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Extending Interactions through Communicative Technology: Bridging Mathematics Classrooms via Skype

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This paper describes how communicative technology between two classrooms located in different sociocultural contexts was used to support mathematics instruction. I analyzed what interactions emerged using the communicative technology, how sociocultural differences were leveraged to construct mathematical knowledge, and how students built this knowledge together across urban and rural classrooms. The results show that reciprocal interactions emerged. Teachers co-designed lesson plans and tasks with consideration of the different classroom social contexts. Based on those teachers' interactions, students had opportunities to justify their ideas and to prepare answers before the connected discussions, and a wide spectrum of ideas was synthesized as collaborative knowledge. These findings suggest that communicative technology has the potential to enhance learning opportunities for students across different social contexts.

Keywords: communicative technology, interactions in mathematics instruction, knowledge building, Skype
MESC Classification: C63, U73
MSC2010 Classification: 97C80, 97U70

I. INTRODUCTION

In mathematics education, researchers have increasingly concentrated on the social context of the mathematics classroom such as equity and socio-economic background (Aguirre, Ingram, & Martin, 2013; Civil, 2006; National Council of Teachers of Mathematics [NCTM], 2014). The context of the classroom is inherently influenced by the general social context that negotiates and constructs mathematical knowledge (Bauersfeld, 1992; Bishop, 1988; Cobb & Smith, 2008; Yackel & Cobb, 1996). This approach originated from the aspects of social constructivism that learning is a construction with social interaction between different contexts (Vygotsky, 1978). Culturally constructed and valued knowledge of students is developed through the interactions, not self-creation.

The interactions in the classrooms are mutual adjustments which consisted of the interactions among teacher, students, and content within the environments (Cohen, Raudenbush, & Ball, 2003; Copp & Smith, 2008; Lampert, 2001). Teachers and students influence each other, they are engaged in mathematical tasks, and they operate the environments which influence their knowledge and attention (Herbst & Chazan, 2012). Specifically, teachers shape their instruction by interpreting and responding to the students and contextualize curricular materials relevant to students' everyday life. Students build up mathematical knowledge situated on environments through the teachers' guidance and the engagement of mathematical tasks. The use of tasks has affordances and constraints in the opportunity of learning. The situated environments also influence these interactions among the elements: teachers, students, and content.

Interactive relationships have an effect on knowledge building which involves creative and sustained work with ideas (Moss & Beatty, 2006; Scardamalia, 2002; Scardamalia & Bereiter, 2006). Knowledge building is a process in which students work collaboratively to improve ideas and to extend the frontiers of public knowledge. Network technology plays a primary role for students to connect with knowledge-creation and collaboration as a building environment. Individual students contribute to give their own idea on the public online space, and they collaboratively construct public knowledge with community discourse by linking their contributions.

While knowledge building principles have been shown to affect how students' development impacts their dispositions and learning in the domain of science, little has been studied about student's knowledge building usage in the learning of mathematics (e.g., Moss & Beatty, 2006). The goal of this study is to explore the interaction in mathematical instruction made possible through the application of knowledge building strategies in connected mathematics classrooms via communicative technology. In particular, this study sought to examine the following research questions: (a) What interactions in the mathematics classroom emerge through the use of communicative technology? (b) How do the different socio-cultural environments influence the interactions among teacher, students, and content? (c) How do students build on knowledge in connected classrooms through communicative technology?

II. THEORETICAL FRAMEWORK

In this section, two theoretical perspectives I draw on are outlined, which are rooted in a Vygotskian tradition. The first is an "instructional interaction" perspective that explains the relationships among teachers, students, and content within situated environments. The second is a “knowledge building” perspective that enables to understand how students are engaged in the process of construction of knowledge via network technology.

1. INTERACTIONS IN MATHEMATICS INSTRUCTIONS

Researchers have contributed to finding the elements of the instructional situation and how they are related to each other in the mathematics classroom. For mapping, the domain of mathematics instruction, Lampert (2001) set teacher, students, and content as the three components of interactions which were inherent in peer partner, small group and whole class. She tried to unpack the complexity of teaching mathematics with mathematical problems and found the relationship with the three elements. In her study, teaching was defined as interactive processes with the connection between students and content. Cohen et al. (2003) encompassed the situated environment with the three elements to propose a new view of mathematics instructional effect and resource.

> “Teaching is a collection of practices, including pedagogy, learning, instructional design, and managing organization. … and the environments of teaching and learning are implicated in the interaction” (p. 124).

