Journal of information and communication convergence engineering
http://jicce.org

J. Inf. Commun. Converg. Eng. 16(4): 213-220, Dec. 2018

Regular paper

Influence Maximization Scheme against Various Social Adversaries

Giseop Noh[1], Hayoung Oh[2*], and Jaehoon Lee[3], ***Member, KIICE***

[1]Division of Software Convergence, College of Engineering, Cheongju University, Cheongju 28503, Korea
[2]DASAN University College in Ajou University, Suwon 16499, Korea
[3]Department of Computer Science and Engineering, Seoul National University, Seoul 08826, Korea

Abstract

With the exponential developments of social network, their fundamental role as a medium to spread information, ideas, and influence has gained importance. It can be expressed by the relationships and interactions within a group of individuals. Therefore, some models and researches from various domains have been in response to the influence maximization problem for the effects of "word of mouth" of new products. For example, in reality, more than two related social groups such as commercial companies and service providers exist within the same market issue. Under such a scenario, they called social adversaries competitively try to occupy their market influence against each other. To address the influence maximization (IM) problem between them, we propose a novel IM problem for social adversarial players (IM-SA) which are exploiting the social network attributes to infer the unknown adversary's network configuration. We sophisticatedly define mathematical closed form to demonstrate that the proposed scheme can have a near-optimal solution for a player.

Index Terms: Novel influence maximization, Social adversaries, Social networks, Word of mouth spreading effect

I. INTRODUCTION

Online social networks (OSNs) have become popular these days, and as OSNs grow, the commercial markets expand their presence in social commerce, online shopping, and so on. Commercial promoters or company owners undertake campaigns to reward the most influential users (power bloggers or reviewers) with prizes or money in order to exploit their ability for viral marketing (i.e., word-of-mouth). The underlying assumption is that when people make their decisions, they are likely to be affected by their friends or colleagues [1]. Sociology describes friendship as formulated based on homophily, which consists of two major forces between friends, selection and social influence [2]. Selection is a process where one chooses friends with similar characteristics, and social influence refers to the process where one modifies his/her own behavior to adapt to friends' behavior. In the 1990s, viral marketing with homophily in social network was studied to understand how new products could effectively spread over social networks [3-5]. Many studies have focused on the ways to maximize influence in OSNs by selecting a given number of seed users.

The influence maximization (IM) problem is formally defined as "the problem of finding a small subset of nodes in a social network that could maximize the spread of influence" [6]. Two probabilistic models are introduced to mathematically represent IM, the independent cascade (IC) [7-9] and the linear threshold (LT) [10, 11]. However, Kempe et al. [1] proved that both the models are NP-hard and proposed a greedy approximation algorithm to ensure the opti-

Received 06 June 2018, **Revised** 14 November 2018, **Accepted** 14 November 2018
*__Corresponding Author__ Hayoung Oh (E-mail: hyoh79@gmail.com, Tel: +82-31-219-3560)
DASAN University College in Ajou University, 206, World cup-ro, Yeongtong-gu, Suwon 16499, Korea.

Open Access https://doi.org/10.6109/jicce.2018.16.4.213 print ISSN: 2234-8255 online ISSN: 2234-8883

mum to within a factor of (1–1/e) (where e is the base of the natural logarithm): start with the empty set followed by repeated addition of an element that gives the maximum marginal gain.

Leskovec et al. [12] proposed an optimized greedy algorithm called ‘Cost-effective Lazy Forward’ which is 700 times faster than a simple greedy algorithm. Wang et al. [13] presented a community-based greedy algorithm for mining top-k influential nodes in mobile social networks. Bhagat et al. [14] maximized product adoption in social networks based on user awareness. Li et al. [15] provided influence diffusion dynamics and influence maximization in social networks with friend and foe relationships.

The basis for the IM is to consider a social network as a graph (directed or undirected) with a set of nodes and edges. The existing researches state solving the IM problems in a network based on the relationships between the users by considering either positive or negative relationships. However, in reality, more than two social networks (i.e., players) can exist (e.g., commercial companies or service providers) within the same market sector. For example, Verizon and AT&T are rivals in the telecommunications market in the US, and RenRen and Facebook could be the plausible adversaries in the global OSN market. In the case of coexistence in the market, the two social players attempt to maximize their market influence against each other.

