-N_1 \le u_1(t), u_2(t) \le N_1$, where the maximum torque output N_1 is set to 0.005 Nm. Moreover, to prevent unusually excessive wheel speed, the wheel speed is implemented with a limiter such that $-N_2 \le \Omega_1(t), \Omega_2(t) \le N_2$, where the maximum wheel

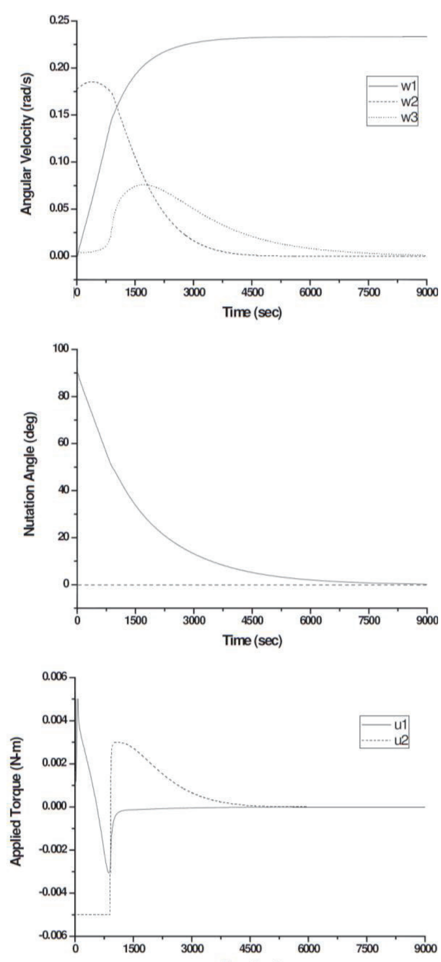

Fig. 2. Simulation results(α=2deg, β=88deg and I_2>I_3>I_1)

speed N_2 is set to 5000 rpm for each wheel as well. In other words, the maximum angular momentum of each wheel is limited to 26 Nms. The spacecraft initially spins about $\hat{\mathbf{b}}_2$ -axis with a spin rate of 0.1771 rad/sec. Let us suppose that the spacecraft inertia property is crucial for the Successful momentum transfer maneuver. The initial spin axis $\hat{\mathbf{b}}_2$ of the spacecraft is chosen as the maximum or minimum moment of inertia axis. Nevertheless, the proposed control law produces a proper tracking control torque command to execute successful momentum transfer maneuver. The performance index is the final nutation angle that needs to be made small enough.

A numerical simulation is performed by using the proposed control law in Eq. (23). The moment of inertia for the spacecraft with two momentum wheels are selected as $[I_1, I_2, I_3]=[85.12, 113.59, 86.24]$kgm^2 and $[I_{w_1}, \$, I_{w_2}]=[0.05, 0.05]$kgm^2, respectively. Note that the initial spin axis $\hat{\mathbf{b}}_2$ of the spacecraft is chosen as the maximum moment of inertia axis, whereas one wheel from two installed in the $\hat{\mathbf{b}}_1$-axis for the nominal spin is aligned along the minimum moment of inertia axis in opposition to that of the platform. The simulation results are displayed in Figs. 2-3. The control input $u_1(t)$ regardless of $u_2(t)$ has the positive value in the initial state, and they gradually converge to zero after appropriate torque profile generation by the nonlinear feedback linearization technique. Therefore, the nominal spin axis $\hat{\mathbf{b}}_1$ seems to absorb the initial angular momentum of the spacecraft effectively. In other words, output functions track reference trajectories with a small error bound. As a result, the final nutation angle converges to a very small value of 0.6 deg over the maneuver time with the negligible residual oscillation. The total kinetic energy decreases from the initial maximum value 220 J since the wheel angular momentum is smaller than 1 Nms based on Eq. (10). Even though the simulation result satisfies the open loop stability conditions in condition 2 of Table 1, it does not meet the stability requirement of characteristic equation in Eq. (29). It means that the spacecraft is likely to be unstable about the small disturbance torque or force at the steady-state in Fig. 2 without continuous active

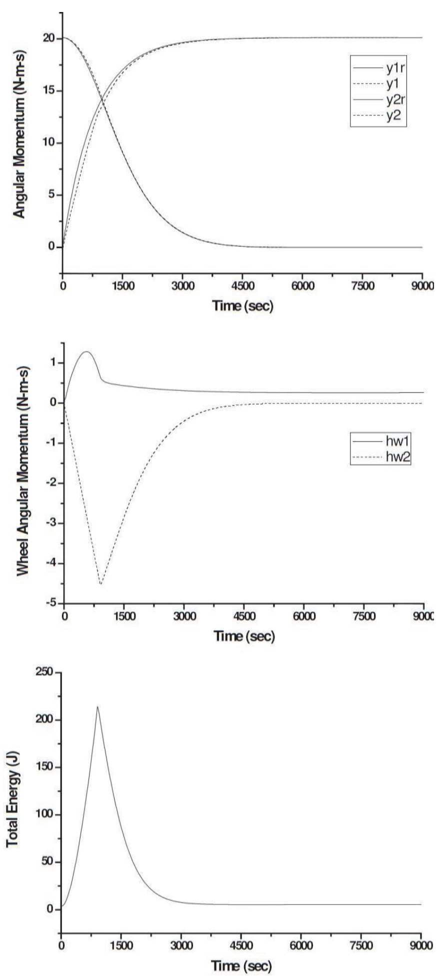

Fig. 3. Simulation results(α=2deg, β=88deg and $I_2>I_3>I_1$)

