International Journal of Internet, Broadcasting and Communication Vol.9 No.1 1-8 (2017)
https://doi.org/10.7236/IJIBC.2017.9.1.1

IJIBC 17-1-1

Quadcopter stabilization using state feedback controller by pole placement method

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Abstract

Nowadays many articles describe the controlling models for four rotor flying vehicle. Basic approaches to the problem of these articles are mathematical expressions describing dynamics of the models of the vehicle and PID control for manipulating the object in 3 dimensional space. Design of control systems is usually started by careful consideration of its mathematical model description. We present a detailed mathematical model for a quad rotor. This paper first considers simulation of quadcopter control based on full state feedback technique with linearization in MATLAB environment and shows the results of the simulations. Finally will be shown experimental results of the state feedback control implemented in real model.

Keywords: *brushless motor, rotor speed, full state feedback control, gain matrix.*

1. Introduction

During last decade with development of small size and high performance microcontrollers were developed many types of unmanned flying vehicles with multiple rotors. Among them a quadcopter takes important role due to its optimality, wide range of application [2]. Quadcopter is an air vehicle with four rotors uniformly located along a circle as shown in Figure1. The machine mass is concentrated in the center of the circle. Each motor produces some force acting on the vehicle as lifting thrust. From Figure 1 seen that rotational direction of two rotors located diagonally is clockwise and other two's direction is counterclockwise. Rotating direction of the rotors not changed in time but their rotating velocity changed according to the movements in 3D space [1- 5]. Direction of movement of the quadcopter controlled by speeds of four rotors which are the control inputs for the model. Several papers describing mathematical models of quadcopter based on state space approach and must be noted that in each of the articles have been used PID feedback control. We will focus only on final state equations of the systems because all equations related to dynamics of quadcopter were derived in below considered papers.

Manuscript Received: Dec. 2, 2016 / Revised: Dec. 18, 2016 / Accepted: Jan. 13, 2017
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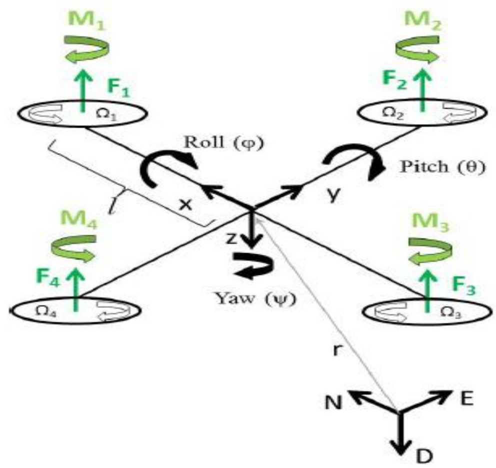

Figure 1. Forces acting on quadcopter and propeller rotational directions

In contrast with the above considered articles and basing on the above information and results of the experiments we set a goal to derive simplified linear dynamic equations for a quadcopter in the state space form with state feedback. As known, one of the main advantages of the state space method is modeling of multiple-input and multiple-output control system.

2. State feedback control

The motion of the quadrotor can be divided into two subsystems which include rotational subsystem (roll, pitch and yaw) and translational subsystem (altitude and x and y position). The rotational equations of motion are derived in the body frame using the Newton-Euler method with the following general formalism.

$$J\dot{\omega} + \omega \times J\omega + M_g = M_b \qquad (1)$$

J-quadrotor's diagonal inertia matrix, ω- angular body rates, M_g-gyroscopic moments due to rotors' inertia, M_b-moments acting on the quadrotor in the body frame.

Total moments acting on the quadrotor becomes.

$$M_b = \begin{vmatrix} lU_2 \\ lU_3 \\ lU_4 \end{vmatrix} = \begin{bmatrix} lK_f(\omega_1^2 - \omega_3^2) \\ lK_f(\omega_2^2 - \omega_4^2) \\ K_m(\omega_1^2 - \omega_2^2 + \omega_3^2 + \omega_4^2) \end{bmatrix} = \begin{bmatrix} l(F_1 - F_3) \\ l((F_2 - F_4) \\ K_z(F_1 + F_3 - F_2 - F_4) \end{bmatrix} \qquad (2)$$

Where K_f and K_M are the aerodynamic force and moment constants respectively and ω_i is the angular velocity of rotor i. Each rotor causes an upwards thrust force F_i and generates a moment M_i with direction opposite to the direction of rotation of the corresponding rotor i.