To address the IM problem between the social adversaries, initially, we have introduced the IM problem for social adversarial players (IM-SA). We have mathematically formulated the IM-SA and proposed a novel approach that can provide a near-optimal solution for a given player. We have exploited the concepts of centrality and of a probabilistic clustering coefficient to infer the unknown adversary’s network structure. To the best of our knowledge, our approach is the first step to formulate the problem in detail and to provide a solution for the IM-SA.

Our contributions are as follows:

We have introduced the IM-SA problem with respect to the influence maximization domain and provided a near-optimal solution using a social network theory.

We have designed and evaluated a novel algorithm to solve IM-SA, and validated our algorithm by generating graph dataset which has two clusters. Our proposed algorithm presents better performance than that of the previously reported greedy scheme [1].

The rest of this paper is organized as follows. In Section II, we have introduced related works on IM problem with the adversarial environment. In Section III, we have mentioned the background and problem definition of the proposed scheme. Section IV presents the design of a system model that includes the adversarial relationship, assumptions, and provides an explanation of the functioning of the proposed scheme. The evaluation and performance analysis are presented in Section V. Finally, a conclusion is presented in Section VI.

II. RELATED WORK

There are few existing studies on IM problem with the adversarial environment and deal with the spread of influence of competing products, opinions, and technologies. In the real world, there are more competitive propagation cases than single influence propagation ones and the fundamental reason for this phenomenon is not only the existence of several promoters in the real market but also that their products and target groups are likely to be similar. For this reason, a few existing studies on IM problem point out that we should identify the effects of the adversary or competitor in IM problem.

Most of the papers on IM problem deal with LT and IC model for the multiple and competitive influence diffusions [16-18]. Borodin et al. [16] used two threshold values for the competitive influence between the two promoter groups. And, the authors present a number of fairly natural and general approaches for the spread of the approximation technology. He et al. [17] studied the influence blocking maximization in OSN under the competitive LT model and the researchers work focused on the ways to block the influence diffusion of an adversary group to the maximum possible extent. Bharathi et al. [18] utilized the IC model to model competitive influence. The researchers introduced first-mover strategies and second-mover strategies for the two-player game in OSN.

A reported study [19] divided influence propagation models into three classes based on adversary’s position. For adversary, there can be two positions, active or passive. The passive adversary is just the position that prevents the conversion of uninfluenced node’s state. On the other hand, active adversary tries to convert all the naïve nodes into his state. Yu et al. [20] stated the perfect example of the passive adversary and referred to it as a blocker. Identification of key blockers in dynamic OSN was the major issue in their work. On the other side, the work by Bharathi et al. [18] could be a nice example of an active adversary. Actually, most of the papers on IM problem with the adversarial environment assume that the adversary group has the active position. We also assume that the adversary group wants to convert naïve nodes’ state to their own state.

Game theory could be one of the solutions for the IM problem with the competitive environment. While the studies [18, 21] assume that the active adversary tries to convert nodes from the naïve state to their own state, Nowak et al. [22] take it for granted that every node has only two states, promoter’s state, and adversary’s state, so that there are no naïve nodes. Our present work is about the evolutionary game theory and it could be perceived as a general approach that describes not only the competition of species in an ecosystem but also IM problem with two adversarial groups. We have also developed our proposed scheme on two-state node environment.

To sum up, related works on IM problem with an adversary can be investigated by several views. First of all, many papers suggest LT model and IC model by considering adversarial condition. Second, recent approaches to solving competitive influence diffusion could be divided based on adversary's attitude i.e. active or passive. Finally, game theory could be applied to IM problem with adversary if the node has two states, the promoter's state or the adversary's state.

III. BACKGROUND AND PROBLEM DEFINITION

A. Betweenness Centrality

In a large network, such as in OSN graph, all the nodes are not treated equally. For example, removal of a bridge node between the two sub-graphs yields two disjoint graphs, signifying changes in the properties of the graph. However, removal of a terminal in a graph causes an only little impact on the structure of the graph. For instance, as can be seen in Fig. 1, removal of a node n_1^5 has no impact on the graph structure while removing a node v divides the graphs into two disjoint graphs.

Betweenness centrality (BC) is a measure of the degree to which vertices lie between other nodes [23]. The BC can have a significant impact on the network by controlling the communication of information with others. For communication between the nodes in different clusters, it is hypothesized that the higher BC node could connect them with each other.

