

Mathematical Performance Model of Two-Tier Indexing Scheme in Wireless Data Broadcasting

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Abstract

Wireless data broadcasting system that can support any number of clients is the effective alternative for the challenge of scalability in ubiquitous computing in IoT environment. In the system, it is important to evaluate quickly the performance parameter, the access time that means how quickly the client access desired data items. In this paper, we derive the mathematical model for the access time in the wireless data broadcast system adopting two-tier indexing scheme. The derived model enables to evaluate the access time without the complicated simulation. In order to evaluate the model, we compare the access time by the model with the access time by the simulation.

Keywords: Wireless data broadcast, access time, mathematical performance model.

1. Introduction

One of the most important challenges of ubiquitous computing in IoT environment is the efficiency for supporting massive mobile clients to access their desired data items, regardless of the number of clients [1-3]. Wireless data broadcast is the effective alternative for the challenge, because it guarantees that arbitrary number of the clients can access their desired data items simultaneously in anytime at anyplace. Figure 1, for example, shows a wireless data broadcast system where a broadcast server disseminates data items from DB over a wireless data broadcast channel through a transmitter. The mobile clients search and download the queried data items from the channel, after accessing it. Thus, the wireless data broadcast system establishes the scalability for the number of the clients, because any number of the clients can access the channel simultaneously [4,5].

The server disseminates data items and index information together in interleaving way, like index IDX in Figure 1. The index information carries the time information that each data item appears on the wireless channel. With the index, the clients predict the time when the queried data items are broadcast, then they download the items selectively from the channel, not continuously listening to the channel until they download all the data items. In order for the clients to access the items efficiently, various index schemes have been developed and reported [4-6]. Whenever each index is developed, we have to evaluate performance parameters caused by the index. Thus, in the broadcast system, it is important to analyze the parameters.

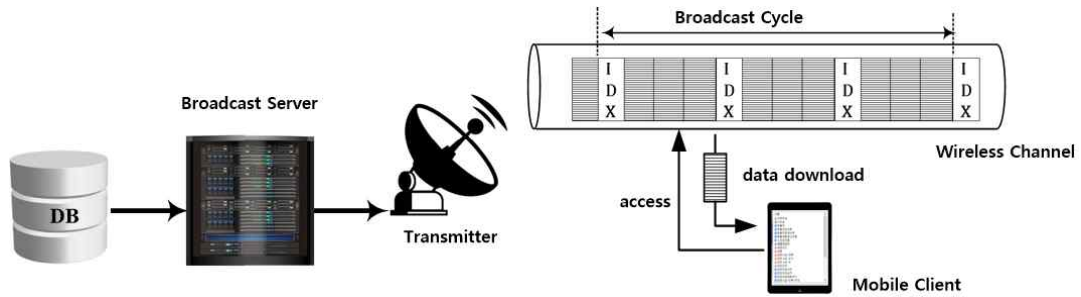


Figure 1. A Wireless Data Broadcast System

The access time is one of the significant performance parameters to be analyzed. The access time is the time duration from beginning that a client tunes into the channel and to finishing that it downloads all items. We can analyze the access time with two ways, simulation and analytic mathematical model. In order to analyze the access time through the simulation, we have to establish a complicated testbed using software stacks and process massive number of queries. Then, we can analyze the access time with the result of the processing. The other way is to derive a mathematical model for the access time in the broadcast system. With the model, we can analyze the access time more easily than the simulation [1, 2].

In this paper, we derive the mathematical model of the average access time in the wireless data broadcast system, adopting the two-tier index, that is organized by partitioning data ID space in two tiers. Here, a client processes a query that accesses multiple data items. It is useful that we analyze how quickly the client accesses its queried data items in the broadcast system.

The rest of the paper is organized as follows. Section 2 describes the two-tier indexing scheme to be adopted to the broadcasting system. In section 3, we derive the mathematical model for the access time and evaluate the model by comparing with the simulation in section 4. Section 5 concludes the paper.