To illustrate the interaction, Cohen and colleagues suggested the instructional triangle diagram (Figure 1) and accounted that the instruction was a stream affected by environments such as other teachers and students, parents and the local district. In this paper, I explored the extended interactions between two classrooms with different socio-cultural background when they were connected via computer-mediated communicative tools.

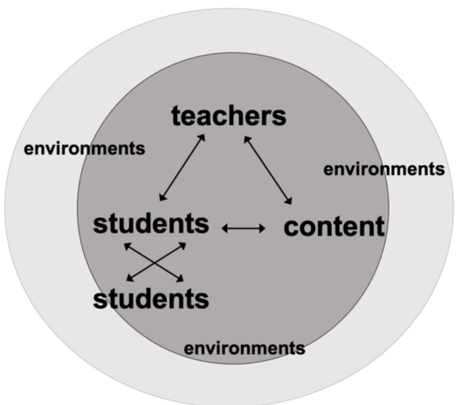

Figure 1. Instructional triangle (Cohen et al., 2003, p. 124)

2. KNOWLEDGE BUILDING AND KNOWLEDGE FORUM

Knowledge building, constructed through social interactions, means not only how people construct knowledge, but also how they utilize knowledge collaboratively. To enhance the opportunity of creating knowledge by students, knowledge building was studied in the domain of science education (Scardamalia & Bereiter, 2006).

As a knowledge building environment, *Knowledge Forum* software has been used for students' learning (Hurme & Jarvela, 2005; Moss & Beatty, 2006; Nason & Woodruff, 2002; Scardamalia & Bereiter, 2006). With the software, students could make notes and organize them like a concept map with a figure. There are two underlying principles embedded in this software: *epistemic agency* and *idea improvement*. This software affords a space to express students' own idea with notes (epistemic agency) and to collaborate with a group toward rigorous ideas (idea improvement).

As the first key principle, *epistemic agency* refers to "the amount of individual or collective control people have over the whole range of components of knowledge building" (p. 118). Knowledge building stemmed from the creation of the epistemic agency and this agency is related to students' own ideas and strategies of problem-solving. Each student and group needs to justify their ideas and refute other's ideas mathematically with evidence, and, finally, a classroom has their own generalized conclusion after the whole discussion (Moss & Beatty, 2006). Knowledge building discourse - refined and transformed knowledge through the discursive practices of communities - includes constructive and collaborative argument (Bereiter, 2002).

Idea improvement is the other core principle in knowledge building (Scardamalia, 2002). Since it is more natural for students to generate new ideas than to revise with peer interactions, the real-time technology might be helpful to give an opportunity to reconnect and edit their ideas. Individuals' ideas are refined through collaborative interactions using network technology, which leads to sustained improvement of these ideas (Scardamalia, 2002). Students could revisit and revise the notes they made previously and this flexibility allowed more time to reflect. In order to improve the ideas, students show deep digital literacy and appropriate time to rethink the idea.

In this study, instead of Knowledge Forum, students utilized Google Docs and Skype as network technology with three levels: group, classroom, and connected classrooms. At the group level, students worked at online worksheets and documents individually and/or collectively. At the classroom level, the students shared the results with the whole group discussion and each classroom's students prepared for the connected discussion via Skype prior to the connection. At the connected classrooms level, both classrooms discussed collaboratively to improve their mathematical understanding. In the next section, I briefly describe the procedures used in the study and show the result found in answer to the research questions with evidence.

III. RESEARCH METHODOLOGY

Netnography was applied to this single explorative case (Yin, 2014) of bridging mathematics classrooms via Skype [BMCS]. The concept of netnography is participant-observational research based on online fieldwork (Kozinets, 2010).

1. PARTICIPANTS

In this study, two 6th grade classrooms were chosen in a metropolitan city. Mr. YH (a pseudonym), who had 10 years of teaching experience, and his 6th-grade students in CS elementary school were selected. CS elementary school was located in an urban area of the city and the number of Mr. YH classroom's students was 31 (the total number of students at the school was 1,483 in the academic year). Mr. YH had initiated the BMCS project for two years and collaborated with Mr. KJ, who had 9 years of teaching experience. Mr. KJ had participated in the BMCS project for 9 months. Mr. KJ also taught 6th graders at YG elementary school in the rural area of the same city and had a smaller class of 16 students (the total number of students was 502 in the academic year). It was anticipated that both urban and rural students might feel the divergence from their contextual environments, even though they were at the same grade level and located in the same metropolitan city (Figure 2).