The BC is formulated to capture the property of connectivity by representing the ratio of the number of shortest paths passing through a given node over all the possible ones between the two nodes to recognize a bridge node. BC is given in [24], as formulated in (1) as

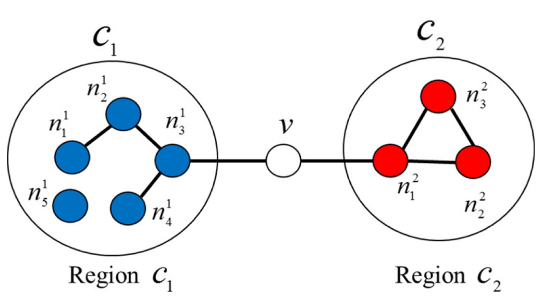

Fig. 1. Node connectivity in a graph.

$$BC(n) = \frac{1}{|V|(|V|-1)} \sum_{s,t \in V} \frac{\sigma(s,t|n)}{\sigma(s,t)}, \quad (1)$$

where $\sigma(s,t)$ is the number of the shortest paths between the nodes s and t, and $\sigma(s,t|n)$ is the number of the shortest paths between the same nodes passing through node n. Nodes n, s, and t are different nodes and elements of V.

B. Clustering Coefficient

The clustering coefficient (CC) is the probability that a particular node and neighboring nodes are connected to each other [25]. It is known that the density of connections between social network nodes is relatively high compared to randomly generated network nodes. Higher CC nodes are more likely to create clusters, which could indicate their impact on the cluster. CC represents the number of connections of a node n for which n has the degree, $deg(n)$, and is essentially the number of edges on the node n. The CC of a node is given in [26] and is represented as (2).

$$CC(n) = \frac{2e_n}{deg(n)(deg(n)-1)}, \quad (2)$$

where e_n is the number of edges between the neighbors of a node n. As a result, for the global structure analysis, the average CC can be computed as follows:

$$CC_{ave} = \frac{1}{|V|} \sum_i^{|V|} CC(i). \quad (3)$$

IV. PROPOSED SCHEME

A. System Model

As discussed in Section 1, we have an OSN that is represented as a graph, $G = <V, E>$, where V is the set of users, and E is the set of edges in G. The graph G has players (p_i) whose goal is to maximize their influence, including within an adversary's region. The i is the index of a player and $2 \leq i \leq k_i$ (k is the number of players). We assume that the number of players is bounded by k and each player has its own region. Let the region for p_i be c_i. Deterministically, $\cup_{i=1}^{k} c_i = G$, as shown in Fig. 2. The p_i has complete knowledge of c_i, which includes the nodes and edges in c_i. However, p_i has no way to know the structure of $G \backslash c_i$. Let p_j be the j-th node in c_i. Every edge is a relationship between the two nodes in G, as represented by $e_{u,v} \in E$ where u and v are any nodes in G. Note that u and v can either belong to the same region or not. For example, p_1 can have complete knowledge of c_1 in terms of the number of nodes that exist and how many edges are connected among n_j^1, where $1 \leq j \leq$

k, as seen in Fig. 2. Also, p_1 can extract the information concerning which nodes from $c_{i \neq 1}$ are connected with c_1. However, p_1 does not know how many nodes and how many edges are configured in $c_{i \neq 1}$.

Problem 1. Given an undirected OSN graph $G = <V, E>$, and clusters in G, a positive integer k and l, and k sub-graphs, find a set S of c_i such that $|S| \leq l$, $S \subseteq c_i$, and $\cup_{i=1}^{k} c_i = G$ where c_i is a cluster in G.

B. Weak Ties and Local Bridge

In social network theory, the relationships between users can be defined as *strong ties* or *weak ties* [27]. Strong ties are a strong relationship, such as those with friends, and weak ties correspond to acquaintances. *Triadic closure* is a well-known principle in sociology that states that "*if two people in a social network have a common friend, then there is an increased likelihood that they will become friends themselves at some point in the future*" [27, 28]. Easley and Kleinberg [27] described the *strong triadic closure* using strong and weak ties such that if a node has two neighbors with strong ties, the two neighboring nodes get connected with either a strong or a weak tie. A more generalized strong triadic closure could be defined such that if nodes n_1 and n_2 have a common neighbor n_3, any neighbor of n_3 is also a neighbor of n_1 and n_2 [29]. In a graph, we notate the two types of edges as a *bridge* and a *local bridge*. The bridge is an edge where if the edge is deleted, the graph is split into two different components. In other words, the local bridge is an edge whose nodes have no common friend [30]. Further, it has been proven that all local bridges are weak ties if a node satisfies the strong triadic closure property [27]. This condition implies that the weak connections play a valuable path in reaching another unknown network.

