

# Sliding Mode Control with Super-Twisting Algorithm for Surge Oscillation of Mooring Vessel System

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## 슈퍼트위스팅 슬라이딩모드를 이용한 선박계류시스템의 동적제어

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**Abstract** : This paper deals with controlling surge oscillations of a mooring vessel system under large external disturbances such as wind, waves and currents. A control synthesis based on Sliding Mode Control (SMC) with a Super-Twisting Algorithm (STA) has been applied to suppress nonlinear surge oscillations of a two-point mooring system. Despite the advantages of robustness against parameter uncertainties and disturbances for SMC, chattering is the main drawback for implementing sliding mode controllers. First-order SMC shows convergence within the desired level of accuracy, in which chattering is the main obstacle related to the destructive phenomenon. Alternatively, STA completely eliminates chattering phenomenon with high accuracy even for large disturbances. SMC based on STA is an effective tool for the motion control of a nonlinear mooring system because it avoids the chattering problems of a first-order sliding mode controller. In addition, the error trajectories of controlled mooring systems implemented by means of STA form in the bounded region. Finally, the control gain effect of STA can be observed in sliding surface and position trajectory errors.

**Key Words** : Mooring vessel system, Sliding Mode Control (SMC), Super-Twisting Algorithm (STA), Chattering, Robustness, Disturbances

**요 약** : 본 논문에서는 바람, 파도, 조류 등의 큰 외란조건에서 선박계류시스템의 계류안정성 확보를 위한 동적제어기 설계를 연구하였다. 선박계류시스템의 비선형 동요를 억제하기 위해 슈퍼트위스팅 알고리즘(STA)을 포함한 슬라이딩 모드 제어(SMC) 기법이 적용되었다. 외란이나 파라미터의 불확실성에 대한 강인성의 장점에도 불구하고, 채터링은 슬라이딩 모드 제어를 적용하는데 주요 단점이 되고 있다. 1차계 SMC는 정확히 제어 목표치에 수렴 하도록 정밀한 제어는 가능하나, 채터링과 같은 파괴적인 현상과 연계되어 적용에 주요한 장애가 된다. 대신에, STA는 큰 외란에도 불구하고 비교적 높은 정확도를 보이며 채터링 현상을 완전히 제거한다. 1차계 슬라이딩 모드 제어기의 채터링 문제를 피할 수 있는 STA기반의 SMC는 비선형 계류시스템의 동적제어를 위한 아주 효과적인 수단으로 판단된다. 아울러, STA로 제어된 선박계류시스템의 위치오차 궤적은 경계 구역 내에서 형성된다. 끝으로, 슬라이딩 표면과 위치제어의 오차결과를 통해 STA의 이득제어 효과도 관측할 수 있다.

**핵심용어** : 선박계류시스템, 슬라이딩 모드 제어, 슈퍼트위스팅 알고리즘, 채터링, 강인성, 외란

## 1. Introduction

Controlling the mooring vessel system including single-point and multi-point mooring systems is crucial for the safety of vessel operation in harsh circumstances. When the mooring vessel system

in case of two-point is harmonically excited, it exhibits the duffing oscillations. It is known that the system responses describe complex dynamical behaviors, including limit cycles and chaotic oscillations (Lee and You, 2018). Few efforts have been made to control the dynamic responses of the duffing system. Using a PID-based controller, Loria et al. (1998) studied the stabilization of duffing equation with uncertainty in all parameters. By employing Sliding Mode Control (SMC) scheme, Kuo et al. (2008)

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investigated the motion suppression of duffing-holmes chaotic system. Their results show the reduction of input chattering which is actually undesirable oscillations with finite amplitude with frequency. Khadra (2016) solved stabilization in a duffing-holmes system with a modified Super-Twisting Algorithm (STA). More recently, Mitra et al. (2017) controlled the primary and sub-harmonic responses of a two-point mooring system using a time-delay state feedback.

In this paper, SMC is applied to control the responses of a two-point mooring vessel system. For decades, SMC has been broadly applied in many engineering fields. The main strengths of SMC are invariability to external disturbances and its robustness to parameter uncertainty (Bigdeli and Ziazi, 2017). Along with the active employment of SMC, the inherent property of SMC so-called chattering effect became the main motivation of emerging second-order sliding mode.