2. Data Broadcast with the Two-Tier Index

Each data item to be disseminated over the wireless channel has its unique number, ID, as a identifier. To organize the index of all the data items for broadcasting, the ID space is partitioned with two tiers, i.e., Tier1 as the upper level and Tier2 as the lower level. The ID space is disjointed into n Tier1 fragments, and each fragment is disjointed m Tier2 fragments. Each Tier2 fragment has its own data set containing data items of which ID is included in the fragment. Figure 2, for example, shows a two-tier ID space Partition $n = 4, m = 2$ for ID space with IDs from 0 to 40. $T1_0, T1_1, T1_2,$ and $T1_3$ are the Tier1 fragments. $T2_0, T2_1, T2_2, \dots, T2_7$ are the Tier2 fragments. Each Tier2 fragment has its own data set. For example, the fragment $T2_0$ has its own data set that contains items with ID from 0 to 5.

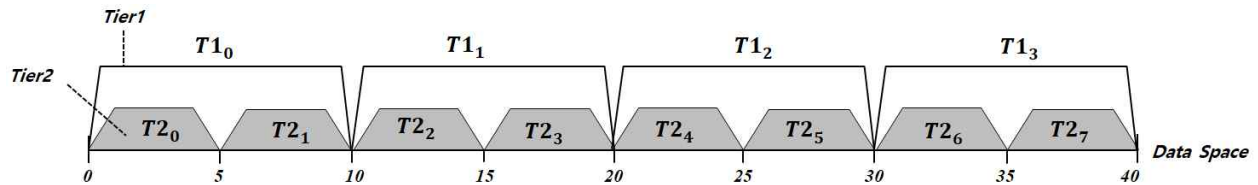


Figure 2. A Two-Tier ID Space Partition

For organizing the index based on two-tier partition, each tier fragment has its own index table. The index table for a Tier2 fragment holds tuples of (k, time) for the data items in its own data set, here, time in the tuple means the broadcast time of data items with ID k . The index table for a Tier1 fragment holds, tuple of (i, time)

for the Tier2 fragment in itself. Here i means the Tier2 fragment number and time is the broadcasting time for the index table for $T2_i$.

The broadcast server disseminates the index table for Tier1 fragments and Tier2 fragments and data items in the interleaving way. On the wireless broadcast channel, the index table for a Tier1 fragment leads the index table for Tier2 fragment, the index for Tier2 leads data items in the fragment. Figure 3 shows the wireless data broadcast channel, constructed two-tier ID space partition shown as Figure 2.

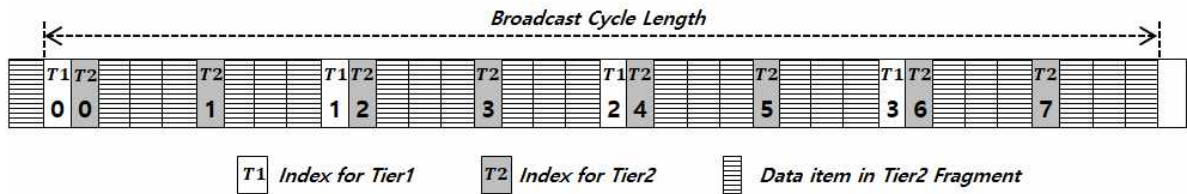


Figure 3. The Structure of the Wireless Data Broadcast Channel

In the broadcasting system, a client process a query Q , accessing multiple data items on the wireless channel. The query Q is a set of data IDs of data items for the client to access. For example, $Q = \{3, 9, 13\}$ means that the client have to download three data items with 3, 9, 13 as its identifier.

3. Mathematical Model for the Access Time

The access time, the time duration from beginning to tune into the channel to finishing data downloads, is the sum of the probe wait time, index wait time, and data wait time. The probe wait time means the time duration from tuning into the broadcast channel until the client meets the first index on the wireless broadcast channel. The index wait time is the time duration until the client meets the last one of the Tier2 indexes to access for processing the given query, after the probe wait time. The data wait time is the time duration that the client completes to download the queried data items after the index wait time. For example, Figure 4 depicts the access time for accessing a data item from the channel, after tuning into the broadcast channel.