We assume that triadic closure is held in an OSN graph with strong arguments (selection and social influence) as discussed previously [2]. Our intuition on this assumption is that if users are the same, their activities stay within the same OSN. For example, if a user is a Facebook user, his/her friends are more likely to be Facebook users and form strong ties. However, if the user can have friends as acquaintances, the friends of his/her acquaintances are more likely to be in a different OSN. We portray an example of local bridges and weak ties in Fig. 3. Note that the acquaintances possibly form weak ties in the same OSN. The local bridges including weak ties may play the role of a conduit in order to maximize influence over an unknown social region (or cluster), thus giving a clue to solving the IM-SA problem. We have described the IM-SA scheme in the following sub-section by considering a weak tie and a local bridge.

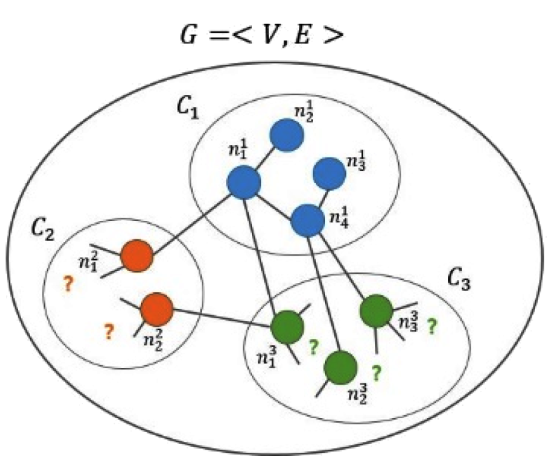

Fig. 2. A graphical example with three social players.

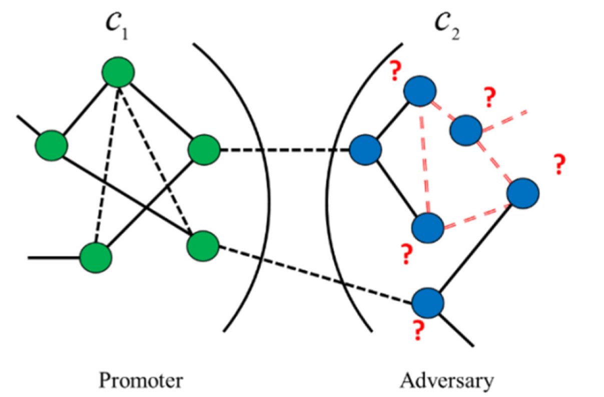

Fig. 3. An example of local bridges (satisfying strong triadic closure property as mentioned previously [27] with strong ties (black solid lines) and weak ties (black dotted lines). The promoter (c_1) knows his region c_1 but does not know about the number of nodes are in the adversary region (c_2) and how the nodes in c_2 are configured.

C. Selecting an Influential User Set

Since our goal was to select l seed users without any knowledge of the adversary's graph structure, we formulated the seed selection scheme with a modified greedy hill climbing (GHC) algorithm, as shown in Fig. 4. We call our scheme as GHC in an adversarial network (GHC-A). To build GHC-A, we had to first consider the properties of G with BC.

To facilitate the local bridge and the weak tie, we need to include the importance of the nodes associated with the local bridges. We formally define the local bridge as follows:

$$e_{u,b}^{lb} = \left\{ \begin{matrix} e = (u,v) \in E | u \in c_i, v \in c_j, u \ and \ v \in G, \\ i \neq j, u \ and \ v \ has \ no \ 1 \ hop \ common \ neighbor \end{matrix} \right\}.$$

Let LB be the set of nodes associated with local bridges. In Algorithm 1, GHC selects a node that can maximize the information cascade (refer to line 3 in Algorithm 1). We con-

Algorithm of GreedyHillClimbing ($\mathbf{l}, \boldsymbol{f}$)

1: Initialize $S := \phi$
2: For i=1 to l do
3: Select a node n such that
$n = argmax_{w \in V \setminus S}(f(S \cup w) - f(S))$
4: $S := S \cup \{n\}$
5: End for
6: Return S

Fig. 4. GHC algorithm with two given inputs (l, f), where l is the cardinality of the seed set and f is the function of an information cascading model.