For one thing, the STA development in various fields has provided chattering attenuation. It is a second-order Sliding Mode Control (2-SMC), since it drives  $\sigma$  (sliding surface),  $\dot{\sigma} \rightarrow 0$  in finite time. Levant (1993) introduced the twisting and super-twisting method featuring a bounded control continuously depending on time. The twisting algorithm needs an additional differentiator, whereas STA does not need it (Utkin, 2013). Lee and Utkin (2007) dealt with the chattering problem in the system with unmodeled dynamics. They designed the controllers with state-dependent or equivalent-control-dependent gain method. Davila et al. (2010) proposed a variable gain STA to ensure the global finite time convergence to the desired sliding surface and chattering suppression. Rivera et al. (2011) investigated the chattering problem of output tracking signals in the under-actuated robotic system. In order to reduce chattering effect and tracking errors, Heng et al. (2017) designed STA with the optimum gain parameters in recent years. It was found that STA was effective in alleviating chattering effect compared to pseudo-SMC.

This paper deals with controlling surge oscillations of a two-point mooring vessel system using SMC. First-order SMC with STA is taken into account for comparing the control performance. The contribution of this work is to present the perfect elimination of chattering even large disturbances by utilizing STA.

## 2. Mathematical Formulation

### 2.1 System Description

The mooring model is introduced by Gottlieb and Yim (1992)

and is further modeled as a single degree of freedom (surge) nonlinear system under external excitations. As depicted in Fig. 1, we assume that the vessel is constrained to move in one dimension (surge). For this system, the control input is the force  $u$  ( $N/kg$ ) that moves the system horizontally and the output is the displacement  $x$  ( $m$ ) of the vehicle system.

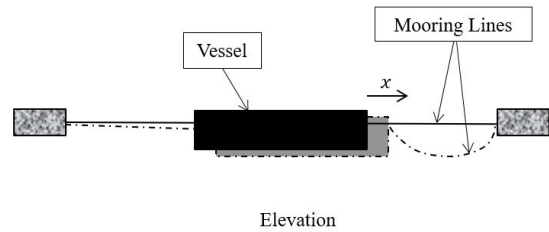


Fig. 1. Two-point mooring system under surge excitations.

Fig. 2 illustrates the phase plane of sliding mode control. The sliding surface ( $\sigma$ ) is in a line in the phase plane. Starting from an initial condition, the state trajectory reaches the time-varying sliding surface in the finite time (Slotine and Li, 1991). An active controller is designed to stay the state trajectories on the sliding surface ( $\sigma = 0$ ). However, control switching may lead to chattering which is a harmful phenomenon of oscillations having finite frequency and amplitude. It is the motion of states oscillating in the neighborhood of a sliding manifold. Despite the advantages of robustness against parameter variations and disturbances in SMC, chattering is the main obstacle in implementing sliding mode controllers.

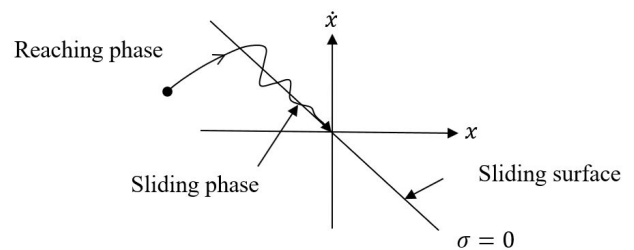


Fig. 2. Phase portrait of SMC.

Fig. 3 shows a block diagram of the mooring system control problem considered in this paper using SMC.  $x_{des}$  describes the desired trajectory,  $e$  is the trajectory error,  $\sigma$  is the sliding variable,  $u$  is the control input and  $x$  denotes output variable. In addition, the block diagram describes the detailed structure of STA.

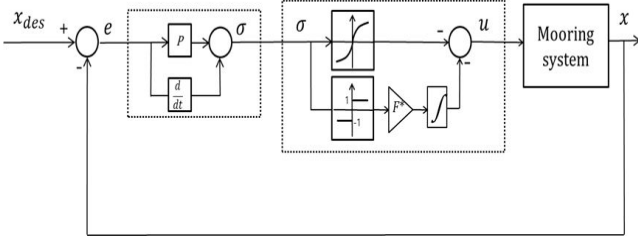


Fig. 3. Block diagram of the SMC with STA scheme.