Defining the state vector of the quadrotor to be, which is mapped to the degrees of freedom of the quadrotor in the following manner

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6 \quad x_7 \quad x_8 \quad x_9 \quad x_{10} \quad x_{11} \quad x_{12}]^T$$
$$X = [\varphi \quad \dot{\varphi} \quad \theta \quad \dot{\theta} \quad \psi \quad \dot{\psi} \quad z \quad \dot{z} \quad y \quad \dot{y} \quad x \quad \dot{x}]^T$$

The rotational equation of motion can be derived

$$\begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}\begin{bmatrix} \ddot{\varphi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix}\begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ J_r\omega_r \end{bmatrix} = \begin{vmatrix} lU_2 \\ lU_3 \\ lU_4 \end{vmatrix} \tag{3}$$

Decomposing the full state space equations, (3), would be derived for the open loop system.

$$\begin{gathered} \dot{x}_1 = x_2 \\ \dot{x}_2 = lk(\omega_4^2 - \omega_2^2)/I_x \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = lk(\omega_3^2 - \omega_1^2)/I_y \\ \dot{x}_5 = x_6 \\ \dot{x}_6 = b(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)/I_z \\ \dot{x}_7 = x_8 \\ \dot{x}_8 = A_x x_1 \\ \dot{x}_9 = x_{10} \\ \dot{x}_{10} = A_y x_3 \\ \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = A_x x_5 + g - \frac{1}{m}(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \end{gathered} \tag{4}$$

where $\omega_1^2,\ \omega_2^2,\ \omega_3^2,\ \omega_4^2$ are square of angular velocity of each rotor which controls the corresponding rotor speed, l, k, b, m, g are radius of quadcopter circle, constants, mass of the object and gravity constant respectively and $I_x,\ I_y,\ I_z$ are inertia for each axis. $[\dot{x}] = [\dot{x}_1, \dot{x}_2 \ldots \dot{x}_{12}]^T$- the first derivative of state variables, $[x] = [x_1, x_2, \ \ldots, x_{12}]^T$ -the internal states. As was described before, control and tracking of quadcopter carried out using PID control method, but in this article we tried to consider usage of full state feedback control and its simulation in MATLAB and implementation. As known, the reference input of the closed loop system is the control input for tracking and control for the quadrotor system. Through the reference input will be dictated 3D motion of sixth degree of freedom in space. Figure 2 illustrates block scheme of the full state feedback.

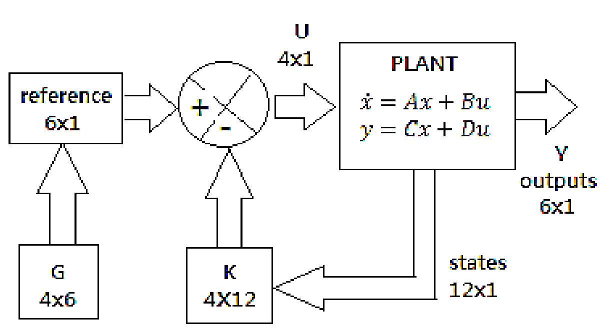

Figure 2. Block scheme of the state feedback

The control input, u, consists of 4 angular velocities, $[\omega]$, of rotors and it is formed by the sum of reference input multiplied by the gain matrix G and internal states multiplied by the feedback matrix K.

Feedback gain matrix K would be defined after pole placement procedure of the characteristic equation of the system. The procedure of pole placement set such that the rise time, overshoot and settling time are set in the same manner as in [7, 8, 9]. We present, intuitively, the settling time for real model of quadcopter about 2 seconds and overshoot about no more 4%. Then using the pole dominant criterion we can choose the poles which are one of many variants as -2+1.96j; -2-1.96j; two -2.0; two -4.0; two -5.0; two -8.0; and two -3.0. For the system the state feedback control references are pitch, roll angles equal to zero and the height of the copter is h. Using state feedback law the relationship between plant input, u, and reference, *ref*, is written as (5).

$$u = G \cdot ref - Kx \tag{5}$$

Assuming that all state variables are readable (measureable) and using state feedback law, (5), and allowing the reference input as ref we get the state feedback control system with reference input as shown in Fig 2. The system on the figure demonstrates quadcopter structure with input $u = [\omega_1^2\ \omega_2^2\ \omega_3^2\ \omega_4^2]^T$ and reference $ref = [\psi_r\ \theta_r\ \varphi_r\ x_r\ y_r\ z_r]^T$. To get full correspondence of feedback and reference input there must be some relation, G (in some literature N_x, N_u), that is expressed as follows.