Figure 2. The scene of bridging mathematics classrooms via Skype

Mr. YH had initially designed and conducted the BMCS project to lessen the social and educational gap between urban and rural areas by using communicative technology such as Skype. The common routine of BMCS consists of 5 phases (Figure 3). First, both classrooms share the same mathematical tasks by connecting via Skype at the beginning of the lesson. Then, each class' students solve the tasks individually or as a group. These solutions would be shared by online collaborative document (e.g., Google Docs). In each class, they have a whole-group discussion to talk about how the students approached the tasks and what they found through the process of problem-solving. Also, they have a time for preparing the connected discussion. At last, they connect with each other via Skype again and share ideas and debate each other. Note that Skype is not used all the time in the routine.

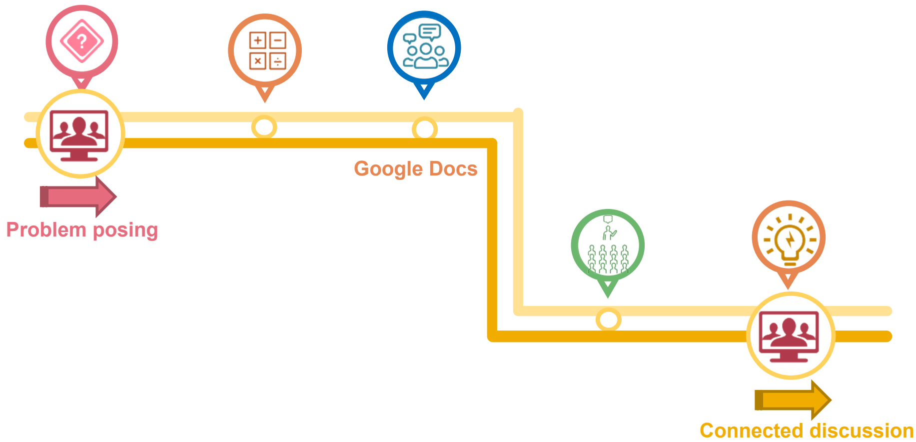

Figure 3. The general procedure of BMCS

Three lessons out of eight were selected with the consideration on diverse mathematical content areas in Mr. YH's YouTube channel (Figure 4): Measurement (Lesson A), Number and Operation (Lesson B), and Statistics (Lesson C). Lesson A dealt with the introduction of the standardized unit for volume by comparing two boxes with non-standardized units between two classrooms. Each group measured two boxes with various units and shared the result. The students recognized the measurement were different depending on their unit size. Then, they considered what kinds of characteristic the standardized unit should have to compare fairly. Lesson B was about the decision on which school could host a multicultural performance by debating mathematically. The third school (DG school) planned to have the performance between two schools (CS and YG school) and two classrooms used mathematical data to support their argumentation. They analyzed the rate of multicultural students, the capacity for the audience, and the distance from nearby schools. Based on this data, they could participate in the Skype discussion and the third school decided which school could host the performance. Lesson C was the last day of the 9-day project in which students planned, promoted and surveyed about which sport was appropriate to be selected for an afterschool sports club shared between CS and YG schools. In the lesson, students were tasked with using survey data from a differently sized population (6th graders in CS school is 327 and in YG is 73). They transformed the raw data into ratio graphs, such as pie chart or bar chart with proportions for each item, and then discussed their findings with the other classroom via Skype to make a final decision regarding the selection of common sports club.

Lesson	Content area	Unit	Main topic	Time
A	Measure	Area and volume of cube	Unit of volume	40’
B	Number and operation	Ratio and rate	Utilizing rate	42’
C	Statistics	Ratio graph	Utilizing ratio graph	47’

Figure 4. The selected lessons of BMCS

2. DATA SOURCES

To conduct the netnography, we collected three types of data: field note data, elicited data, and archival data (Emerson, Fretz, & Shaw, 2011; Kozinets, 2010). As a first step, field note data was collected through Mr. YH’s YouTube channel. I observed and participated in the YouTube live broadcast on the basis of real-time communication to watch the lessons of BMCS. After joining the live broadcasting, I watched the lesson video clips repeatedly and made field notes from the lessons (about 40-minutes each).

Second, elicited data, co-created through personal and communal interactions (Salmons, 2015), included interviews with Mr. YH before and after the lessons. The interviews lasted approximately 45 minutes each. The first interview was to comprehend the features of the BMCS. The second and third were focused on how the teachers prepared the lessons and how the implementation of teaching practices was aligned with their intended lesson plans for each lesson.

Third, archival data, directly copying from pre-existing online documents, consisted of students’ activity results, lesson plans, teachers’ reflection reports, and teachers’ corresponding threads. Students’ online worksheets were accumulated on the online shared drive. Mr. YH planned the lesson with the collaboration of Mr. KJ and they made thorough lesson plans. They met online and offline to discuss and plan specific lessons. The reflection journals were also collected, which might improve and prepare future lessons.