A general dynamical model for variable structure systems (VSS) is described by

$$\dot{x}(t) = h(x, t, u) \quad (1)$$

where  $x(t) \in R^n$  is the state vector,  $u(t) \in R^m$  is the control input, and  $h$  is the vector of continuous nonlinear functions in the mooring system. The VSS controller is intended to achieve tracking of the desired trajectory  $x_{des}$  by the actual response  $x$  of the mooring system. The controller design problem consists of choosing the input  $u_i^+, u_i^-$  with  $m$ -dimensional vector  $s$  ( $s \in R^m$ ) (Utkin, 1977).

$$u_i = \begin{cases} u_i^+(x, t), & \text{if } s_i(x) > 0 \\ u_i^-(x, t), & \text{if } s_i(x) < 0 \end{cases} \quad i = 1, \dots, m. \quad (2)$$

Thus, the variable structure system technique is a form of discontinuous nonlinear control.

The nonlinear mooring system given in equation (1) is further considered as follows:

$$\dot{x}(t) = f(x) + \eta(x)u \quad (3)$$

$$y(t) = g(x) \quad (4)$$

where  $f$ ,  $\eta$ , and  $g$  represent the continuous nonlinear functions in the system.

It is noted that the control variable  $u$  is determined by (2) and  $y$  denote the output variable. A typical form for the sliding surface ( $\sigma$ ) is the following, which depends on just a single parameter ( $p$ );

$$\sigma(t) = \left( \frac{d}{dt} + p \right)^k e \quad (5)$$

$$k = 1, \quad \sigma = \dot{e} + pe \quad (6)$$

$$k = 2, \quad \sigma = \ddot{e} + 2p\dot{e} + p^2e \quad (7)$$

### (1) First-order SMC

For the application of variable structure control in equation (2), the control action is discontinuous across the sliding manifold  $\sigma(t) = 0$ ,

$$u(t) = -F^* \text{sgn}(\sigma) \quad (8)$$

where  $F^*$  is a sufficiently large positive constant which represents the amplitude of the discontinuous control input. that is

$$u(t) = \begin{cases} -F^*, & \sigma > 0 \\ F^*, & \sigma < 0 \end{cases} \quad (9)$$

Since the control input in SMC scheme in equation (8) basically includes the sign function, it induces undesirable chattering signals. This approach causes high oscillation of mechanical parts and heat loss in power electrical circuit involving high control activity.

### (2) SMC with a STA

The algorithm of super-twisting is based on the second-order SMC. This algorithm can be utilized to reduce chattering considerably. Note that the chattering problem is of great importance when exploiting VSS. This approach is very robust since it does not use any information of the time derivatives. The mooring control  $u$  is computed by integration of the discontinuous control generated by a sliding controller. In this paper, STA is described by

$$u(t) = -\lambda \sqrt{|\sigma|} \text{sgn}(\sigma) + \theta \quad (10)$$

$$\dot{\theta}(t) = -W \text{sgn}(\sigma) \quad (11)$$

$$\lambda = \sqrt{F^*}, \quad W = 1.1 F^* \quad (12)$$

where  $\lambda$  and  $W$  are the control gains to be designed,  $\dot{\theta}$  describes the leakage term of STA; it is a second-order sliding mode algorithm, and  $F^*$  is a positive constant to be taken sufficiently large to ensure good tracking performance. This control scheme

guarantees the appearance of a 2-sliding mode  $\sigma = \dot{\sigma} = 0$ , which attracts the trajectories in finite time (Shtessel et al., 2017).

### (3) Control synthesis for a two-point mooring system

As illustrated in Figure 1, the dynamical system is a simplified model of mooring vessel for surge motion. The dynamical model with a unit mass of vessel is described based on the equilibrium of the small motion under external excitations (Lee and You, 2018). By introducing the time form of  $\tau = \omega t$ , the equation of motion is now transformed to the non-dimensional model, in which  $\omega \in R^+$  is the excitation angular frequency (rad/s) and  $t$  is the time in seconds (s). The nonlinear behavior of two-point mooring system with an time-varying external disturbance  $d$  can be explained as

$$\omega^2 \ddot{x} + \delta_1 \omega \dot{x} + \mu_1 x + \mu_2 x^3 = u(\tau) + d(\tau) \quad (13)$$

where  $\dot{x} = \frac{dx}{d\tau}$  and  $\ddot{x} = \frac{d^2x}{d\tau^2}$ .