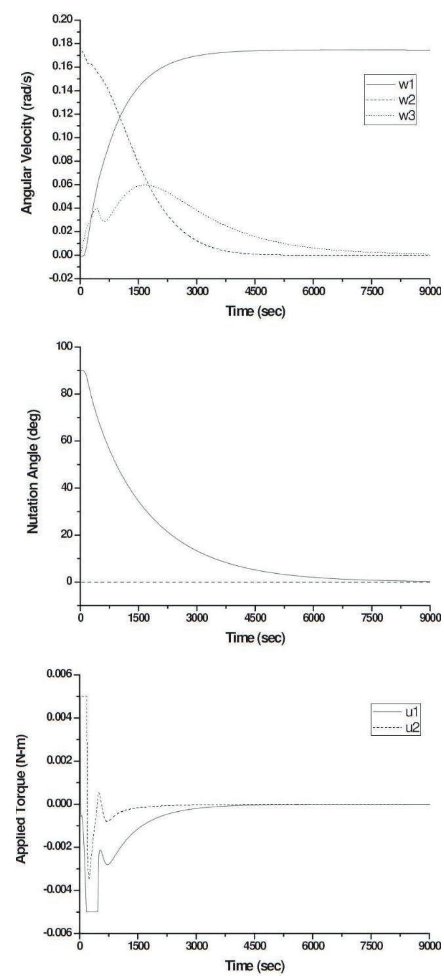

Fig. 4. Simulation results(α=2deg, β=88deg and $I_1>I_3>I_2$)

control. Moreover, two wheels are still not saturated to the maximum speed. (see Fig. 3).

Next, the suggested control strategy is utilized once again for the case of the unstable moment of inertia configuration, namely, the initial spin axis $\hat{\mathbf{b}}_2$ of the spacecraft is chosen as the minimum moment of inertia axis, whereas one wheel from two installed in the $\hat{\mathbf{b}}_1$-axis for the nominal spin is aligned along the maximum moment of inertia axis in opposition to that of the platform. Therefore, the moment of inertia data for the spacecraft model are chosen as [I_1, I_2, I_3]=[113.59, 85.12, 86.24]kgm^2. The simulation results are shown in Figs. 4-5. The control input $u_1(t)$ regardless of $u_2(t)$ has negative value in the initial state in opposition to the first simulation result, and they also gradually converge to zero after appropriate torque profile generation by feedback linearization. Therefore, the nominal spin axis $\hat{\mathbf{b}}_1$ seems to absorb initial angular momentum of the spacecraft effectively. As a result, the final nutation angle converges to a very small value, 0.6 deg, over the maneuver time with negligible residual oscillation. The wheel angular momentum $h_{w_1}(t)$ becomes finally negative value larger than 1 Nms to satisfy the constant total angular momentum requirement. Therefore, the total kinetic energy level maintains the first maximum value 225 J through the simulation time in opposition to the first simulation result. Even though the simulation result satisfies the open loop stability conditions in condition 1 of Table 1, it does not meet the stability requirement of characteristic equation in Eq. (29). It is also noted that the spacecraft is likely to be unstable about the small disturbance torque or force at the steady-state in Fig. 4 without continuous active control.

Finally, Figs. 6-7 present the simulation results under the existence of rotor misalignment in both wheels, namely, the alignment angles of two wheels are α=2deg and β=88deg, respectively. The proposed control strategy produces two proper tracking control torque commands $u_1(t)$ and $u_2(t)$ to execute a successful momentum transfer maneuver. As a result, the final nutation angle error converges to a very small value, 0.6 deg, over the maneuver time. Furthermore, it is also guaranteed by conducting many simulations with