$$G = -B^{-1}(A - BK)C^{-1} \tag{6}$$

Inserting (6) into standard state space equations would be created closed loop feedback equations of the control system that is follows as (7).

$$\begin{gathered} \dot{x} = (A - BK)x + BGu \\ y = Cx \end{gathered} \tag{7}$$

It is necessary to discretize the model as if it was a real model based on digital microcontroller system. It is well known, that discretized version of continuous time system is derived through the Laplace transform and Z transform and using the corresponding MATLAB functions equations (5), (6), (7) are converted into discrete time form of (8).

$$\begin{gathered} P_d = \exp(T_s[p]) \\ K_d = \mathrm{c2d(K)} \\ G_d = B_d^{-1}(A_d - B_d K_d)C_d^{-1} \\ x[n+1] = (A_d - B_d K_d)x[n] + B_d G_d u[n] \\ y[n] = C_d x[n] \end{gathered} \tag{8}$$

where P_d is poles on Z plan, A_d, B_d, C_d, K_d, G_d are discrete time matrices and *x[n], y[n], u[n]* are values of the states, outputs and control inputs at *n-th* time step correspondingly.

3. Simulation result

The simulation has been done in two steps. First is open loop control and second is closed loop control with reference. In the simulation some constant values as drag coefficients A_x= 0.25, A_y = 0.25, A_z = 0.25 and inertia I_m = 3.357e − 5, b = 1.14e − 7, k = 2.98e − 6 are taken from [2]. Values of mass of the copter, m, distance from center of the copter to any of the rotor center, l, are could be chosen randomly. The sample time step t_s=0.01 sec and duration of simulation time is within 10 seconds. Reference input was designed such that starting at position (0, 0, 0) the copter launches up and moves one circle of motion with radius equals to 8 meter and remains at position (2.6, -6.3, 80). This position To move as described here the reference is *ref*=[zeros(size(t)) zeros(1,length(t)) zero(1,length(t)) 8*sin(2*pi*t/T) 8*cos(2*pi*t/T) -80*ones(1,length(t))]

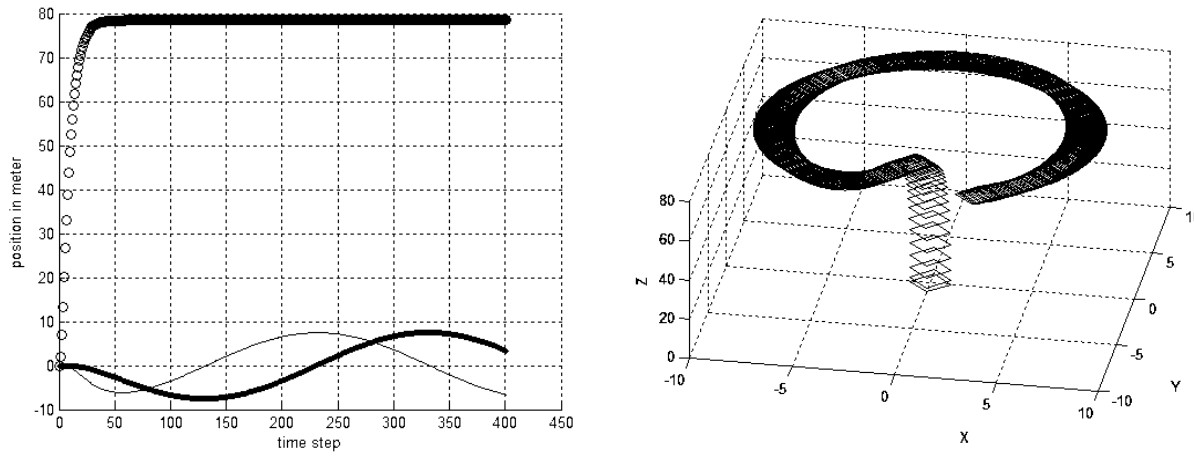

Figure 3. Timing diagram of X, Y, Z position, Trajectory of 3D tracking

4. Implementation

Our mission is use of state feedback control by pole placement method and make the quadcopter to hover in the midair. For this reason, we conducted some experiments on applied voltage versus speed ratio and lifting force to determine the response time of BLDC (Fig. 4). The experiment is done using propeller Detrum BM2810CD-KV1000 BLDC and Detrum ESC 30A ESC.

