

Consensus of Leader-Follower Multi-Vehicle System

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Abstract

According to the characteristics of salvo attack for the multiple flight vehicles (MFV), the design of cooperative guidance law can be converted into the consensus problem of multi-vehicle system through the concept of multi-agent cooperative control. The flight vehicles can be divided into leader and followers depending on different functions, and the flight conditions of leader are independent of the ones of followers. The consensus problem of leader-follower multi-vehicle system is researched by graph theory, and the consensus protocol is also presented. Meanwhile, the finite time guidance law is designed for the flight vehicles via the finite time control method, and the system stability is also analyzed. Whereby, the guidance law can guarantee the line of sight (LOS) angular rates converge to zero in finite time, and hence the cooperative attack of the MFV can be realized. The effectiveness of the designed cooperative guidance method is validated through the simulation with a stationary target and a moving target, respectively.

Key words: Leader-follower, Multiple flight vehicles, Consensus, Cooperative guidance

1. Introduction

With the quick development of the aerospace technology, conventional combat strategy using a single flight vehicle cannot satisfy the demands of combat mission in complicated combat fields. In order to improve the hit accuracy and enlarge the defense area, cooperative combat system using multiple flight vehicles (MFV) has been a research focus in recent years. Cooperative combat system can involve in a variety of fields, including cooperative path planning [1], formation control [2], cooperative guidance [3], multiple sensor and data fusion [4], etc. Cooperative guidance is the key technology in cooperative attack, which plays an important role in the cooperative combat.

Salvo attack is a typical combat mode in the cooperative combat, requiring MFV from different positions and directions to attack the target simultaneously [5]. Salvo attack can not only greatly improve the hit probability, but also enhance the effectiveness of attacks. Cooperative guidance for MFV has been getting increasing attentions in recent years. An impact time control guidance law (ITCG) was proposed, so

MFV can reach the target at the specified time simultaneously [6]. Wei X. built a three dimensional guidance model using vector calculation and designed a distributed cooperative guidance strategy [7]. In [8], the cooperative control problem of multi-missile systems was addressed, and a two-stage control strategy was proposed, aiming at simultaneous attack from a group of missiles at a stationary target. The coordinate variable is applied to MFV salvo attack in [7], in which a universal two-layer guidance structure was proposed, and the coordinate strategy can be realized by centralized and distributed coordinate control respectively.

In salvo attack, all of the flight vehicles should be guided and controlled to attack the target simultaneously. Salvo attack is similar to multiple agent cooperative control, in networks of agents (or dynamic systems), “consensus” means to reach an agreement regarding a certain quantity of interest that depends on the state of all agents. A “consensus algorithm” (or protocol) is an interaction rule that specifies the information exchange between an agent and all of its neighbors in the network. The consensus theory has been applied to several fields, including formation control for

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satellites, cooperative control for UAVs, multiple robots system distribution control, etc.[10]-[12]. Besides, an interesting area in multi-agent systems is the consensus of a group of agents with a leader, where the motion of the leader is independent in the group and the agents are necessary to follow its leader. This type of configuration can be found in large number of biological systems, which helps in designing the multi-agent systems. Leader-following consensus theory has been widely applied in the fields of distribution tracking [13][14], flocking algorithms[15],etc. Therefore, the research results of multi-agent cooperative control will be great significance for consensus research of leader-follower multi-vehicle system.

In recent years, finite time control has been fully developed and used in combination with multi-agent cooperative control. Two non-continuous finite time consensus protocols for continuous system were presented in [17]. On this basis, a general form of finite time consensus protocol was proposed in[18], in which consensus protocol can be in the form of continuous state feedback according to the selected parameter. The asymptotical stable control technology is frequently used in previous studies on cooperative guidance, but in salvo attack, the requirement for controlling the attack time is higher. Therefore, how to make the state of leader-follower MFV system converge in finite time is worth investigating.

Motivated by the problems discussed above, a novel cooperative guidance strategy is proposed for leader-follower multi-vehicle system. The consensus problem is investigated through graph theory, and consensus protocol is presented. To guarantee the LOS angular stability, the finite time guidance law is designed for the flight vehicle via finite time control method. Simulation results are provided to demonstrate the effectiveness of the proposed approach.

The rest of the paper is organized as follows: the basic knowledge on graph theory and finite time control is given in Section 2. Section 3-4 describes the kinematics of MFV and formulation of cooperative guidance. In Section 5, a finite time consensus protocol is proposed to ensure that MFV arrive at the target simultaneously, and a finite time guidance law is designed to guarantee that LOS angular rates converge to zero in finite time. Simulation examples and analysis are shown in Section 6. Results and conclusions are summarized in Section 7.

2. Preliminaries

2.1 Graph Theory

Graph theory has been widely applied to the consensus

research of multi-agent system. The weighted graph $G=(v, \varepsilon, A)$ is used to represent the communication topology, $v=\{1, 2, 3, \dots, n\}$ is the vertex set, and $\varepsilon \subset v \times v = \{(i, j): i, j \in v\}$ is the edge set. $A=[a_{ij}] \in R^{n \times n}$, $i, j=1, 2, 3, \dots, n$, is the weighted adjacency matrix, where a_{ij} is the weight of edge from i to j . Depending on the direction of edge, the graph can be divided into directed graph and undirected graph. The graph is called an undirected graph if and only if $(i, j) \in \varepsilon \Leftrightarrow (j, i) \in \varepsilon$, and $a_{ij}=a_{ji}>0$. Otherwise, it is called a directed graph. Vertex j is the neighbor of vertex i , the neighbor set of vertex is denoted by $N_i = \{j \in v: (i, j) \in \varepsilon, j \neq i\}$.

2.2 Finite Time Control Theory

In the traditional control method, when the closed-loop system satisfies *Lipschitz* continuous characteristic, usually, only guarantees that the guidance system state tends to zero when the time approaches infinity. Therefore, it is incomplete in theoretical point of view [10]. In the last stage of guidance, the state of flight vehicles changes rapidly such that the time-optimal feature should be guaranteed in the guidance systems. Therefore, system converged in finite time is attributed to time-optimal control.

The finite time stability is defined as state of the system reaching equilibrium in a finite time, and then stabilized at the equilibrium point. Because fractional power exists in the finite time controller, the finite time controller has better robustness and disturbance rejection performance than non-finite-time control systems. Therefore, finite time control has been a research focus in recent years [19-21].

Considering the following nonlinear system [10],

$$\dot{x} = f(x), \quad f(0) = 0, \quad x \in R^n, \quad (1)$$

where $f(x): U \rightarrow R^n$ is continuous in an open neighborhood U of the origin.

Definition 1. Considering the system (1), the origin $x=0$ is given as a finite-time-stable equilibrium of (1) if and only if the system is of finite-time convergence and Lyapunov stability.