Algorithm of GreedyHillClimbingAdversary ($\boldsymbol{l}, \boldsymbol{f}, \boldsymbol{\Gamma}$)

1: Initialize $S := \phi$
2: Verify the set of nodes associated local bridges (LB)
3: For j=1 to l do
4: Compute N_j^f and N_j^S
5: Compute $SCORE_j^f$ and $SCORE_j^{lb}$ using (4) and (5), respectively
6: Compute $SCORE_j^{tot}$ for all nodes $n_j^i \in c_i$ using (6)
7: Select a node n such that
$n = argmax_{n_j^i \in c_i}(SCORE_j^{tot})$
8: $S := S \cup \{n\}$
9: End for
10: Return S

Fig. 5. GHC-A exploiting local bridges with node characteristics (BC and CC).

sider the impact of the local bridge with a weighting function so that GHC-A can maximize the spread of influence, including the promoter's as well as the adversary's region. We introduce the weighting function on $e_{u,v}^{lb}$ as follows.

Let us define three metrics to build GHC-A such that

- N_j^f: The number of activated nodes after running f, using the seed set (S), and including a node n_j^i, where c_i is the region of the promoter.
- N_j^S: The number of activated nodes belonging to LB after running f using a new seed set (S) including a node n_j^i, where c_i is the region of the promoter.
- $SCORE_j^f$: The normalized score of a node n_j^i such that

$$SCORE_j^f = \frac{N_j^f}{\max(\{N_k^i | k \in S \cup n_j^i\})}. \quad (4)$$

- $SCORE_j^{lb}$: The strength of LB, which is the aggregate of BC and CC if $n_j^i \in$LB and $n_j^i \in c_i$, where c_i is the region of the promoter. The mathematical form is defined as

$$SCORE_j^{lb} = \sum_{k=1}^{N_j^S}(BC(n_k^i) + CC(n_k^i)) \quad (5)$$

- $SCORE_j^{tot}$: The total score considering the IM in promoter's region and LB.

$$SCORE_j^{tot} = \frac{\Gamma}{\Gamma + N_j^f} \times SCORE_j^f + \frac{N_j^S}{\Gamma + N_j^f} \times SCORE_j^{lb}. \quad (6)$$

The modified algorithm (i.e., GHC-A) using (4)–(6) is depicted in Fig. 5. Note that the Γ is the key weighing factor in GHC-A model. By controlling the Γ, we monitor the impact of a seed node between promoter's region and adversary's region beyond the LBs. With the increase in the Γ, more priority is given to $SCORE_j^{lb}$. Otherwise, $SCORE_j^f$ gets more priority than $SCORE_j^{lb}$. Therefore, to investigate the impact of BC and CC, we can use (6) by controlling the Γ. We analyze the Γ by varying the size of Γ as mentioned in Section IV.

D. Optimality of Proposed Algorithm

In this subsection, we provide the optimality of our algorithm which can guarantee a factor of 1-1/e of the optimal solution, where e is the base of the natural logarithm. After obtaining mathematical proof of GHC and GHC-A, we provide optimal lower bound of our algorithm. Since the basic structure of GHC and GHC-A is same except for the three additional computational steps for computing N_j^f, N_j^S, $SCORE_j^f$, $SCORE_j^{lb}$, $SCORE_j^{tot}$ (please refer lines # 4,5,6 in Fig. 5), we have proven the optimality of Algorithm 1 (GHC).

Let $f(\cdot)$ be the function such that find a number of activated nodes with given input set.

We call $f(\cdot)$ in '*monotonic*' if and only if $S \subseteq T \rightarrow f(S) \leq f(T)$. Also, we define '*sub-modularity*' if and only if *for all* $S \subseteq T$, $f(S \cup \{v\}) - f(S) \geq f(T \cup \{v\}) - f(T)$, where v is a node of graph G.

Finding optimal k set with a number of nodes to maximize influence is the same as a set covering problem (SCP) finding k sets completely cover universe set U, which is known as NP-Hard. Let OPT be the optimal set. GHC is monotone and sub-modularity with the Algorithm 1 line #3 in Fig. 4.