3. DATA ANALYSIS

In order to explore the interactions that emerged from BMCS, I analyzed the collected data to identify new kinds of interactions in the mathematics classroom by the bifocal lens: macro and micro. From the macro lens, the new interactions between two classrooms were identified in terms of Teacher-Content-Teacher and Teacher-Content-Student. Each analytic focus was used to examine whether the identified interactions were aligned with connecting to environments as advocated by Cohen and colleagues (2003). On the other hand, from the microlens, Student-Content-Student interactions were subsequently categorized in terms of students' collaborative knowledge construction-social agreement, acknowledgment of different contexts- based on core principles of knowledge building (i.e., epistemic agency and idea improvement).

Also, to pinpoint how different contexts affect interactions in the mathematics class, lesson plans, interview data, and the teachers' reflection reports were used. I made open coding from the oldest lesson (Corbin & Strauss, 2008). Whenever I noticed the change in the following lesson, we added additional coding. The open coding included a collaborative lesson plan, mathematical task, motivation, preparation for the whole discussion, support from technology, and mathematical justification. These codes were inductively categorized into collaborative planning, mathematical task, and preparation for argument. To confirm the categories, the teachers' reflection reports were compared and contrasted after implementing the lesson plans.

IV. FINDINGS

To explore the new interactions and knowledge building in connected mathematics classrooms via Skype, I concentrated on the emerging patterns and how the interactions changed through the use of BMCS across three lessons.

The results are presented in three primary sections. The first section (Teacher-Content-Teacher) presents the findings related to co-planning lessons and tasks to show how the interaction was adapted by applying communicative technology. The second section (Teacher-Content-Student) presents the findings related to how students' extended learning opportunities were influenced by different social environments. The third section (Student-Content-Student) presented findings related to how social agreements worked to support knowledge building practices across classrooms. Each of the following sections starts with descriptions of common patterns found across the evidence and then moves to an in-depth analysis.

1. TEACHER-CONTENT-TEACHER WITHIN ENVIRONMENTS

1) Collaborative Lesson Plans

Every instruction starts from teachers' lesson planning. When teachers decide to use technology in the classroom, this planning is getting more complex since the use of technology might affect other interactions with students, content, and mathematical activity (Zbeik et al., 2007). This is why the use of technology needs an appropriate rationale for the exact time and with specific purposes. The following comments describe Mr. YH's strong idea about using technology for the meaningful learning in post-interview of Lesson A:

> In teaching practice, the teacher has to have a purpose for all activities. Why do they need to collaborate together via Skype? When? Where? It was very difficult to determine the necessity. So, these questions should be solved before planning the lessons. Unless a teacher has a rationale for using such technology, students also cannot find the necessity or reason, too. In order to make a meaningful connection, it was really difficult to choose the appropriate time and specific activity. That was the biggest worry for us.

To overcome these concerns, teachers made a timetable of the lesson specifically about timing, frequency, and tentative subjects for collaboration. When it comes to connecting time, in lesson A, the two classrooms were connected one time at the end of the lesson as general routine. It took only took 3 minutes out of 40. In lesson B, the three classrooms were connected 4 times. The total time was 12 minutes out of 42. In lesson C, the two classrooms participated in the same project, and they were connected 2 times in the lesson. The total time was 7 minutes out of 47. The time of connection and the number of connected classrooms were dependent on the mathematical content relationship. In lesson B, the time was the longest since both classrooms had a debate with their mathematical data and analysis. On the other hand, lesson A had the shortest time for the connection since the main activity for the connected discussion was to share the idea of the standardized unit for volume.

From the observation of the lessons, the teachers planned and co-operated mathematical activities to make well-structured lessons by the limited connections. Since two teachers used the same lesson plans in two different classroom contexts for BMCS, they needed collaboratively to set the time for the connections and to consider other classrooms' situation. In the pre-interview of lesson A, Mr. YH focused on that point when planning for mathematical instructions:

> It can be very dangerous to connect only for a connection. That's why we need to plan the lesson together, so two teachers should regularly meet in person and use a messenger before connecting. The key point of the lesson is when to connect and

> how to develop the discussion via Skype. Considering the characteristics of learners and the timetable, we make an appointment to be connected for certain times and activities. Of course, we can't apply this teaching method to every unit and period. Therefore, we tend to choose the appropriate period and carefully to prepare for the period.