Finite-time convergence: for $\forall x_0 \in U_0 \subset R^n$, there exists a continuous function $T(x): U_0/\{0\} \rightarrow (0, +\infty)$, solution $x(t, x_0)$ of system (1) satisfies: $x(t, x_0) \in U_0/\{0\}$ and $\lim_{t \rightarrow T(x_0)} x(t, x_0) = 0$ for all $t \in [0, T(x_0))$; and $x(t, x_0) = 0$ for $t > T(x_0)$. The origin is given as a globally finite-time-stable equilibrium if it is a finite-time-stable equilibrium with $U=U_0=R^n$.

Lemma 1 [22]. Considering the system (1), supposing there exists a continuous function $V(x): \hat{U} \subset U$ such that the following conditions hold:

(i) $V(x)$ is positive definite.

(ii) There exist real numbers $\eta > 0$ and $0 < c < 1$, and an open neighborhood $\hat{U} \subset U$ of the origin such that

$$\dot{V} + \eta V^c \leq 0.$$

Then the origin is a finite-time-stable equilibrium of system (1). If $\hat{U} = U = R^n$ and $V(x)$ is proper, then the origin is a globally finite-time-stable equilibrium.

2.3 Finite Time Consensus of Leader-follower multi-agent system

Considering a leader-follower multi-agent system composed of one leader and multiple followers, the dynamics of the followers is

$$\dot{x}_i(t) = u_i(t), i = 1, 2, \dots, n, \quad (2)$$

where, $x_i(t) \in R$ is the state of the i th follower, $u_i(t) \in R$ is the control input. The state of the leader is in dependent of the ones of the followers.

Definition 2. The leader-follower multi-agent system (2) is known to achieve the finite-time consensus, if for any initial states, there is a constant $T_0 \in [0, +\infty)$, such that

$$\lim_{t \rightarrow T_0} \|x_i(t) - x_0(t)\| = 0, \quad (3)$$

and

$$x_i(t) = x_0(t), \forall t \geq T_0, i \in I. \quad (4)$$

Lemma 2 [23]. Let $x(t)$ be a solution of $\dot{x} = f(x)$, $x(0) = x_0 \in R^n$, where $f: U \rightarrow R^n$ is continuous, U is an open subset of R^n , and $V: U \rightarrow R$ is a local Lipschitz function such that $D^+V(x(t)) \leq 0$, where D^+ denotes the upper Dini derivative. Then with denoting the positive limit set as $\Theta^+(x_0)$, $\Theta^+(x_0) \cap U$ is contained in the union of all the solutions that remain in $S = \{x \in U : D^+V(x) = 0\}$.

Next, the homogeneity with dilation is given for the finite time convergence analysis.

Function $V(x)$ is homogeneous of degree $\kappa \in R$ with the dilation $r = (r_1, r_2, \dots, r_n)$, $r_i > 0$, $i = 1, 2, \dots, n$, if for any $\varepsilon > 0$

$$V(\varepsilon^{r_1} x_1, \varepsilon^{r_2} x_2, \dots, \varepsilon^{r_n} x_n) = \varepsilon^\kappa V(x). \quad (5)$$

If $r_1 = r_2 = \dots = r_n = 1$, the dilation is trivial.

For the following n dimensional system

$$\dot{x}(t) = f(x), x = (x_1, x_2, \dots, x_n)^T \in R^n, \quad (6)$$

a continuous vector field $f(x) = (f_1(x), f_2(x), \dots, f_n(x))^T$ is homogeneous of degree $\kappa \in R$ with the dilation $r = (r_1, r_2, \dots, r_n)$, for any $\varepsilon > 0$,

$$f_i(\varepsilon^{r_1} x_1, \varepsilon^{r_2} x_2, \dots, \varepsilon^{r_n} x_n) = \varepsilon^{\kappa + r_i} f_i(x), i = 1, 2, \dots, n. \quad (7)$$

System (6) is called homogeneous if its vector field is

homogeneous. Furthermore,

$$\dot{x} = f(x) + \tilde{f}(x), \tilde{f}(0) = 0, x \in R^n, \quad (8)$$

is said to be locally homogeneous of degree $\kappa \in R$ with respect to the dilation $r = (r_1, r_2, \dots, r_n)$, if $f(x)$ is homogeneous of degree $\kappa \in R$ with respect to the dilation $r = (r_1, r_2, \dots, r_n)$ and \tilde{f} is a continuous vector field satisfying

$$\lim_{\varepsilon \rightarrow 0} \frac{\tilde{f}_i(\varepsilon^{r_1} x_1, \varepsilon^{r_2} x_2, \dots, \varepsilon^{r_n} x_n)}{\varepsilon^{\kappa + r_i}} = 0, \forall x \neq 0, i \in I. \quad (9)$$

For convenience, let $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in R^n$, and $\text{sig}(x)^\alpha = |x|^\alpha \text{sign}(x)$, where $\text{sign}(\cdot)$ denotes the sign function, and $|x|$ denotes the absolute value of the real number x .

$$\text{sign}(x) = \begin{cases} 1, & x > 0; \\ 0, & x = 0; \\ -1, & x < 0. \end{cases}$$

Lemma 3 [24]. Provided that system (2) is homogeneous of degree $\kappa \in R$ with the dilation (r_1, r_2, \dots, r_n) , the function $f(x)$ is continuous and $x=0$ is asymptotically stable. If the homogeneity degree $\kappa < 0$, then the equilibrium of system (2) is finite time stable. Moreover, if (9) holds, then the equilibrium of system (8) is locally finite time stable.

3. Multiple Flight Vehicles (MFV) Kinematics

Considering the situation where MFV attack against a target, the planar engagement geometry is shown in Fig. 1. X-O-Y is the inertial reference coordinate system, supposing there are one leader and multiple followers, M_l represents the leader, and M_i represents the i th follower, T represents the target. Subscript l, i, t represent the state of the leader, i th flight vehicle, and target respectively. V, A, θ, λ_D and R denote the speed, normal acceleration command, heading angle, LOS angle, and the distance between flight vehicle and target, respectively.

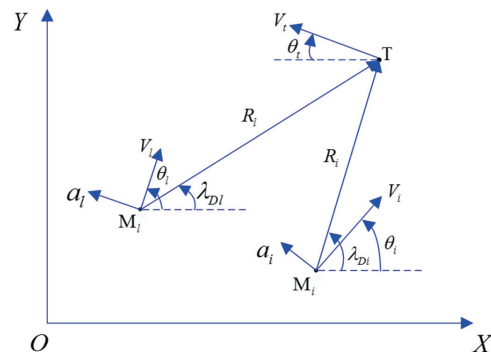


Fig. 1. Planar engagement geometry.

According to the relative kinematics shown in Fig.1, the relative kinematic equations of the leader are given as follows:

$$\begin{cases} \dot{R}_l = V_t \cos(\lambda_{Dl} - \theta_l) - V_l \cos(\lambda_{Dl} - \theta_l) \\ R_l \dot{\lambda}_{Dl} = -V_t \sin(\lambda_{Dl} - \theta_l) + V_l \sin(\lambda_{Dl} - \theta_l) \end{cases} \quad (10)$$

In order to simplify the derivation, let $V_{Rl} = \dot{R}_l$, $V_{\lambda l} = R_l \dot{\lambda}_{Dl}$, where V_{Rl} denotes relative velocity component along the LOS, $V_{\lambda l}$ denotes relative velocity component normal to the LOS.