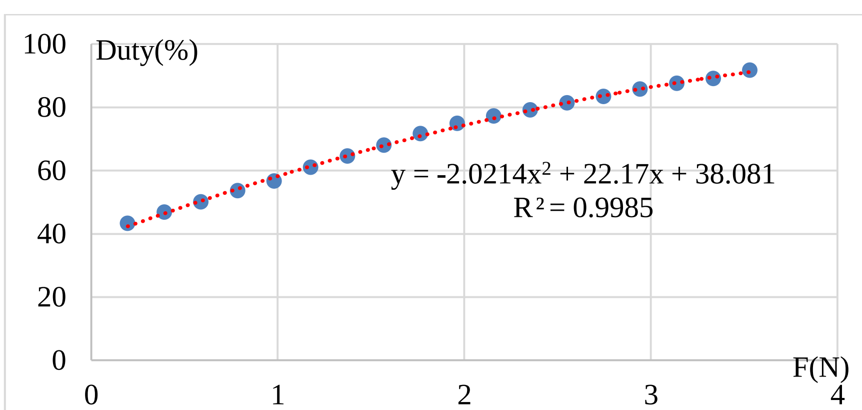

Figure 4. Lifting force and duty cycle of PWM relation

Overall, equation (4) of motion to be used in our six degree of freedom simulator. However, they are not appropriate for control design. They are too complicated to gain significant insight into the motion. Because the equations of the system under control is highly nonlinear it is necessary to linearize. As seen from the above equations for the quadcopter that the state matrix elements consist of sine, cosine functions which mean the mathematical model is nonlinear [4]. It is known that at small angles $(approximately\ -25^{\circ} < \theta < 25°)$ the functions $sin(\theta) \approx \theta$ and $cos(\theta) \approx 1$. It follows that the rotation matrix R equals to the identity matrix. Considering above situation and summarizing all information described in the previous section the dynamic equations for the quadcopter hover mode can be written in the following state space form.

$$\begin{gathered}
\dot{x_1} = \dot{\varphi} = x_2 \\
\dot{x_2} = \ddot{\varphi} = \frac{l}{I_x} U_2 = \frac{l}{I_x}(F_1 - F_3) \\
\dot{x_3} = \dot{\theta} = x_4 \\
\dot{x_4} = \ddot{\theta} = \frac{l}{I_y} U_3 = \frac{l}{I_y}(F_2 - F_4) \\
\dot{x_5} = \dot{\psi} = x_6 \\
\dot{x_6} = \ddot{\psi} = \frac{l}{I_z} U_4 = \frac{K_z}{I_y}(F_1 + F_3 - F_2 - F_4)
\end{gathered} \tag{8}$$

The matrix form of (8) is (9).

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ \frac{l}{I_x} & 0 & -\frac{l}{I_x} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{l}{I_y} & 0 & -\frac{l}{I_y} \\ 0 & 0 & 0 & 0 \\ \frac{K_z}{I_z} & -\frac{K_z}{I_z} & \frac{K_z}{I_z} & -\frac{K_z}{I_z} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} \tag{9}$$

From Fig. 4 we determine force produced by propeller. Actual experimental environment for quadcopter is shown in Figure 5.

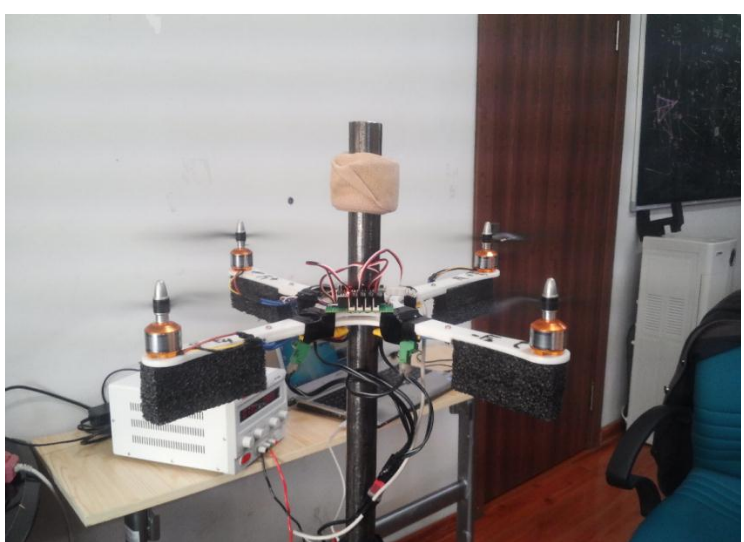

Figure 5. The experimental environment of the quadcopter system.