The model parameters in the equation (13) are:  $\delta_1$  controls the amount of damping,  $\mu_1$  represents the linear stiffness;  $\mu_2$  controls the amount of nonlinearity in the restoring force (Banik and Datta, 2010). The state variables are defined as  $x_1 = x$  and  $x_2 = \dot{x}$ . The governing equation (13) can be rewritten into the state-space representation given in the equations (14) and (15) as follows:

$$\dot{x}_1 = x_2 \quad (14)$$

$$\dot{x}_2 = 1/\omega^2 (u + d - \delta_1 \omega x_2 - \mu_1 x_1 - \mu_2 x_1^3) \quad (15)$$

$$y = x_1 \quad (16)$$

A control synthesis based on SMC can be categorized into two phases. At the first step, according to the equation (6), the tracking error vector is defined with  $e = x - x_{des}$  where  $x$  is the state vector of surge displacements, and  $x_{des}$  for the desired state vector. The sliding surface is expressed as

$$\sigma(t) = \dot{e} + p e = \dot{x} - \dot{x}_{des} + p(x - x_{des}) \quad (17)$$

where  $p$  is a positive constant which defines the convergence rate of the vessel system. Once the sliding surface is designed, the

control law  $u$  is can be taken as

$$u(\tau) = u_1 + u_2 \quad (18)$$

where the continuous control law  $u$  is constituted by two terms. In order to eliminate chattering problem, STA can be used as follows:

$$u_1 = -\sqrt{F^*} \sqrt{|\sigma|} \operatorname{sgn}(\sigma) \quad (19)$$

$$\dot{u}_2 = -1.1 F^* \operatorname{sgn}(\sigma) \quad (20)$$

This control approach is utilized to solve the chattering problem with improving control accuracy simultaneously.

## 3. Simulation tests

The proposed SMC with super-twisting has been applied to suppress surge oscillations. The parameter values for the vessel model are listed in Table 1.

When no control action is employed or  $u(\tau) = 0$  in equation (13), the mooring system exhibits a periodic motion and aperiodic motion depending on the initial conditions. In this case, the periodic excitation is given by  $d(\tau) = D \sin(\tau)$ , where  $D$  is the amplitude of the external disturbance. For the case of chaotic behavior, the phase portrait and time series curve for  $\omega=0.7$  (rad/s) and  $D=1.1$  are illustrated in Fig. 4. The chaotic solutions reveal wandering solutions of irregularly oscillating types without a uniform pattern (Lee and You, 2018). In order to achieve the oscillation suppression under large disturbances, the sliding mode controllers are applied to this system. The external disturbances  $d(\tau)$  such as the waves, wind, and currents are given as

$$d(\tau) = 3 \sin 4\tau + 2 \cos 5\tau + 10 [N.kg^{-1}] \text{ or } [m.s^{-2}] \quad (21)$$

Fig. 5 depicts the control activity using the first-order SMC. This algorithm can be utilized to the autopilot controller for some kinds of actuators to steer or to propel vessel (e.g., rudder or propeller). However, the actuators will be broken since the control input is chattering back and forth with the full range. Compared to this controller, STA provides a smoother curve without excessive control activity, as illustrated in Fig. 6. Instead of the discontinuous control, STA gets rid of chattering with a smoother

transition.

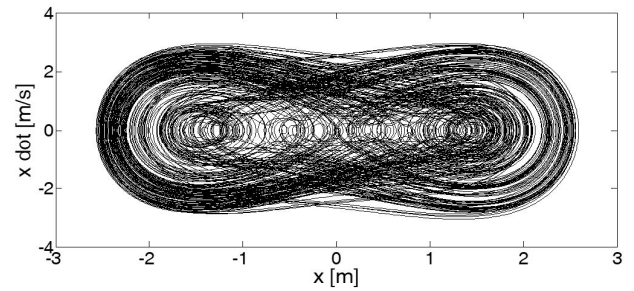
The dynamic responses of state variables are shown in Fig. 7 for comparison between two controllers. As a result of the two controllers, it is known that STA achieves the desired level of accuracy without chattering. It can be seen that the mooring system needs 15 seconds to come back to the desired level by means of STA. Even if the first-order SMC control law provides good dynamic responses, chattering occurs for control activity and system responses. In general, the proposed STA still provides good responses against large external disturbances while retaining continuity of control activity.

Figs. 8 and 9 represent the comparison between two controllers for the state variables and sliding surface. In contrast with first-order SMC, STA provides that the surge oscillations converge towards the desired level and the maximum peak points drop sharply. The sliding surface  $\sigma$  is driven to zero within a finite time by the continuous STA. It can be clearly observed that chattering phenomenon is indeed eliminated by STA.