Taking the derivative of the relative kinematics equation of the leader, (10) can be transformed as

$$\begin{cases} \dot{V}_{Rl} = \frac{V_{\lambda l}^2}{R_l} + w_R - u_{Rl} \\ \dot{V}_{\lambda l} = -\frac{V_{Rl} V_{\lambda l}}{R_l} + w_{\lambda} - u_{\lambda l} \end{cases}, \quad (11)$$

where w_R and u_{Rl} represent the acceleration component along the LOS of target and leader, respectively. w_{λ} and $u_{\lambda l}$ represent the acceleration component normal to the LOS of target and leader, respectively.

Similarly, the relative kinematics equation of follower can be described as

$$\begin{cases} \dot{V}_{Ri} = \frac{V_{\lambda i}^2}{R_i} + w_R - u_{Ri} \\ \dot{V}_{\lambda i} = -\frac{V_{Ri} V_{\lambda i}}{R_i} + w_{\lambda} - u_{\lambda i} \end{cases}, \quad (12)$$

where u_{Ri} represent the acceleration component along the LOS of follower, $u_{\lambda i}$ represent the acceleration component normal to the LOS of follower.

4. Problem Formulation of Cooperative Guidance

In cooperative guidance of multiple flight vehicle, flight vehicles from different positions and directions attack the target simultaneously via salvo attack, which can not only enhance the overall combat capability, but also improve the hit probability. Therefore, salvo attack has a very important

engineering significance for high-value target attack.

Based on the requirement to complete different tasks, the flight vehicles can be divided into two classes: leader and followers. Followers accomplish their mission under command of the leader. Leader-follower strategy can reasonably distribute different types of flight vehicles, decreasing the redundant detection equipment, thus saving the cost. The effect of the flight vehicles can be maximized by collaboration work, and hence the purpose of “1+1>2” can be achieved.

“Consensus” means that the individual in a multi-agent system can reach an agreement regarding a certain quantity of interest depending on the state of all agents by communicating with neighbors. Research for consensus problem has been more detailed since multi-agent cooperative control problem has been attracting more attentions in recent years, and consensus algorithms has been widely applied in some new fields. MFV is required to attack the target simultaneously in cooperative guidance, so the states of flight vehicles should reach agreement. Therefore, cooperative guidance can attribute to the consensus problem.

Figure 2 shows two equivalent forms of consensus algorithms: (a) a network of integrator agents in which agent i receives the state x_j of its neighbor, agent j , if there is a link (i, j) connecting the two nodes; and (b) the block diagram for a network of interconnected dynamic systems all with identical transfer functions $P(s)=1/s$. The collective networked system has a diagonal transfer function and is a multiple-input multiple-output (MIMO) linear system.

Research on consensus of multi-agent system has been carried out widely, in which leader-following consensus is an important consensus problem. Leader-following consensus theory has been applied to the fields of distribution tracking, flocking algorithms, mobile sensor networks [16], etc. Compared to the cooperative control with same labors, the superiority of leader-follower strategy is that one or several leaders with central roles can collaborate with simple and low cost followers to accomplish the mission. Therefore, this strategy can not only save the cost, but also improve

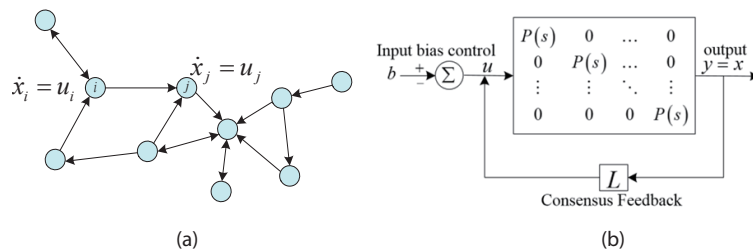


Fig. 2. Two equivalent forms of consensus algorithms

the robustness and adaptability of the whole system. In the mission of cooperative guidance with complicated and variable combat environment, the probability exposed to the enemy would increase if all the flight vehicles turn on their homing devices. Nevertheless, leader-follower strategy can decrease this probability, moreover, it can adjust the combat scheme according to the communication among flight vehicles, and hence enhance the battle performance of system.

Leader-follower structure is adopted in this paper, all the followers can receive the state information of the leader, and communicate with their neighbor. The states of followers need to be adjusted continuously in order to achieve agreement of all flight vehicles. The design of cooperative guidance law contains two key technologies: firstly, the LOS angular rates are required to converge; secondly, the arrive-time of flight vehicles should be coordinated to attack the target simultaneously.

From the relative kinematics model described above, it is worth noting that the state of leader is independent of the one of follows, and followers can keep consistent with the leader. The cooperative guidance model is presented taking follower-target relative kinematics model as an example. In the LOS coordinate system, the relative kinematics equations are described as

$$\begin{cases} \dot{R}_i = V_{Ri} \\ \dot{V}_{Ri} = V_{\lambda i}^2 / R_i + w_{Ri} - u_{Ri} \\ \dot{V}_{\lambda i} = -(V_{Ri} V_{\lambda i}) / R_i + w_{\lambda i} - u_{\lambda i} \end{cases} \quad (13)$$

In consensus problem of leader-follower multi-vehicle system, states of the flight vehicles are required to reach agreement. A new variable denoting time-to-go T_{goi} is introduced to represent the time required to arrive the target for the i th flight vehicle, and the following algorithm is adopted to estimate the time-to-go as it has a high calculation precision in the terminal guidance.

$$T_{goi} = -\frac{R_i}{V_{Ri}} \quad (14)$$

Take the derivation of (14), the dynamic of T_{goi} change can be obtained,

$$\dot{T}_{goi} = -1 + \frac{V_{\lambda i}^2}{V_{Ri}^2} - \frac{R_i}{V_{Ri}^2} u_{Ri} \quad (15)$$

where u_{Ri} is the control quantity of time-to-go changes. To realize attacking the target simultaneously, the time-to-go of flight vehicles should be controlled such that T_{goi} could reach agreement.