Mr. YH and his partner continuously contacted each other in many ways to implement the lesson plans. The teachers' communication played an important role to increase the precision for timing issue when they used communicative technology. As an evidence in his own words, Mr. YH expressed that 'the key point' was 'when and how to discuss via Skype'. The purpose of the BMCS lesson is to extend interactions beyond one classroom to discuss mathematical ideas. For the meaningful connection, Mr. YH also argued that teachers should fully understand the reason why they need to connect for mathematics instruction. The teachers did not merely try just to connect their classrooms without spatial constraint, but also to make a virtual space to create public knowledge by discussing mathematical opinions which might be varied from different context-based classrooms.

2) Mathematical Task

In the lesson plan, the teachers cooperated to design interactive mathematical tasks suited to the contexts of each classroom. Mr. YH and Mr. KJ spent most of the time developed meaningful tasks for connected classrooms. The tasks in lesson B and C were reflected on the social aspects around the classrooms such as the demographics of the classroom population and the preference of each classroom. The task of lesson A has compared the two boxes without a standardized unit, lesson B was to select a school for a multicultural performance, and lesson C was to discuss which sports club they would establish.

As the example of a task which reflects students' different environments, the task of lesson C was one of the activities from the 9-day sports club project between two schools. The following excerpt from the field notes presents an activity for students to remember their overarching goal for this project in the pre-interview of lesson C:

> He reminds the students what the day is today. For 9 days, the students from the 2 schools have had the same project: which sports would be selected as the sports union club between 2 schools? The teacher shows the pictures for what they did for 9 days on the screen. The pictures are about the posters which were made by the teacher and students in CS School.

Both teachers would like for students to experience the comparison with different populations. From the mathematical perspective, when students learn ratio graphs such as pie chart, it might be not easy to devise a task to fit students' own context. However, the use of the different contexts between two classrooms can allow students to understand how the ratio graphs are different from other graphs. For example, 1% of 1000 means 10, but 1% of 100 means 1. The same 1% has a different numerical value depending on the total population. From this task, students have an opportunity to develop different conceptual understanding of the ratio graphs (e.g., even though the percentage for two situations is the same, the different population causes the numerical value.)

In addition to mathematical focus, when the teachers designed the mathematical task, they tried to reflect on students' contexts. The following excerpt from the pre-interview of lesson A shows the process for the teachers to determine what the topic of the project would be:

> When YG school was connected for determining the survey topic initially, the students' favorite subject was a favorite singing group. However, in order to connect two different schools, we should focus on the different result in the ratio graphs between two schools. In our pilot survey, the proportion of the favorite singing groups was similar in both classes. That is, the topic was not fit for our teaching goals. For selecting the theme of the problem, we should consider students' backgrounds, socio-economic, religion, politics, and so on., …, Therefore, we have chosen sports club for after school because our students' most popular subject for 6th grade is Physical Education.

In problem-solving, everyday-life contexts can influence students' motivation (Streefland, 1991). The teachers wanted to select a topic based on students' interests (e.g., favorite singing group). While this topic was relevant to students, it was not closely connected to the instructional goal since the singing group topic did not have diversity in the preliminary survey. As an effort to consider the sociocultural contexts of schools, the teachers finally chose the sports club task for the BMCS lesson, which was related to both the mathematical learning goal as well as students' interests, social contexts and/or regional surroundings.

2. TEACHER-CONTENT-STUDENTS WITHIN ENVIORNMENTS

1) Monitoring as Preparation for Whole-Group Discussion

One important facet of a teacher's expertise in mathematics instruction is how to facilitate classroom discussion by selecting, sequencing, and connecting students' mathematical thinking (Stein & Smith, 2011). During the BMCS lessons, these teaching practices were implemented much easier with the use of technology. In BMCS, the teachers built up various levels of discussions: group, whole-group, and beyond the classroom. To make a connection between the group level and the whole-group level, the teachers could make a fundamental starting point for the classroom discussion with monitoring on each student's problem-solving strategies before the whole-group discussion. The teachers used a tablet PC to select and gather students' solutions purposefully. The following is an example of an excerpt from one observation in Lesson C:

> As a next activity, students are asked to make two ratio graphs and compare each. They work individually and draw ratio graphs with creative diagrams, not only a traditional pie chart. After the teacher's direction, he walks around and monitors individual activity. Some students say they do not have enough time. Others ask which type of graph is more appropriate for visualizing the data. The teacher picks some students' graphs and takes pictures with a tablet to share with others. The students move to their own group to analyze the graph together.

Mr. YH gathered specific ideas to apply to the whole-group discussion. He tried to figure out what kinds of strategies were used by his students and to understand why they were struggling with the tasks. The evidence the teachers collected was used for planning the sequence of strategies to share with other classmates.