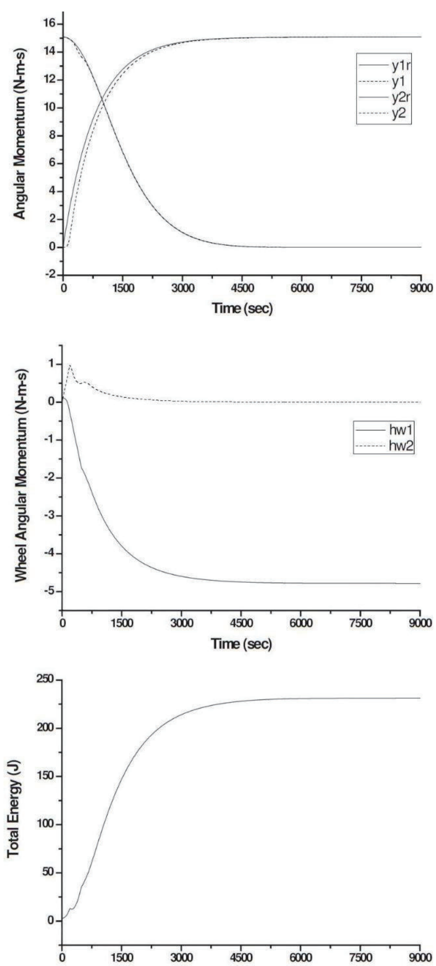

Fig. 5. Simulation results(α=2deg, β=88deg and I_1>I_3>I_2)

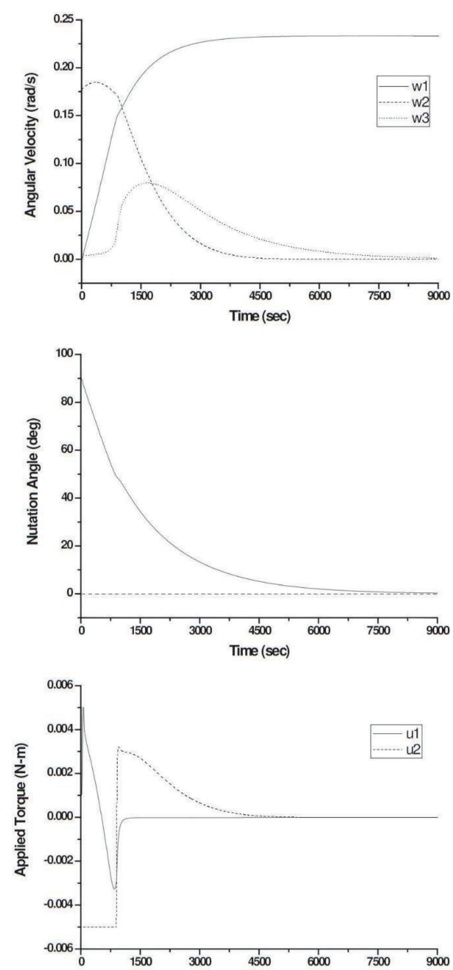

Fig. 6. Simulation results(α=2deg, β=88deg and I_2>I_3>I_1)

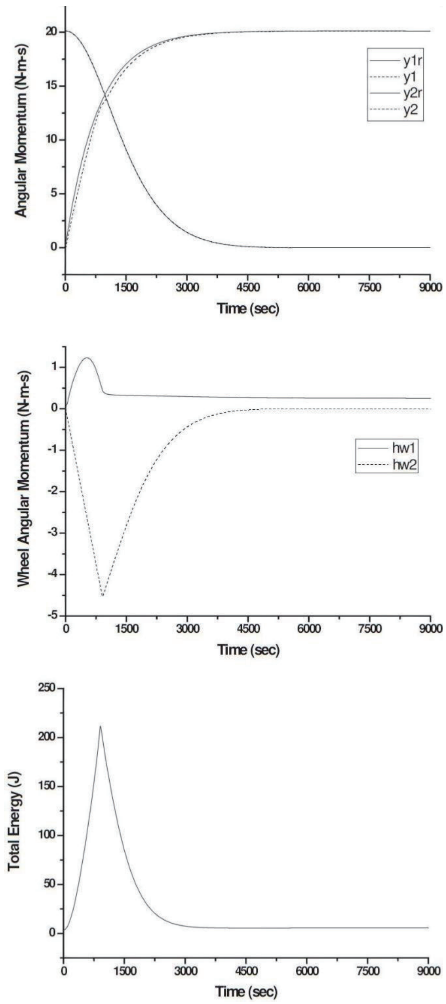

Fig. 7. Simulation results(α=2deg, β=88deg and $I_2>I_3>I_1$)

various small misalignment initial errors that the nutation angle error is converged to a small error bound.

6. Conclusions

In this paper, the momentum transfer control for the nutation angle regulation was proposed for the spacecraft with two momentum wheels. The feedback linearization technique was used for the two wheels to make orthogonal to the principal axes and then the asymptotic stability of the system was proven .The proposed control law was able to make the nutation angle be in a very small bound. In this paper, it is noted that the added wheel does not contribute to the initial attitude acquisition maneuver once it is installed in the initial spin axis. However, it is proven that the added wheel overcomes the rotor misalignment problem effectively by the proposed control law. Note that most of the shapes in the class of cube satellites are standardized so that the off-diagonal term of MOI is almost zero. Consequently, the suggested control system which is based on the critical assumptions can be applicable to the generic cube satellites directly without much concern about the limited design conditions.

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