In the process of terminal cooperative guidance, acceleration component along the LOS is generally regarded

as zero, i.e. $w_{Ri}=0$. The second equation in (13) can be simplified as

$$\dot{V}_{Ri} = \frac{V_{\lambda i}^2}{R_i} - u_{Ri} \quad (16)$$

Assuming

$$\hat{u}_{Ri} = \frac{V_{\lambda i}^2}{V_{Ri}^2} - \frac{R_i}{V_{Ri}^2} u_{Ri} \quad (17)$$

Substituting (17) into (13) and (16) and combining with (15), the cooperative guidance model is obtained as follows,

$$\begin{cases} \dot{R}_i = V_{Ri} \\ \dot{V}_{Ri} = \frac{V_{Ri}^2}{R_i} \hat{u}_{Ri} \\ \dot{V}_{\lambda i} = -\frac{V_{Ri} V_{\lambda i}}{R_i} - u_{\lambda i} + w_{\lambda i} \\ \dot{T}_{goi} = -1 + \hat{u}_{Ri} \end{cases} \quad (18)$$

It can be seen from the cooperative guidance model that two matters need considering. On the one hand, the time-to-go T_{goi} need to reach agreement of the team by control command \hat{u}_{Ri} . Furthermore, substitute $V_{Ri} = \dot{R}_i$ and $V_{\lambda i} = R_i \dot{\lambda}_{Di}$ to (18), yielding

$$\ddot{\lambda}_{Di} = -\frac{2\dot{R}_i}{R_i} \dot{\lambda}_{Di} + \frac{w_{\lambda i}}{R_i} - \frac{u_{\lambda i}}{R_i} \quad (19)$$

Therefore, on the other hand, the LOS angular rates $\dot{\lambda}_{Di}$ should converge to zero by control command $u_{\lambda i}$. Therefore, consensus problem of leader-follower multi-vehicle system can transform to the problem of time-to-go adjusting when LOS is stable.

5. Design of cooperative guidance law for time consensus

The cooperative attack of a leader with multiple followers is investigated in this paper. The communication topology

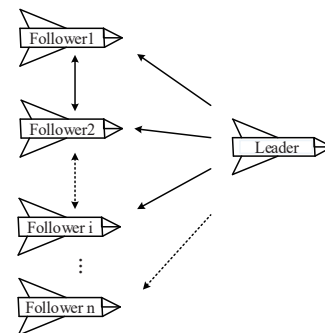


Fig. 3. Communication topology between flight vehicles

between flight vehicles is shown in Fig. 3.

The arrow represents the direction of information flows. The leader can transform its state information to followers, and followers can communicate with their neighbor, but the state of followers cannot affect the leader. In the practical environment, the leader vehicle is always equipped with complex detect device, and the follower vehicles with the simple detect device, which not only saves the cost but also lightens the weight.

Firstly, the control quantity along LOS is designed to guarantee the time-to-go of flight vehicles can reach agreement.

The expression of the new control quantity \hat{u}_{Ri} is described as (17). Commonly, $V_{\lambda i} \ll V_{Ri}$, so the first item in (17) is approximately regarded as zero, i.e. $\frac{V_{\lambda i}^2}{V_{Ri}^2} \approx 0$.

Then, (17) can be simplified as

$$\hat{u}_{Ri} = -\frac{R_i}{V_{Ri}^2} u_{Ri}. \quad (20)$$

Supposing the i th follower and the target will rendezvous at T_{fi} , and the current time is t , so

$$T_{fi} = t + T_{goi}. \quad (21)$$

Taking derivative of (21) one can get $\dot{T}_{fi} = 1 + \dot{T}_{goi}$, and combining with $\dot{T}_{goi} = -1 + \hat{u}_{Ri}$ in (18), the dynamic of time-to-go is obtained as

$$\dot{T}_{fi} = \hat{u}_{Ri}. \quad (22)$$

From (22) we know that the dynamic of time-to-go is equivalent to the first-order leader-follower multi-agent system in (2), so the multi-agent consensus theory can be applied to (22). According to the leader-follower multi-agent consensus theorem and the finite time control theory, a control protocol is designed as follows for the i th follower when communication topology is connected,

$$\hat{u}_{Ri} = \sum_{j=1}^n a_{ij}(t) \varphi_1 \left(\text{sig}(T_{fi} - T_{fj})^\alpha \right) - b_i(t) \varphi_2 \left(\text{sig}(T_{fi} - T_{go})^\alpha \right), \quad (23)$$

such that the system state T_{fi} can reach agreement in finite time.

Replace $T_{fi} - T_{fj}$ in (23) with $T_{goj} - T_{goi}$, the finite time consensus protocol of leader-follower multi-vehicle system is

$$\hat{u}_{Ri} = \sum_{j=1}^n a_{ij}(t) \varphi_1 \left(\text{sig}(T_{goj} - T_{goi})^\alpha \right) - b_i(t) \varphi_2 \left(\text{sig}(T_{goi} - T_{go})^\alpha \right), \quad (24)$$

where $a_{ij}(t)$ is the adjacency weight between follower flight vehicle i and j , $b_i(t) \in R$ is the adjacency weight between follower i and leader at time instant t , φ_1 is a continuous odd function satisfying $T_{go} \varphi(T_{go}) > 0$ ($\forall T_{go} \neq 0$), and $\varphi(T_{go}) = c_l T_{go} + o(x)$ around $T_{go}=0$ for some constant $c_l > 0$, $l=1, 2$.

Suppose that all the vehicles of the multi-vehicle system under consideration share a common state space. At any time, each vehicle updates its current state based on the information received from its neighbors. Undirected graphs are used to model communication topologies. Each vehicle is regarded as a node. Each edge (v_i, v_j) or (v_j, v_i) corresponds to an available information link between vehicle i and vehicle j . In reality, the communication topology usually switches due to the link failure or creation. To describe the variable topologies, define a piecewise constant switching function $[0, \infty) \rightarrow P = \{1, 2, \dots, m\}$, where m denotes the total number of undirected graphs with all possible communication. The communication graph at time t is denoted by \bar{G}_σ . Consider an infinite sequence of nonempty, bounded and contiguous time interval $[t_k, t_{k+1})$, $k=0, 1, 2, \dots$, such that the communication topology \bar{G}_σ switches at t_k and it does not change in time interval $[t_k, t_{k+1})$, $k=0, 1, 2, \dots$. Suppose that the communication topology \bar{G}_σ switches between topologies $(\bar{G}_1, \bar{G}_2, \dots, \bar{G}_m; \bar{G}_1, \bar{G}_2, \dots, \bar{G}_m; \dots)$ periodically in the order. Then there are m connection components with the corresponding sets of graph $\bar{G}_1, \bar{G}_2, \dots, \bar{G}_m$ sequentially in each time interval $[t_{km}, t_{(k+1)m})$, $k=0, 1, 2, \dots$.

In each time interval $[t_{km}, t_{(k+1)m})$, system (24) can be decomposed into the following m subsystems

$$\dot{T}_{goi} = \sum_{j=1}^n a_{ij}(t) \varphi_1 \left(\text{sig}(T_{goj} - T_{goi})^\alpha \right) - b_i(t) \varphi_2 \left(\text{sig}(T_{goi} - T_{go})^\alpha \right) \quad (25)$$

$$t \in [t_{km+r-1}, t_{km+r}), k=0, 1, 2, \dots, m, i \in I.$$

Let (25) can be rewritten as

$$\dot{\delta}_i = \hat{u}_i = \sum_{j=1}^n a_{ij}(t) \varphi_1 \left(\text{sig}(\delta_j - \delta_i)^\alpha \right) - b_i(t) \varphi_2 \left(\text{sig}(\delta_i)^\alpha \right), \quad (26)$$

$$t \in [t_{km+r-1}, t_{km+r}), k=0, 1, 2, \dots, m, i \in I.$$

Theorem 1. Suppose that communication topology of followers is undirected, and the system topology with leader is connected. Then under protocol (25) with $0 < \alpha < 1$, the continuous odd function φ_1 satisfies $T_{go} \varphi(T_{go}) > 0$ ($\forall T_{go} \neq 0$), and $\varphi_1(T_{go}) = c_l T_{go} + o(T_{go})$ around $T_{go}=0$ for some constant $c_l > 0$, $l=1, 2$, so the leader-following finite time consensus can be achieved.