2) Preparation for Connected Argument

To bridge the whole-group discussion with the connected discussion, the teachers gave opportunities to prepare the connected discussion between two classrooms. This type of discussion was unique compared to other traditional discussions. Since it would be difficult to anticipate what was going on in the opposite classrooms, even though two classrooms began with the same mathematical task. Therefore, this preparation was a meaningful experience for students to think about other class' ideas which were based on different contexts. To solidify their opinions, the students prepared rationales to support their statements prior to the connected discussion, and also discussed anticipated questions, as described in the field note excerpt from Lesson C:

> In the whole discussion, Mr. YH encourages students to discuss the questions: "What would the opposite class ask in the connected discussion? What should our

> class answer the expected questions? What is a possible justification?" He continues to emphasize students should discover mathematical ideas from the data and graph…. After the group discussion, one member of group A says, "The largest number in CS school is baseball, and the largest in YG school is dodgeball." The teacher asks the reason, and the student responds, "The baseball team of our school is very famous, and it is common for elementary school students to love dodgeball like YG school." …Group B finalizes their opinion of sports club selection: "We've come to the conclusion to choose dodgeball. Dodgeball is largest in school B and the second largest in our school. The deviation of percentage about dodgeball is only 7. Therefore, we will select dodgeball."

The teacher noted that his students should prepare how to ask and answer in the connected discussion with the other class prior to the connection. These preparations made students strengthen their idea and to think critically about the possible opinions of students from the other class. For developing an argument, the students established the foundation of their assertion from analyzing the data and thinking about the situated environment. In the above excerpt, group A proposed contextual evidence for why baseball was popular sports in CS school, and group B justified their idea with mathematical evidence from the data analysis. Consequently, two different classrooms could participate in the connected discussion on a specific mathematical point of view with prepared questions and answers in advance.

This preparation process for connected discussion also made the students interested in the social context of the other classroom. The connection via communicative technology influenced to extending the thinking space from the single classroom environment to multiple classroom environments. The students could think about not only the similarity and the difference between classrooms but also what made such a difference from their environments. The following conversation is the example of what kinds of interest students had at the whole-group discussion in Lesson B:

> Teacher: Do you have any remaining questions to YG school?
> Student A: I am curious why their proportion of multicultural students is so high.
> Teacher: Sure, you can. What else?
> Student B: About the public transportation, do they have any bus or subway? And how many? How long time takes to get to their school?

The students had time for anticipating potential questions and answers prior to the actual connection via Skype. Some of these questions could be related to the social background of the opposite classroom. The students might be curious about the social factors (e.g., the high proportion of the multicultural students in YG school) and the infrastructure near the school (e.g., public transportation). These interests for the social aspect would be good assets for the connected discussion and situate the students in the given task with a meaningful understanding of the contexts.

3. STUDENTS-CONTENT-STUENTS WITHIN ENVIRONMENTS

1) Introducing Tasks by Peers

General lessons start with a warm-up activity. The motivation from this activity is sustained through the lesson development and wrap-up. Teachers should take consideration into how to make student motived and engaged in mathematical activities across the whole lesson. Since BMCS has no warm-up activity, the introduction of the tasks has an important role across the whole process. One way to introduce a task for high engagement into the lesson was a problem posing by peers. In lesson B, the third classroom's students in the other school (DK elementary school) posed the mathematical task as follows:

> CS, YG, and DK elementary School came out on one screen. DK elementary school students were wearing traditional Asian apparel. Looking at the screen, they greeted each other. Students in DK school sang a song, "I love you, I love you and me. Hello everyone, we are a multicultural cultural club of DK school." And they presented today's task, "We are planning the show that is scheduled during the week of Multicultural Education. But we should decide only one place between CS or YG school. We will decide where to go after watching YouTube live lesson." Then, the video call was disconnected.

The students were more interested and engaged in visualized materials rather than written textbook problems. Both teachers knew that their students were the visual generation, so the teachers determined to introduce the task of the day by using the third classroom's students. This video call from the third classroom had the effects on the concentration and motivation. Since the video call would not be repeated, the students should be careful to understand the problem situation and to figure out what kinds of resources they could use to solve the problem. In addition, a third classroom's student said that DK school would make a judgment after watching the life lesson on YouTube. Due to the competition, students from two schools could feel more motivated, too. This small difference influences the students' engagement and motivation in the mathematical classroom.