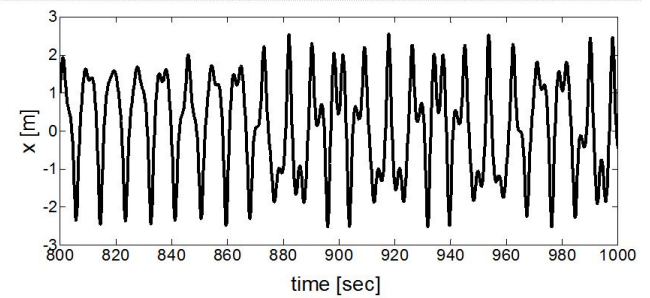
Fig. 10 illustrates the position and velocity errors of the controlled mooring system. The starting and ending point can be seen in the phase plane and the error trajectories are forming in the bounded manifold. It is known that the duffing system exhibits complex nonlinear behavior depending on the initial condition. The controlled the mooring system shows insensitive to that because the surge motion is subject to the surface equation itself. Figs. 11 and 12 illustrate the effect of the gain  $F^*$  of the super-twisting controller. When  $F^*$  is increased, the dynamic responses of the mooring system reaches the desired level faster. Finally, it should be noted that the chattering elimination is of great importance when exploiting the benefits of a sliding mode controller to a wide range of applications including oscillation suppression.

Table 1. Specifications of mooring system (Shah et al., 2005)

Parameters	Values
Structure type	Vessel
Mass of moored vessel	$1.2 \times 10^5 \text{ kg}$
Specific gravity of concrete	2.4
Number of anchored mooring lines	2
Young's modulus of mooring lines	$20.595 \times 10^9 \text{ N/m}^2$
$\delta_1$	$0.01 \text{ s}^{-1}$
$\mu_1$	$0.0213 \text{ s}^{-2}$
$\mu_2$	$0.319 \text{ m}^{-2} \text{ s}^{-2}$



(a) phase portrait of the mooring motion



(b) time series for frequency ratio  $\omega=0.7 \text{ rad/s}$  and  $D=1.1$

Fig. 4. Phase portrait for surge oscillations (a) and time history curve (b) for  $\omega=0.7 \text{ rad/s}$  and  $D=1.1$  (without controller).

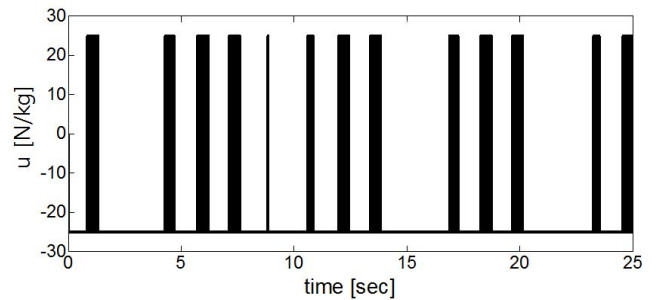


Fig. 5. Control input using first-order SMC.

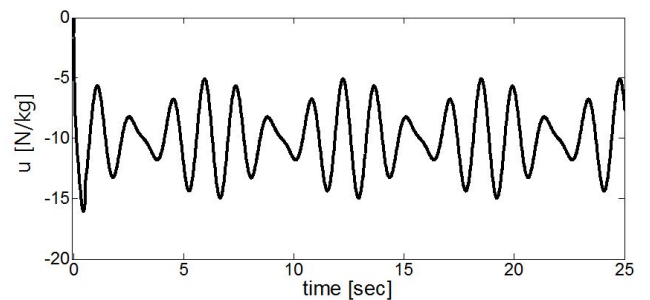


Fig. 6. Control input using STA.

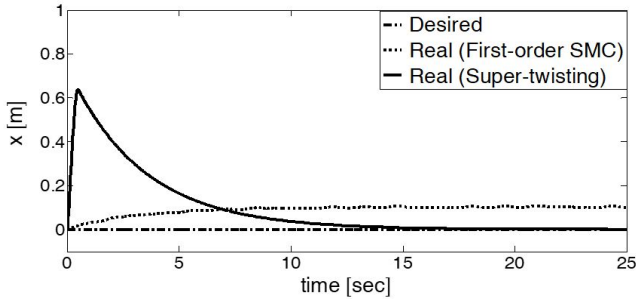


Fig. 7. Comparison of surge oscillation with displacement.