Proof. Take the *Lyapunov* function candidate $V = \frac{1}{2} \sum_{i=1}^n \delta_i^2$. Since the connection weight $a_{ij}(t)$ and $b_i(t)$ are invariant in each time interval $[t_{km+r-1}, t_{km+r})$, along the trajectory of system (26),

$$\begin{aligned}\dot{V} &= \sum_{i=1}^n \dot{\delta}_i \dot{\delta}_i = \sum_{i=1}^n \dot{\delta}_i \left[\sum_{v_j \in N_i(t)} a_{ij}(t) \varphi_1(\text{sig}(\delta_j - \delta_i)^\alpha) - b_i(t) \varphi_2(\text{sig}(\delta_i)^\alpha) \right] \\ &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n a_{ij}(t) (\delta_i - \delta_j) \varphi_1(\text{sig}(\delta_j - \delta_i)^\alpha) - \sum_{i=1}^n b_i(t) \delta_i \varphi_2(\text{sig}(\delta_i)^\alpha) \quad (27) \\ &\leq 0, t \in [t_{km+r-1}, t_{km+r}), k=0, 1, 2, \dots, r=1, \dots, m, i \in I.\end{aligned}$$

Since the communication topology is connected in each interval $[t_{km+r-1}, t_{km+r})$, $k=0, 1, 2, \dots, r=1, \dots, m$, there is at least one $r \in \{1, 2, \dots, m\}$ such that $b_i(t) > 0$ or $j \in I$ such that $a_{ij}(t) > 0$ for any $i \in I$. Then $\dot{V} = 0$ if and only if $\delta_i = \delta_j = 0$, i.e., $T_{goi} = T_{goj} = T_{goi}$ for all $i, j \in I$, $i \neq j$. By Lemma 2, the origin of system (26) is globally asymptotically stable.

Next, we prove that system (26) is locally homogeneous of degree $\kappa=2(\alpha-1)$ with dilation $(\underbrace{2, 2, \dots, 2}_n)$.

Since the topology is invariant in each time interval and the given odd function $\varphi_l(T_{go})$ satisfies $\varphi_l(T_{go}) = c_l T_{go} + o(T_{go})$, $l=1, 2$, system (26) can be rewritten as

$$\begin{aligned}\dot{\delta}(t) &= \sum_{j=1}^n a_{ij} \left(c_1 \text{sig}(\delta_j - \delta_i)^\alpha + o(\text{sig}(\delta_j - \delta_i)^\alpha) \right) - b_i \left(c_2 \text{sig}(\delta_i)^\alpha + o(\text{sig}(\delta_i)^\alpha) \right) \\ &= \left[c_1 \sum_{j=1}^n a_{ij} \text{sig}(\delta_j - \delta_i)^\alpha - c_2 b_i \text{sig}(\delta_i)^\alpha \right] + \left[\sum_{j=1}^n a_{ij} o(\text{sig}(\delta_j - \delta_i)^\alpha) - b_i o(\text{sig}(\delta_i)^\alpha) \right] \quad (28) \\ &= \tilde{f}_i(\delta) + \bar{f}_i(\delta)\end{aligned}$$

where

$$\begin{aligned}\tilde{f}_i(\delta) &= c_1 \sum_{j=1}^n a_{ij} \text{sig}(\delta_j - \delta_i)^\alpha - c_2 b_i \text{sig}(\delta_i)^\alpha \\ \bar{f}_i(\delta) &= \sum_{j=1}^n a_{ij} o(\text{sig}(\delta_j - \delta_i)^\alpha) - b_i o(\text{sig}(\delta_i)^\alpha)\end{aligned} \quad (29)$$

When $r_1=r_2=\dots=r_n=R$, there is

$$\begin{aligned}\tilde{f}_i(\varepsilon^R \delta_1, \varepsilon^R \delta_2, \dots, \varepsilon^R \delta_n) &= c_1 \sum_{j=1}^n a_{ij} \text{sig}(\varepsilon^R \delta_j - \varepsilon^R \delta_i)^\alpha - c_2 b_i \text{sig}(\varepsilon^R \delta_i)^\alpha \\ &= \varepsilon^{\alpha R} \left[c_1 \sum_{j=1}^n a_{ij} \text{sig}(\delta_j - \delta_i)^\alpha - c_2 b_i \text{sig}(\delta_i)^\alpha \right] \quad (30)\end{aligned}$$

From $\varepsilon^{\alpha R} = \varepsilon^{R+\kappa}$, one can get $\kappa=R(\alpha-1)$. When $R=1$, $\kappa=\alpha-1$. This is the trivial dilation. When $R=2$, $\kappa=2(\alpha-1)$.

Through the above analysis, $\delta_i(t) \rightarrow 0$, $\forall i \in I$, as $t \rightarrow \infty$. Then for any $\zeta > 0$, there is a constant $t_0 > 0$, such that for all $t \geq t_0$, $|\delta_i| < \zeta$. For any $\zeta(t)$, $\zeta > 0$ is bounded for all $t \geq t_0$. Then there is

$$\begin{aligned}\lim_{\varepsilon \rightarrow 0} \frac{\tilde{f}_i(\varepsilon^R \delta_1, \varepsilon^R \delta_2, \dots, \varepsilon^R \delta_n)}{\varepsilon^{k+\eta}} &= \lim_{\varepsilon \rightarrow 0} \frac{\sum_{j=1}^n a_{ij} o(\text{sig}(\varepsilon^R \delta_j - \varepsilon^R \delta_i)^\alpha) - b_i o(\text{sig}(\varepsilon^R \delta_i)^\alpha)}{\varepsilon^{k+\eta}} \\ &= \lim_{\varepsilon \rightarrow 0} \frac{\sum_{j=1}^n a_{ij} o(\varepsilon^{\alpha R} \text{sig}(\delta_j - \delta_i)^\alpha) - b_i o(\varepsilon^{\alpha R} \text{sig}(\delta_i)^\alpha)}{\varepsilon^{k+R}} \quad (31) \\ &= 0, \forall i \in I\end{aligned}$$

Hence (28) is locally homogeneous of degree $\kappa=2(\alpha-1)$ with dilation $(\underbrace{2, 2, \dots, 2}_n)$. That is, system (26) is locally

homogeneous of degree $\kappa=2(\alpha-1) < 0$ with dilation $(\underbrace{2, 2, \dots, 2}_n)$.