More interestingly, each classroom's students collected the statistical information of their school by mutual interactions. At the beginning of the lesson C, two classrooms exchanged the statistical information of their own schools such as the number of total students and the number of multicultural students. Rather than teachers give mathematical tasks to the student directly, the students could construct the task and problem situation in BMCS. The following excerpt shows how the students share the information for solving the task in lesson B:

> Student A presented the data of YG school: "The total number of students is 73. The number of multicultural students is 7, … the number of chairs is 90." Student B presented the data of CS school: "The total number of students is 1483. The number of multicultural students is 9, …, the number of chairs is 500." Students were writing the data into the worksheet when the other school students told specific information about each school.

In real life, it is not easy to collect meaningful data from the field. Students might spend much time searching for such information on the Internet. However, this peer interaction was not just reading the statistical number which they wanted to know, but collaborating by sharing the information (Figure 5). The students might think that the task was related to the real world since the task was posed by their peers and the statistical information was authentic.

Schools		Total students		Multi-cultural students		Total schools in the district		Schools within 2 miles	
CS	YG	1483	73	9	7	55	25	5	0

Figure 5. The student's worksheet for lesson B

2) Social Consent

By using communicative technology and shared online document, students experienced a social agreement process through the group level and the whole-group level discussions. In lesson C, one issue that arose was how to handle the sum of percentages was 101 rather than 100. On the screen, students shared their table by taking a picture and uploading it to Google Docs. The teacher and students saw the results slide by slide. In the lesson, students discussed this with each other as shown in the following field note excerpt:

Teacher: Some groups' sum of percentage is 100, but others are 101. What's happening?'

Student A: After calculating the percentage, we got the 101, but didn't change to 100 because 101 is a more precise value.

Student B: When we draw the ratio graph, we cannot draw it with the sum 101.

Teacher: What should we do? What will you change?

Student C: We can subtract from 'etc.' category because that is not important for the decision.

Student D: I think that we subtract the same value from every item fairly.

In knowledge building, one primary principle is idea improvement (Scardamalia & Bereiter, 2006), which could be supported through community discourse. Each group needed to discuss whether they agreed with the offered opinion because they were unsure how to draw the graph when the percentage sums were different. The students justified their own group's opinion with rationale (e.g., "because that is not important for the decision"). Also, the students accessed the shared online document, which served as an online space to exercise epistemic agency and revised whenever they wanted. Generally, students used this online document to accumulate available resources for the discussion, and it provided an opportunity to externalize the mathematical justification about the result. Although this specific problem came from rounding up, students were able to clarify their understanding about the sum of percentage through the discussion.

Part of the rationale behind BMCS is that when students in one classroom reach consensus with another demographic group, their ideas could be more meaningful to a greater number of people. In lesson C, the following example emerged from the final connection via Skype:

> To share the comprehensive conclusion, Student E (CS school) says, "We have the conclusion to choose dodgeball. Dodgeball is the largest in school YG and second largest in our school. The gap is only 7%. Therefore, we will select dodgeball." Student F (YG school) also answers, "The sum of each percentage is highest in dodgeball. Therefore, we want to choose dodgeball, too." Finally, Mr. YH comments, "Our conclusion is the same. As a result, dodgeball is tentatively selected as the sports club."

In the first connection, they exchanged information between the two schools. After the activities and discussions about the ratio graph, they debated with each other by using the data and providing a rationale for their ideas. This provided opportunities to learn not only the application of a ratio graph (Figure 6) but also how to solve an everyday-life problem using mathematics knowledge of statistical surveys and graphs. Each class' students approached using a percentage operation, not the raw numerical values. CS School student used subtraction ("the gap is only 7%") and YG student used addition ("the sum of each percentage") to make a reasonable decision. Both schools' students consistently had the opportunity to think about how the students in the other schools' idea beyond their classroom were related to their own idea. Consequently, individual students' ideas created a foundation for group discussion, and this discussion extended across both classrooms involved in BCMS. This example suggests that mathematical knowledge could be collaboratively synthesized through the same learning content with communicative technology.

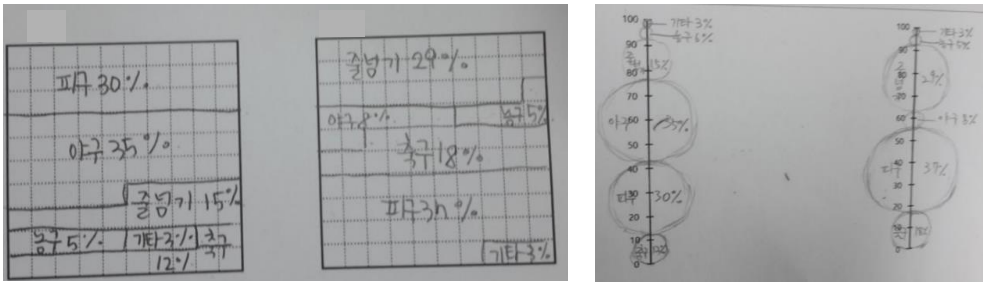

Figure 6. Two ratio graphs for comparison

V. CONCLUSION AND DISCUSSION

The purpose of this study was to find out what interactions among teachers, students, and content within connected environments emerged and how communicative technology and shared online documents supported knowledge building in mathematics learning. In the case reported here, the BMCS lessons provided new opportunities for students to interact with other classroom's students from the guidance of co-designed lesson plans by teachers, including preparing the connected discussions and considering the different socio-cultural contexts of the other school.