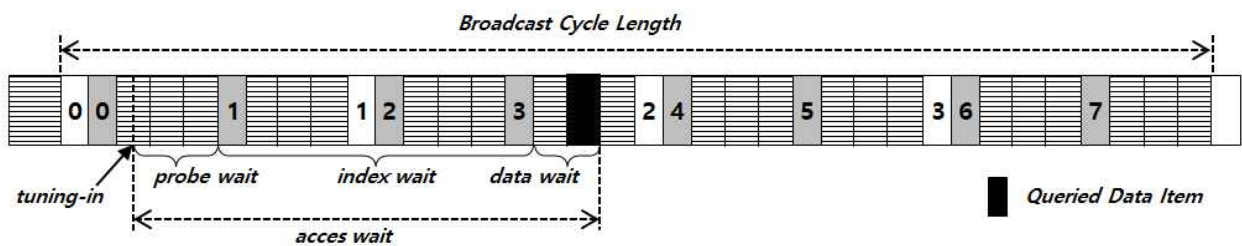


Figure 4. An Example for the Access Time

Thus, we can denote the average access time T_a as the sum of the average probe wait time T_{pw} , the average index wait time T_w , and the average data wait time T_{dw} as below.

$$T_a = T_{pw} + T_w + T_{dw} \quad (1)$$

We consider the wireless broadcast system with an index for n Tier1 fragment and n Tier2 fragment.

3.1 Average Probe Wait Time Model

The location on the wireless channel where the client tunes into affects the probe wait. The client tunes into the channel at the time broadcasting a Tier1 index or during a Tier2 fragment on the channel. The average probe wait T_{pw} is the sum of the multiplication of probe wait in a Tier1 index by the probability that the position of the client tuning-in is the Tier1 index and the multiplication of probe wait in a Tier2 fragment by probability in the fragment. The average probe wait T_{pw} is derived as follows:

$$T_{pw} = \frac{1}{2L} (nS_{ix1}^2 + n^2S_{ier2}^2) \quad (2)$$

Where, L is the length of the broadcast cycle and S_{ix1} means the size of the Tier1 index. Also, S_{ier2} is the size of the Tier2 fragment.

3.2 Average Index Wait Time Model

The index wait time is affected by the client's tuning-in location in the channel. The tuning-in location of the client is one of the three cases as follows:

- Case1: tuning in the middle of overlapped Tier2 fragments with a given query
- Case2: tuning in the middle of non-overlapped Tier2 fragments with the given query
- Case3: tuning in the middle of the non-overlapped Tier1 fragments with the given query

We can consider that the average index wait time T_w is the sum of the average index wait times for the three cases. The average index wait time T_w is derived as follows:

$$T_w = T_w^1 + T_w^2 + T_w^3 \quad (3)$$

Here, T_w^1 , T_w^2 , and T_w^3 mean the average index wait time for Case1, Case2, and Case3, respectively.

Firstly, the average index wait time for Case1 T_w^1 is derived as follows:

$$T_w^1 = \frac{S_{ier2}}{L} (L - S_{ier2} + S_{ix2}) N_{ier1} N_{ier2} \quad (4)$$

Here, S_{ix2} means the size of the Tier2 index. N_{ier1} and N_{ier2} mean the number of Tier1 fragments and the number of Tier2 fragments containing the IDs of queried data items.

Secondly, the average index wait time for Case2 T_w^2 is derived as follows:

$$T_w^2 = \left\{ \frac{S_{ier2}}{L} \left(\frac{(n-N_{ier2})(n-N_{ier2}-1)}{2} S_{ier2} \right) + \frac{S_{ix1}}{L} (nS_{ier1} + N_{ier2}S_{ier2}) \right\} N_{ier1} \quad (5)$$

Here, S_{ier1} is the size of the Tier2 fragment

Lastly, the average index wait time for Case3 T_w^3 is derived as follows

$$T_w^3 = \frac{(n-N_{\text{tier } 2})S_{\text{tier } 2} + (n-N_{\text{tier } 1})S_{\text{tier } 1}}{L} \left(\frac{(n-N_{\text{tier } 2}) + n(n-N_{\text{tier } 1})}{2} + S_{\text{idx } 1} S_{\text{idx } 2} \right) \quad (6)$$

3.3 Average Data Wait Time Model

The average data wait time T_{dw} , the required time for completing to download all the queried data items from the wireless channel.

$$T_{dw} = \frac{NS_{data}}{2n^2} \quad (7)$$

Here, N is the number of queried data items and S_{data} is the size of the data item.

4. Evaluation

We evaluate the derived model of the access time for the data set of 5000 items in uniform distribution. For the evaluation, we compare the access time by the model with the access time by simulation. We implemented a testbed for the simulation using SimJava, the discrete time scheduler, based on JAVA. The testbed consists of the broadcast server, the wireless channel, and the client. We compare the model with the simulation for the case that the client accesses multiple N data items.