Similarly, under protocol $\hat{u}_i = \tilde{u}_i + \bar{u}_i$, where

$$\begin{aligned}\tilde{u}_i &= c_1 \sum_{v_j \in N_i(t)} a_{ij}(t) \text{sig}(T_{goj} - T_{goi})^\alpha - c_2 b_i(t) \text{sig}(T_{goi} - T_{goi})^\alpha, \\ \bar{u}_i &= \sum_{v_j \in N_i(t)} a_{ij}(t) o(\text{sig}(T_{goj} - T_{goi})^\alpha) - b_i(t) o(\text{sig}(T_{goi} - T_{goi})^\alpha).\end{aligned} \quad (32)$$

System (22) is locally homogeneous of the degree $\kappa=2(\alpha-1) < 0$ with dilation $(\underbrace{2, 2, \dots, 2}_n)$.

Combining with the analysis above, one can get system (26) is globally asymptotically stable and locally homogeneous of the degree $\kappa=2(\alpha-1) < 0$ with dilation $(\underbrace{2, 2, \dots, 2}_n)$. By lemma

3, system (26) is locally finite time stable. Thus the origin of (26) is globally finite time stable, because if the equilibrium of a nonlinear system is globally asymptotically stable and locally finite time convergent, then it is globally finite time stable. Therefore, $T_{goi} - T_{gol} \rightarrow 0 (\forall i \in I)$ in finite time can be achieved. Theorem 1 is proved.

Note that in the process of terminal cooperative guidance, as the angle between the LOS direction and the axial direction of flight vehicle body is not zero, so different angles of attack can be obtained by adjusting their attitude, and the acceleration along the LOS can be provided by the aerodynamic fins. Furthermore, due to the air resistance and gravity, the acceleration along the LOS is not zero for the flight vehicle under the combined effect.

Another important technology of designing cooperative guidance law is to guarantee LOS angular rates converge to zero, resulting in hitting the target.

Design a finite time guidance law

$$u_{\lambda_i} = -N \dot{R}_i \lambda_{Di} + w_{\lambda_i} + \beta |\lambda_{Di}|^\eta \text{sgn} \lambda_{Di}, N = \text{const.} > 2, \quad (33)$$

where N is the effective navigation ratio, β and η are real numbers, $\beta > 0$ and $0 < \eta < 1$.

A theorem is described as follows for guidance law.

Theorem 2 In the process of terminal cooperative guidance, LOS angular rates converge to zero is achieved under the guidance law (33) described above. The bigger β value is, the faster the convergence rate can be. And the bigger η value is, the faster of the convergence rate can be. The reasonable value range of η is $0 \leq \eta < 1$.

Proof: substitute the finite time guidance law into the following flight vehicle-target relative kinematics equation

$$\ddot{\lambda}_D = -\frac{2\dot{R}(t)}{R(t)} \dot{\lambda}_D - \frac{1}{R(t)} u_{\lambda_i} + \frac{1}{R(t)} w_{\lambda_i}, \quad (34)$$

so

$$\ddot{\lambda}_D = \frac{(N-2)\dot{R}(t)}{R(t)} \dot{\lambda}_D - \frac{\beta |\dot{\lambda}_D|^\eta \text{sgn} \dot{\lambda}_D}{R(t)}, \quad (35)$$

and

$$\begin{aligned} \dot{\lambda}_d \left[\ddot{\lambda}_d + \frac{\beta |\dot{\lambda}_d|^\eta \operatorname{sgn}(\dot{\lambda}_d)}{R(t)} \right] &= \frac{(N-2)\dot{R}(t)}{R(t)} (\dot{\lambda}_d)^2 \\ &= -\frac{(N-2)|\dot{R}(t)|}{R(t)} (\dot{\lambda}_d)^2 \leq 0 \end{aligned} \quad (36)$$

Select a smooth and positive definite function

$$V_1 = (\dot{\lambda}_d)^2.$$

In the process of terminal cooperative guidance, there are

$$\dot{R}(t) < 0, 0 < R(t) < R(0), \quad \forall t > 0. \quad (37)$$

Taking the derivative of V_1 , and combining with (36) and (37), one can get

$$\dot{V}_1 \leq \frac{-2\beta}{R(t)} V_1^{\frac{1+\eta}{2}} < \frac{-2\beta}{R(0)} V_1^{\frac{1+\eta}{2}}, \quad \forall t > 0. \quad (38)$$

It can be noticed from lemma 1 that LOS angular rate $\dot{\lambda}_d$ could converge to zero in finite time t_r , and t_r satisfies

$$t_r < \frac{|\dot{\lambda}_d(0)|^{1-\eta} R(0)}{\beta(1-\eta)}. \quad (39)$$

One can find that the bigger β value is, the faster convergence rate can be obtained. Obviously, if the guidance system is controllable, initial LOS angular rate satisfies

$$|x(0)| \ll 1(\text{rad/s}). \quad (40)$$

So the bigger η value is, the faster convergence rate can be obtained. **Theorem 2** is proved.

Table 1. Initial Parameters of Flight Vehicles

Flight vehicle	position/m	head angle/(°)	velocity/(m·s ⁻¹)
Leader	(-10504,-12000)	35	300
Follower1	(-12100,-10100)	40	300
Follower2	(-13000,-8100)	55	300

Table 2. Simulation results of Instance 1-4

Maneuver type	Vehicle	Impact time	Miss distance
Static	Leader	46.27s	0.0710m
	Follower 1	46.27s	0.0675m
	Follower 2	46.27s	0.0623m
Slow speed	Leader	43.10s	0.0120m
	Follower1	43.09s	0.2750m
	Follower 2	43.09s	0.0322m
High speed	Leader	40.52s	0.0313m
	Follower1	40.48s	0.0712m
	Follower 2	40.45s	0.0334m
Sinusoid	Leader	40.51s	0.0137m
	Follower1	40.47s	0.0813m
	Follower 2	40.43s	0.0265m

6. Simulation Results and Analysis

The cooperative attack of three flight vehicles is investigated in this paper. The three flight vehicles have different roles, one leader and two followers. Followers attack the target under the guide of the leader, and the followers can communicate with the leader and each other mutually. Four instances are considered to test the designed cooperative strategy.

At the initial moment, the state parameters of flight vehicles are given in Table 1, and the following four cooperative attack conditions are selected to test the effectiveness of the proposed cooperative guidance algorithm:

Instance 1: the target is stationary, $v_t=0\text{m/s}$;

Instance2: the target moving at a slow speed, $v_t=30\text{m/s}$;

Instance3: the target moving at a quicker speed, $v_t=60\text{m/s}$;

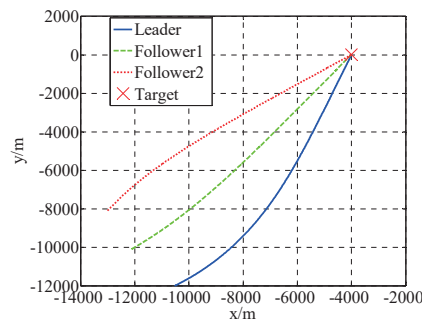
Instance4: the target moving with a sinusoidal speed, $v_t=30\sin(t)\text{m/s}$;

Simulation results were carried out using the proposed cooperative guidance law based on the consensus theory and the finite time control technology. Fig. 4-7 and Table 2 show the simulation results for Instance 1-4.