Below shown constants and values of technical parameters of the quadrotor used for our simulation and experiments.

$g = 9.81$	$[\frac{m}{s^2}]$ gravity constant
$m = 0.09$	$[kg]$ mass of motor
$M = 0.8 - (4m)$	$[kg]$ mass of body
$I_x = \frac{1}{2}M(a^2 + b_2) + 2l^2m$	$[kg\ m^2]$ inertia in x axes of disc body frame
$I_y = \frac{1}{2}M(a^2 + b_2) + 2l^2m$	$[kg\ m^2]$ inertia in y axes of disc body frame
$I_z = \frac{1}{2}M(a^2 + b_2) + 4l^2m$	$[kg\ m^2]$ inertia in z axes of disc body frame
% Force produced by propeller	$[N]$ $F = K_t\omega^2$
$C_t = 0.1154$	thrust coeff
$q = 1.225$	$[\frac{kg}{m^3}]$ air density coeff
$D_p = 0.256$	$[m]$ diameter of propeller
$\pi = 3.14$	
$K_t = C_t\frac{\rho D^4}{4\pi^2}$	$[kg\ m^2 rad^2]$ Force coeff.
% Moment produced by propeller	$[Nm]$ $M = K_m\omega^2$
$C_p = 0.0743$	drag coeff.
$K_m = C_t\frac{\rho D^5}{8\pi^2}$	$[kg\ m\ rad^2]$ moment coeff
$K_z = \frac{K_m}{K_t}$	$[m]$ z axes moment level lengh

Figure 6 shows angular values of roll, pitch and yaw read from MEMS sensors during the experiments in the hovering mode.

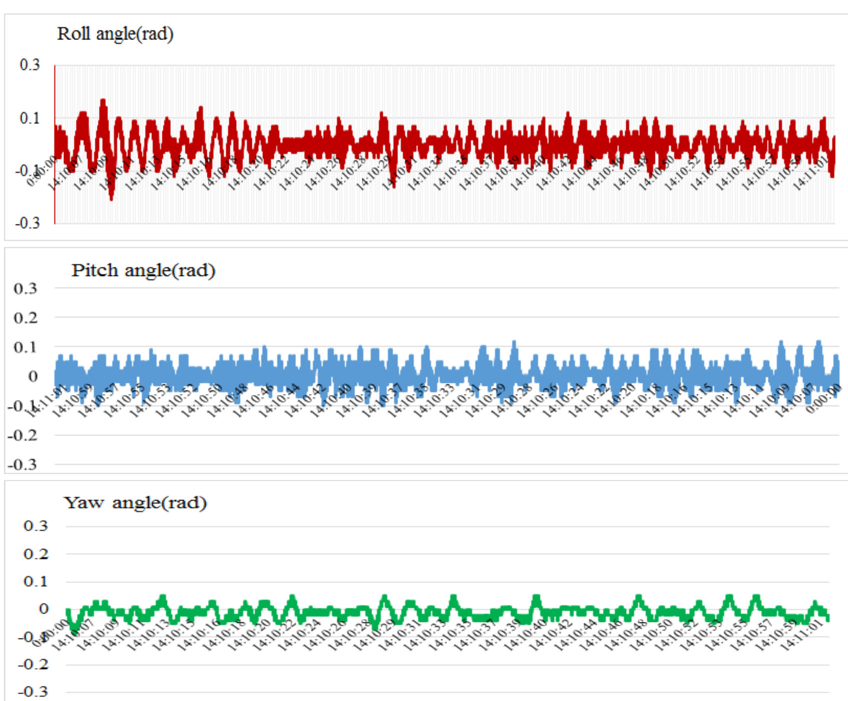

Figure 6. The experimental result of the quadcopter system.

5. Conclusion

We demonstrated the simulation of the model based on full state feedback control in MATLAB environments and it's implementation in real model. In the simulation was used simplified mathematical model considering such simplification as linearization, zeroing of some nonlinear functions. The simplified model was simulated in open and closed loop modes. Although the model was highly simplified the simulation results show relatively realistic dynamics. During the simulation there was chosen reference inputs as vertical launch and circular motion. Simulation results are shown graphically. Also our experimental results show good stabilization of the quadcopter in the hovering mode. Further work will be focused on 3D movement of the air vehicle by tracking of the reference input.

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