Figure 5 shows the access time in the unit of bytes by the derived model and simulation $N=1$ and $N=10$. Figure 5(a) depicts the access time for the case $N=1$. The difference of the access time by the model and the simulation is about 2%. This means the model can evaluate the access time as a performance parameter without the complicated simulation that have to set up a testbed. Figure 5(b) shows the access time for the case $N=10$. The figure also shows the similar difference with Figure 5(a).

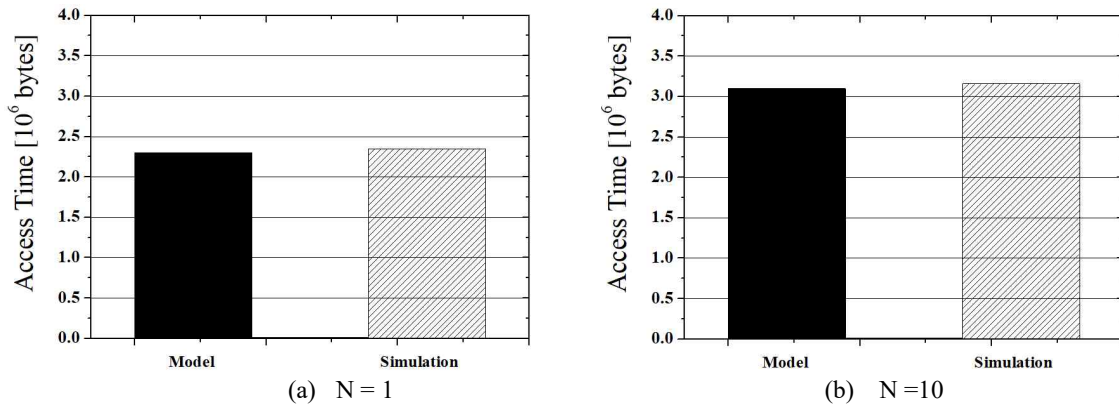


Figure 5. The Access Time by the Model and Simulation

5. Conclusion

In the wireless data broadcast system, it is important to quickly evaluate the performance parameter, the access time. For the quick evaluation of the access time in the wireless data broadcasting, we derive the mathematical model for the access time. The derived model for the access time reflects the broadcast system adopting two-tier index by data ID space partition. The derived access time model is the sum of three terms,

the probe wait time, index wait time and data wait time. We derive the mathematical model for the three terms. We evaluate the access time model by comparing the access time by the simulation. In the evaluation, the access time by the derived model shows the difference of about 2% from the access time by simulation. That means the derived model enables to evaluate the performance of the wireless data broadcasting without the complicated simulation.

Acknowledgement

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References

- [1] Z. Lu, Y. Shi, W. Wu, and B. Fu, "Data Retrieval Scheduling for Multi-Item Requests I Multi-Channel Wireless Broadcast Environments," *IEEE Transactions on Mobile Computing*, Vol. 13, No. 4, pp. 752-765, April 2014.
- [2] Z. Lu, W. Wu, and B. Fu, "Optimal Data Retrieval Scheduling in the Multichannel Wireless Broadcast Environments," *IEEE Transactions on Computers*, Vol. 62, No. 12, pp. 2427-2439, December 2013.
- [3] G.G.Md.N. Ali, V.C.S. Lee, E. Chan, M. Li, K. Liu, J.Ly, and J. Chen, "Admission Control-based Multichannel Data Broadcasting for Real-time Multi-item Queries," *IEEE Transactions on Broadcast*, Vol. 60, No. 4, pp. 589-605, April 2015.
- [4] Y.Chang, Q. Liu, and X. Jia, "A Data Rate and Concurrency Balanced Approach for Broadcast in Wireless Mesh Network," *IEEE Transactions on Wireless Communication*, Vol. 13, No. 7, pp. 3556-3566, July 2014.
- [5] S. Im, M. Song, S.W. Kang, J. Kim, C.S. Hwang, and S. Lee, "Energy Conserving Multiple Data Access in Wireless Data Broadcast Environments," *IEICE Transactions on Communications*, Vol. E90-B, No. 9, pp. 2629-2633, September 2007.
- [6] S. Im, and H. Hwang, "An Index based on Irregular Identifier Space Partition for Quick Multiple Data Access in Wireless Data Broadcasting," *IEICE Transactions on Information and Systems*, Vol. E99-D, No. 11, pp. 2809-2813, November 2016.