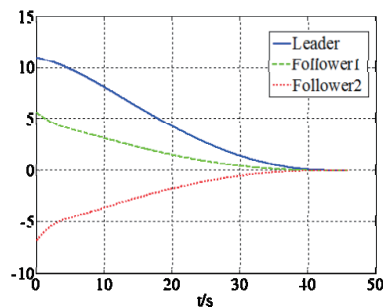
Figure 4-7 shows the simulation results of Instance 1-4, respectively. Fig.(a) shows the trajectories of flight vehicles, Fig.(b) shows the control command normal to LOS, $u_{\lambda d}$, which is controlled to guarantee LOS angular rates converge to zero. Fig. (c) shows the distance between flight vehicles and target, R_i . Fig.(d) shows the velocity of the three flight vehicles, v_i . Fig.(e) shows the time-to-go of three flight vehicles, T_{goi} .

In Instance 1, suppose the target is a stationary point with position , the designed cooperative guidance law is adopted. Three vehicles launched from different positions will arrive at the target at different time without the control of impact time, which will lead to failure of the combat mission. Instead, they can communicate mutually by the topology, then salvo attack is achieved by adjusting the cooperative guidance command in real time. From the results, it can be observed from Fig.(b) that all control commands could converge to zero in the final phase, thus all of the flight vehicles could hit the target. As shown in Fig.(c), it can be seen that the distances to the target are different at the initial moment. The leader is the farthest to the target, and two followers are closer to the target

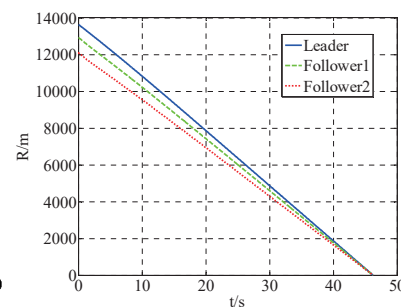
than the leader. The distance to the target of the three flight vehicles can tend to zero at the same time under control of the cooperative guidance law, which means that the three flight vehicles can hit the target simultaneously, and the guidance time is 47.01s. Therefore, the time-to-go consensus is realized by velocity changes of flight vehicle. Fig.(d) shows all the initial velocities of three flight vehicles are $u_{i,0}$, the velocity of the leader is invariable since the state of the leader is independent of the ones of followers. Two followers are closer to the target as shown in Fig. (c), so they decrease their velocity through the control command along LOS to wait the leader, such that all the flight vehicles can attack the target simultaneously, obviously. Fig.(e) further verifies this conclusion. It can be



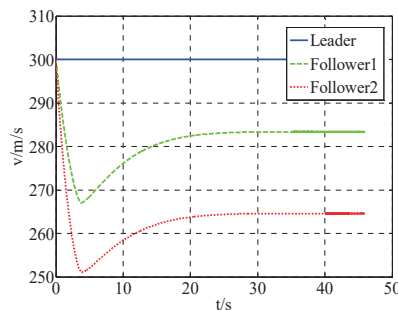
(a) Trajectories of flight vehicles



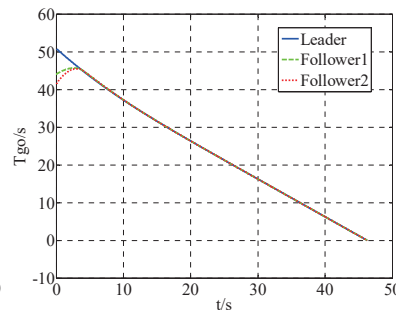
(b) Control command normal to the LOS



(c) Distance between flight vehicles and target



(d) Velocity of three flight vehicles



(e) Time-to-go of three flight vehicles

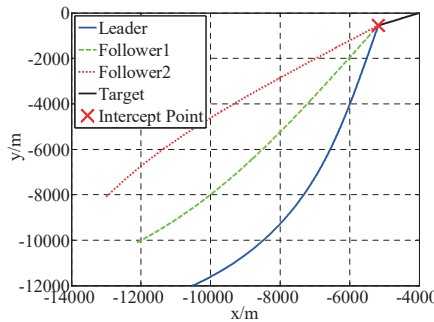
Fig. 4. Simulation results for Instance 1

seen from Fig. (d) that the three curves become flat after the prophase adjusting, which indicates the demand is satisfied. Through adjusting the time-to-go of followers under control of the consensus protocol, the curve of followers can overlap with the ones of leader at around 3s as shown in Fig.(e), which indicates the time-to-go of three flight vehicles could reach agreement.

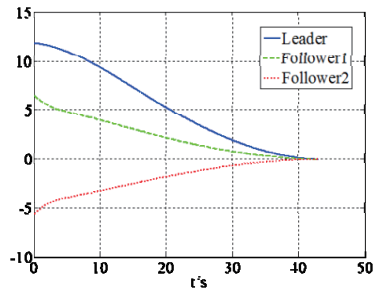
Instance 1: stationary target

In Instance 2-4, the intercept for the incoming target with different velocities was investigated using the designed cooperative guidance law. Fig.(a) shows the trajectories of flight vehicles during intercepting the incoming target. Similarly, three flight vehicles launched from different positions and directions, and they can intercept the incoming target simultaneously under the proposed cooperative guidance law. For the simulation

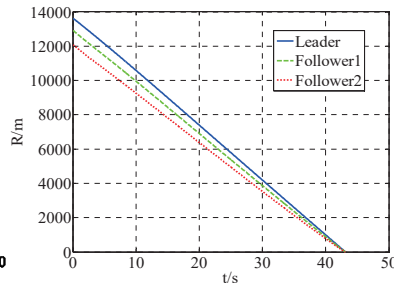
results of intercepting the incoming target, Fig.(b) shows the control command normal to LOS, Fig. (c) shows the distance between flight vehicle and target, Fig. (d) shows the velocity of three flight vehicles, Fig.(e) shows the time-to-go of three flight vehicles. Commonly, as the velocity of flight vehicles is bigger than the ones of the target, the LOS angular rates become stable around zero means the flight vehicle could hit the target. From Instance 2-4 in Fig.(b), it can be observed that all the control commands could converge to zero in the final phase, which guarantees the three flight vehicles could hit the target. Where, the control command comes to zero in the final phase when the velocity of the target is 30m/s. When the velocity increases to 60m/s, the control command has slight fluctuations in the final phase, and the control command shows a sine curve when the velocity of the target changes sinusoidally.