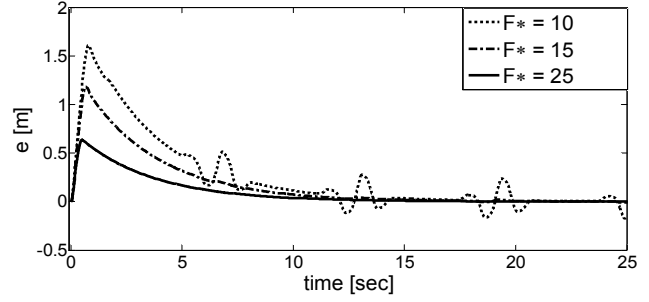


Fig. 11. Control gain effect of position errors using STA.

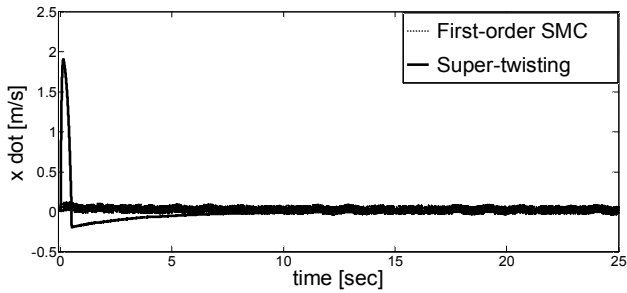


Fig. 8. Comparison of surge oscillations with velocity.

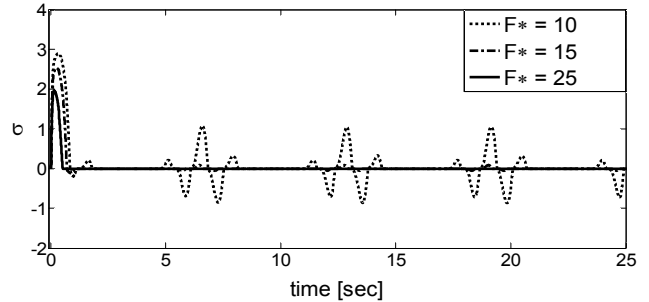


Fig. 12. Control gain effect of sliding surface using STA.

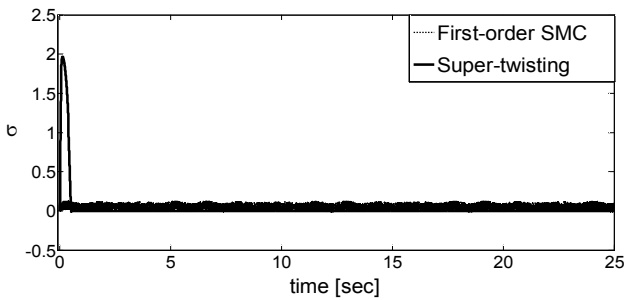


Fig. 9. Comparison of sliding surface.

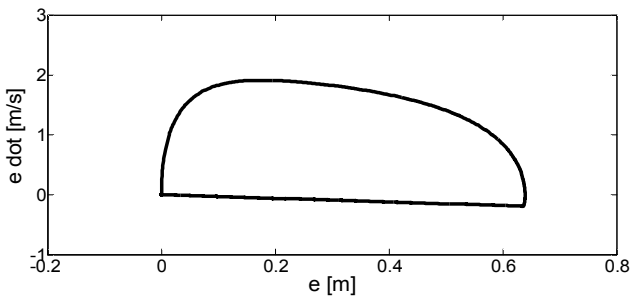


Fig. 10. Phase plane of the controlled motion using STA.

#### 4. Conclusions

The robust SMC with STA has been presented for controlling surge oscillations of a two-point mooring system under large external disturbances. In common with the strengths of SMC, the control system provides good dynamic results guaranteeing accuracy, robustness and simple implementation.

Even though the state variables of first-order SMC converge towards the desired level, this control algorithm usually cannot prevent chattering due to the discontinuous structure. However, the super-twisting controller provides the desired level of accuracy and it eliminates chattering phenomenon successfully. Even the large external disturbances are excited, the super-twisting controller still compensates it. Super-twisting controller is an effective method for the oscillation suppression of nonlinear mooring system since it solves chattering of first-order SMC. In addition, this paper describes the phase plane of the controlled mooring system and the control gain effect of the super-twisting controller. The error trajectories are forming in the bounded region by means of STA. Future work will be extended to the adaptive gain of STA for controlling surge oscillations of mooring motion.

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