Let $T = \{w_1, w_2, ..., w_k\}$ be the optimal solution of size k,
$S = \{n_1, n_2, ..., u_k\}$ be the solution obtained by GHC,
$S_i = \{n_1, n_2, ..., u_i\}$ be the solution obtained by GHC with i nodes,
$S_{i+1} = S_i \cup \{u_{i+1}\}$ be the solution obtained by GHC adding u_{i+1}

into S_i,

$\delta_{i+1} = f(S_i+1) - f(S_i)$ be the marginal gain at the i-th iteration.

$$f(S_{i+1}) = f(S_i) + \delta_{i+1}. \quad (7)$$

Since $f(T) - f(S_i) \le f(T \cup S_i) - f(S_i)$

$$\le \Sigma_{n_j \in G \wedge n_j \notin S} f(S_i \cup \{n_j\}) - f(S_i) \le k \cdot \delta_{i+1},$$

$$\delta_{i+1} \ge \frac{f(T)-f(S_i)}{k}. \quad (8)$$

By using (8), we rewrite (7) as follows

$$f(S_{i+1}) \ge f(S_i) + \frac{1}{k}(f(T)-f(S_i)),$$

$$f(S_{i+1}) \ge \left(1-\frac{1}{k}\right) f(S_i) + \frac{1}{k} f(T). \quad (9)$$

The final form of optimality by induction using (9) is as follows

$$f(S) = f(S_k) \ge \left(1-\left(1-\frac{1}{k}\right)^k\right) f(T)$$

$$\approx \left(1-\frac{1}{e}\right) f(T) \approx 0.63 \cdot \mathrm{OPT}. \quad (10)$$

Since the basic algorithm structures are the same between GHC and GHC-A, the optimality of GHC-A is trivial. Also, our algorithm is near-optimal and guarantees the minimum 63% of the optimal solution.

V. EVALUATION

A. Simulation

We simulate our GHC-A scheme using Eq. (6), which reflects the influence level of a promoter's cluster as well as adversary's influence on LBs. We first generate two clusters in a graph in this paper. We argue that two clusters are enough to validate our algorithm (GHC-A) since if the effectiveness of IM is valid between clusters, GHC-A is scalable on a more complex graph including more than three clusters. We leave the extensive simulation with a more complex graph structure as our future work. We choose the IC model to simulate the IM-SA. Note that the LT model can also be applicable to our scheme. We generate 150 nodes for each of the two clusters (promoter and adversary clusters). We set the ratio of LB between the two clusters as 0.2. To verify the impact of Γ, we vary the value of Γ from 1 to 17.

In our simulation, we were able to monitor two aspects of OSN structure. First, we could identify the number of activated nodes by GHC and GHC-A. If the number of activated nodes using GHC-A is larger than that of GHC, we conclude that GHC-A is more effective with respect to IM problem. Second, we can check the impact of Γ by monitoring the number of activated nodes when we use GHC-A.

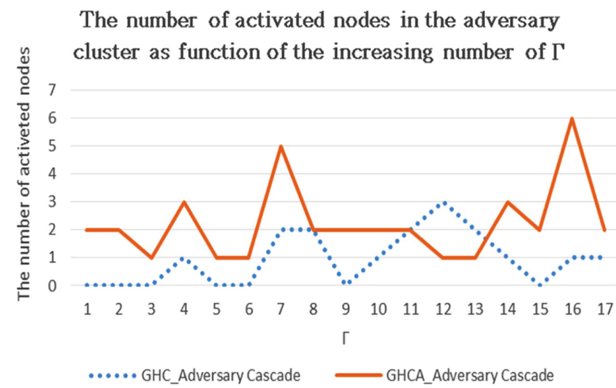

Fig. 6. The number of activated nodes in the adversary cluster as a function of the increasing number of Γ.

To make our simulation more statistically reliable, the simulations were repeated 20 times for each Γ setting, and the outcomes were averaged. The results are represented in Fig. 6.