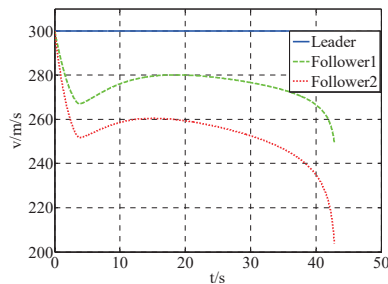
(a) Trajectories of flight vehicles



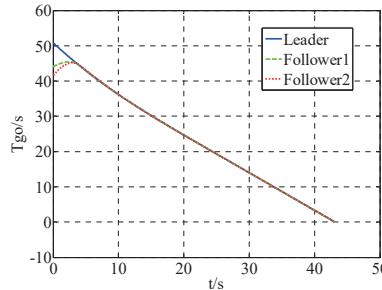
(b) Control command normal to the LOS



(c) Distance between flight vehicle and target



(d) Velocity of three flight vehicles



(e) Time-to-go of three flight vehicles

Fig. 5. Simulation results for Instance 2

It can be seen from Fig.(c) that the distances to the target of the three flight vehicles could come to zero at the same time, which indicates that the consensus of leader-follower multi-vehicle system is achieved. Furthermore, the intercept time will be shorter as the velocity of the target increases. Therefore, the designed cooperative guidance method is also effective for intercepting a moving target. Fig.(d) shows the velocity of the leader is also constant, and followers could adjust their velocities such that the state of the system can reach agreement. The velocity of flight vehicles is controlled by command along to LOS. But the velocity curves of followers no longer tend to flat, they change continuously as the target is moving. As the velocity of the target increases, the amplitude value of the vehicle velocity is bigger. It can be seen that the time-to-go could

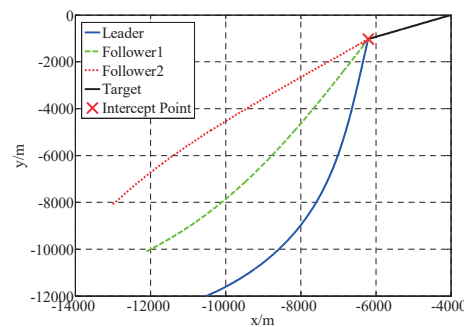
reach agreement when the target is moving as shown in Fig.(e), the time-to-go curves of followers tend to the ones of the leader at around 4.5s in Instance 2-4, it takes longer time to reach agreement compared to the stationary target attack. From the simulation results, it can be concluded that the designed cooperative guidance law based on finite time consensus theory is effective for intercepting a moving target, but the consensus time is longer than the instance of stationary target.

Instance 2: moving target

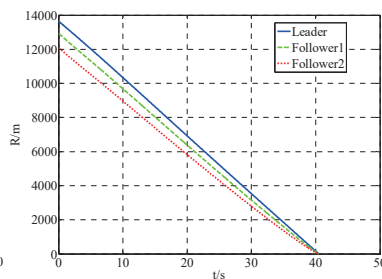
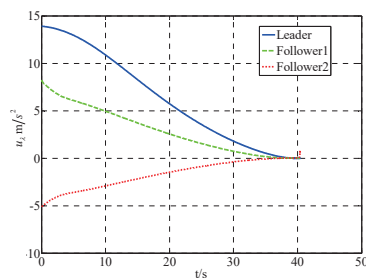
Three flight vehicles cooperative intercept of a moving target with the flight velocity 30m/s.

Instance 3: moving target at a quicker speed

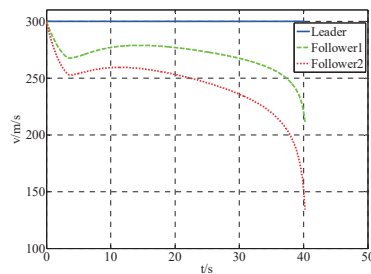
Instance 4: moving target with velocity changes at sinusoid law



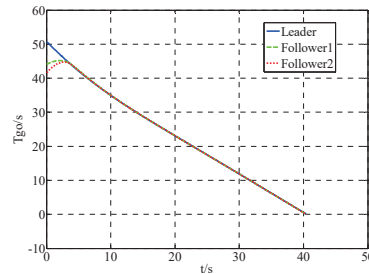
(a) Trajectories of flight vehicles



(b) Control command normal to the LOS (c) Distance between flight vehicle and target

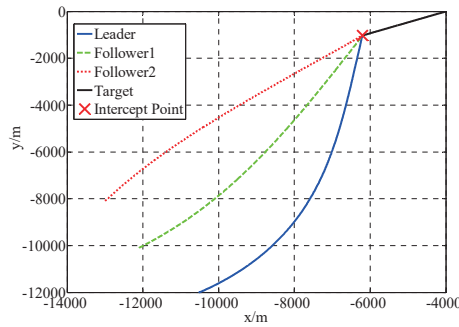


(d) Velocity of three flight vehicles

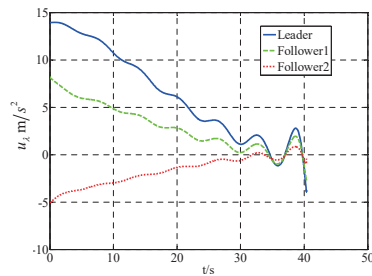


(e) Time-to-go of three flight vehicles

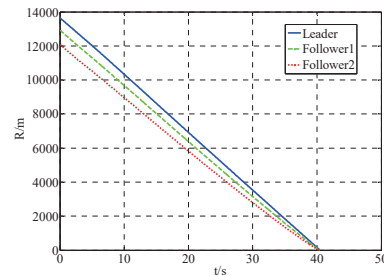
Fig. 6. Simulation results for Instance 3



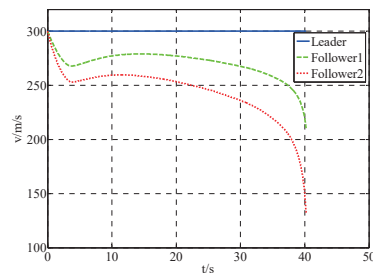
(a) Trajectories of flight vehicles



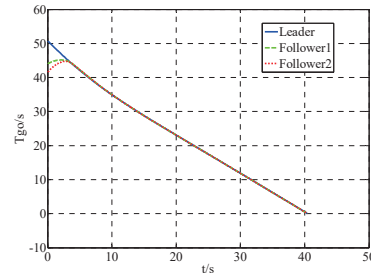
(b) Control command normal to the LOS



(c) Distance between flight vehicle and target



(d) Velocity of three flight vehicles



(e) Time-to-go of three flight vehicles

Fig. 7. Simulation results for Instance 4

7. Conclusion

A cooperative guidance strategy in salvo attack for leader-follower multi-vehicle system is proposed in this paper. The consensus protocol of leader-follower multi-vehicle system is designed by the finite time consensus theory, therefore, the time-to-go of flight vehicles can reach agreement, and the finite time stability of the system is analyzed. Meanwhile, a finite time guidance law is designed by the finite time control technology, so the LOS angular rates can converge to zero. The consensus problem is resolved by the proposed method, thus guarantee MFV hit the target simultaneously. It can be concluded that the finite time control theory has a great advantage in consensus problem of MFV. Salvo attack can be

achieved by the proposed approach, but the consensus time is longer for intercepting a moving target compared to the stationary target instance.

There are some future studies related to this work. One interesting study is the consensus protocol considering topology transformation and communication delay. In addition, how to design a reasonable estimation method to estimate the target states in cooperative guidance is also worth researching.

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