First, students could access other classroom's mathematical idea beyond their own classroom level. This bridged relationship is aligned with Vygotsky's idea (1978) that mathematical learning is the process of social interactions. Students could build up their mathematical understanding based on the individual and group level problem-solving. This is facilitated by the teachers' monitoring before the whole-group discussion. For the preparation on the connected discussion, students could have the unique time for advocating the converged idea in their own classrooms and making a tentative refutation on the possible opinions from the other classroom (Lannin, Ellis, Elliot, & Zbiek, 2011).

Second, BMCS provided opportunities for students to focus on social contexts, particularly sociocultural difference. Given that “The teacher guides the development of a community of validators and thus encourages the devolution of responsibility” (Yackel & Cobb, 1996, p.473), the teachers devised contextualized tasks such as different sized populations or students’ different interests in sports by incorporating communicative technology to extend their community discourse. Students might be interested in not only the mathematical content but also other classroom’s sociocultural context. Furthermore, they were curious about where this difference came from.

As a result, I propose a new instructional framework for BMCS (see Figure 7). In this framework, both teachers were reciprocally interacting with each other, resulted in the emergence of new Teacher-Content-Teacher interactions. For example, the teachers planed together from the context-based task to wrap-up activity, including a careful discussion about how and when to connect via Skype. Also, at the Teacher-Content-Student interactions, students were encouraged to anticipate a plausible question from the other class and prepare to refute a different opinion. Students-Content-Students interaction not only occurred within a classroom but also across classrooms as they used communicative technology to discuss the shared task with the other classroom. Therefore, one powerful possibility of the BMCS approach is that helps students perceive this new environment as a new opportunity to extend their mathematical ideas and allows teachers to take advantage of it to promote learning by using digital technology meaningfully.

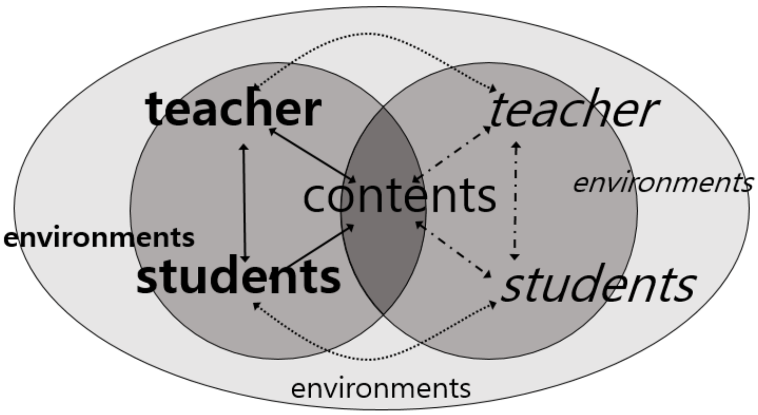

Figure 7. Transformed interactions with Skype

In terms of knowledge building, there was evidence that students’ mathematical ideas were influenced by interactive communication. For example, when students discussed how to handle situations of whether the sum of percentages in the graph was appropriate with 101 rather than 100, the students improved their mathematical concept on *whole* and *parts* in ratio graph through the activity via Google Docs. With co-designed tasks, students engaged in constructing and improving collaborative knowledge rather than transmitting unchangeable mathematical truth. Also, their discussion via communicative technology (e.g., Skype) appeared to support sharing information and finding common understanding; that is, students had learning opportunities to solve context-based task through the blended learning experience in both online and offline environments.

This study seems difficult to generalize with the constraints of the single case, limited data, and regional restriction. However, this teaching method is still in the beginning phase for researchers and practitioners. I wish this study would be extended to other content areas or disciplines. Technology help our students experience without the limitation of spatial and temporal issues (Kaput, 1992). With communicative technology and online documents, students can have more access on authentic environments and living experiences. Students also can easily connect to other classrooms to collaborate with each other. I might say that the efficiency of technology eventually contributes to having an effect on our mathematical learning and teaching situation. Based on a collaboration of different level subjects, we are able to do advanced our mathematics instruction.

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