B. Analysis

As shown in Fig. 6, the dotted-blue line shows the result of the existing cascade algorithm (GHC); the solid-orange line shows the cascading result using GHC-A. The x-axis is the varying value of Γ. Except for the range of Γ as described previously [11, 13], our scheme showed better performance than GHC. We note that the GHC-A achieve two factors of GHC in terms of the activated nodes and the threshold value. Theoretic guarantee of GHC and GHC-A (the minimum number of activated nodes) is the same as shown in IV-C. However, if we use additional information reflecting network structure (BC and CC), the result of the performance can be increased (refer to Fig. 6.) Namely, we found that there exists the sub-optimal threshold of Γ around 10. If the system designer knows the OSN structure, he may deploy the different strategies by facilitating the Γ in order to increase the performance of IM-SA. The finding optimal Γ is not trivial. We choose the IC model to simulate the IM-SA performance. We consider the analytic study on Γ as one of our future work. However, it was noted that our scheme works better than the traditional IM approach by controlling Γ.

The time complexity and running time of GHC-A can be derived from Fig. 5. From line #3, the algorithm iterates l times. And the inner loop of the computation of N_j^f, N_i^f and $SCORE_j^f$ is constant c. The computing time of $SCORE_j^{lb}$ and $SCORE_j^{tot}$ is $N_j^S \times c$. In line #7, $argmax_{n_j^i \in c_i}(SCORE_j^{tot})$ requires the number of the cardinality of c_i($i.\ e.$, $|c_i|$). If we consider $|c_i|$ as $[|V|/2]$ in G by taking average nodes of two clusters, we can count the number of iterations as follows. Let $[|V|/2]$ be Λ.

$$\text{1st iteration of for loop: } \Lambda \times N_j^S \times c, \quad (11)$$

$$\text{2nd iteration of for loop: } (\Lambda - 1) \times N_j^S \times c, \quad (12)$$

$$\vdots$$

$$(\Lambda - 1)\text{-th iteration of for loop: } 2 \times N_j^S \times c, \quad (13)$$

$$\Lambda\text{-th iteration of for loop: } 1 \times N_j^S \times c. \quad (14)$$

If we sum up all the iterations of the for loop, the computation time is

$$\Sigma_{i=1}^{\Lambda} i \times N_j^S \times c = \frac{\Lambda \times (\Lambda + 1) \times N_j^S \times c}{2} = \frac{\left[\frac{|V|}{2}\right] \times \left(\left[\frac{|V|}{2}\right] + 1\right) \times N_j^S \times c}{2}. \quad (15)$$

Note that the denominator of $[|V|/2]$ can vary. However, the upper bound of rthe unning time is still valid in our case. The total computational time is

$$l \times \frac{\left[\frac{|V|}{2}\right] \times \left(\left[\frac{|V|}{2}\right] + 1\right) \times N_j^S \times c}{2} \approx l \times |V|^2. \quad (16)$$

Therefore, the time complexity and running time is $O(l \times [V]^2)$. If we take small size of seed users, the running time is reduced to $O(|V|^2)$.

VI. CONCLUSION

The influence maximization problem has been studied under a number of domains to describe the effects of the "word of mouth" for the promotion of new products. In reality, however, more than two social groups exist in the same market sector. In this paper, we first introduce the Influential Maximization (IM) problem for Adversarial social players, the IM-SA. Through a performance evaluation based on mathematical analysis, we demonstrate that the proposed scheme can provide the sub-optimal solution for a given player.

ACKNOWLEDGMENTS

This work was supported by the Ajou University research fund and Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (No. 2017R1D1A1B03035557) and by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. NRF-2018R1C1B5083091).

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Giseop Noh
received B.S. degree in Industrial Engineering from R.O.K Air Force Academy in 1998 and an M.S. degree in computer science from University of Colorado Denver in 2009, and Ph.D. degree in Computer Engineering in Seoul National University. He is currently an assistant professor in Cheongju University. His current research interests include recommender system, big data analysis, online information managements and social network analysis.

Hayoung Oh
received Ph.D. degree in Computer Science from Seoul National University in 2013. From 2002 to 2004, she joined the Shinhan Financial Group as a developer in applied research. In 2010, she was with UC Berkeley as a researcher. Since 2016, she has been with Ajou University as a professor in DASAN University College for the Education of Software. Her research interests include social and wireless networking.

Jaehoon Lee
received B.S. degree in Information & Communication Engineering from Dongguk University in 2011 and an M.S. degree in computer science from Seoul National University in 2013. He is currently a Ph.D. candidate at Seoul National University. His current research interests include recommender system, big data analysis and social network analysis.