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Designing Rehearsals for Secondary Preservice Teachers in Mathematics Methods Course

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ABSTRACT. This study identifies elements involved in designing rehearsals for improving preservice teachers' capacity to teach mathematics. Observation of a secondary mathematics methods course and regular interviews with the teacher educator following each class were used in this research. After characterizing what is considered and enacted in rehearsals as a way to help preservice teachers practice the work of teaching mathematics, I illustrate them with examples from the observations and interviews. I then discuss the challenge of dual contexts-the teacher education classroom and the secondary mathematics classroom-and dual perspectives-the mathematical and pedagogical-in designing and enacting rehearsals. I conclude with implications for mathematics teacher education.

Ⅰ. Practice Demands in Mathematics Teacher Education

As the notion of core teaching practices for helping students learn mathematics are brought to the fore, the demand for mathematics teacher education programs to focus on practice has increased (e.g., Conference Board of the Mathematical Sciences, 2012; National Council of Teachers of Mathematics & Council for the Accreditation of Educator Preparation, 2012). Despite indication of international convergence, the pedagogies of teaching professional practice still remain idiosyncratic and learned on the job. In principle teacher educators can draw from their personal teaching experiences, potentially the experiences of colleagues in

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different contexts, and reports from practitioner and research literature to determine the substance of the curriculum, such as including units on instructional explanations or leading discussions. These are good resources. However, it still remains for the teacher educators to synthesize these resources, and to reason as best they can when making decisions about how to organize, set up, and assess activities intended to prepare preservice teachers to teach their future K-12 students well. One reason for difficulties in organizing activities for teacher education, even given indications of the goals of teacher education, may be the relatively uncharted territory of frames for decision making about activities for teacher education.

Pedagogies of teacher education are a young but important field for the improvement of teacher education. Shared pedagogies can accelerate teacher educators' capacity to articulate and work on the problems of instruction in teacher education (e.g., Elliott, Kazemi, Lesseig, Mumme, & Kelley-Petersen, 2009). Such problems include modeling core teaching practices demanded by teaching and situating it in the context of teaching. For the growing body of literature on pedagogies in teacher education, this study focuses on "rehearsals," or simulations of teaching practices, that were used for the purpose of improving skill at core practices. In this article, I examine: what can be considered in designing a rehearsal to create approximations of practice about secondary mathematics teaching practice?

This study draws data from observations on a mathematics methods course for secondary preservice teachers and interviews with the teacher educator of the course. For the investigation, this study examined rehearsal design and enactment of decision making, and then investigated what functioned individually and with each other. This research moreover illustrates what works in designing and implementing rehearsals with examples from the observations and interviews. This research concludes with a discussion of the role of challenges of dual contexts and dual perspectives in teacher education.

Ⅱ. Background

As Lampert (2010) articulates, "practice" is used in many ways. "Teaching practice" can be used to contrast with theory, and "teaching practices" can refer to the constituent components of teaching practice in the first sense. Such constituent components might include leading a discussion, diagnosing and responding to K-12 students' errors, or choosing and using representations (e.g.,

Grossman, Hammerness, & McDonald, 2009). Yet a third meaning is "to practice", as in to rehearse. This article examines preservice teachers' practicing of teaching practices, such as in a rehearsal to prepare them for teaching practice in contrast with learning teaching theory. Though teacher educators differ somewhat in their criteria for appraising the centrality of a particular teaching practice, there is agreement in a focus on teaching practices that are somehow central to the work of teaching. Their criteria include that the practice occurs frequently, that preservice teachers must employ and can learn the practice, and that skillful deployment of the practice will improve K-12 students' learning or the ability for preservice teachers to learn from K-12 students' thinking (Lampert, 2010). Professional preparation in teacher education is intended to ready preservice teachers for the daily work of teaching. Coursework can supplement fieldwork by providing learning opportunities where preservice teachers have greater freedom to experiment with and engage in deliberate practice. Pedagogies for such opportunities include approximations of practice, which are opportunities for preservice teachers to "engage in practices that are more or less proximal to the practices of a profession" (Grossman, Compton, et al., 2009, p. 2058). Approximations can approximate practice to varying extents, balancing authenticity with opportunities to stumble, reflect, or strategize.

As a social setting for building preservice teachers' commitment to teach confidently, furthermore, rehearsals involve preservice teachers publicly and deliberately practicing how to teach mathematics (Lampert et al., 2013). It offers a setting where a teacher educator can use performance as a site for leading the cooperative elaboration of work of teaching. A preservice teacher is responsible for teaching a provided mathematical topic in rehearsal while the teacher educator simulated as a school student in ways that intentionally represent the intellectual and social range of actions that might be anticipated in an actual class (Lampert & Graziani, 2009). Other preservice teachers participate with their understanding of the secondary students they will teach. The simulation can represent the multiple relationships with secondary students and content that might be in play, as well as the routine and spontaneous instructional interactions that must be managed in teaching (Lampert et al., 2013, p. 229). The teacher educator performs as simulated student and manages the rehearsing preservice teacher and the other preservice teachers to examine what the preservice teacher does. The assessment about the rehearsal provides rehearsing preservice teachers a group of colleagues within which to figure out what they recognize the work of teaching in the rehearsal.

Rehearsals support the learning of skillful teaching, whether skillfulness is

described in terms of mediating K-12 student learning (e.g., Lampert et al., 2013) or in terms of teachers' ability to learn about teaching and learning through teaching (e.g., Boerst, Sleep, Ball, & Bass, 2011). Grossman, Hammerness, and McDonald (2009) hypothesized that approximations such as rehearsals would allow teacher educators to focus on specific strategies. Ghousseini (2009) found that rehearsals afford opportunities to develop preservice teachers' judgment and skill in teaching practices; she examined the pedagogies of modeling and rehearsing on a context involving teaching multiplicative reasoning. Kazemi, Franke, and Lampert (2009) observed the importance of how the rehearsed practice is decomposed, in what order the constituent components are emphasized, and what representations best support learning. Lampert et al. (2013) examined rehearsals in the context of instructional activities with number and operation across three sites. Instructional activities are a sequence of activities that involve modeling of a teaching practice via a specific mathematics activity by a teacher educator, planning classes, and public rehearsals with fellow preservice teachers; these activities build up to enacting the specific mathematics activity with children. They also identified work that teacher educators and preservice teachers do together and opportunities for preservice teachers to learn from rehearsals, focusing especially on how teacher educators and preservice teacher take up the teaching practice of eliciting and responding to K-12 students' thinking.

Rehearsals in teacher education have been studied primarily with elementary and middle school mathematical contexts with highly experienced teacher educators. The literature on rehearsals has focused on descriptions of and principles for enactments. This study agrees with that the principles identified by these scholars, such as a commitment to teachers' need to learn to "elicit, observe, and interpret K-12 students' reasoning, language, and arguments and to adjust their instruction accordingly to promote learning" (Lampert et al., 2013, p. 227), are integral to teacher education. Furthermore, because teaching does, teacher education also must intertwine the perspectives of pedagogy and mathematics (Ball & Forzani, 2007; Darling-Hammond, 2006a; Grossman, 2006). This study seeks to elaborate some of the objects that "bound" the scope of an approximation. Approximations by definition do not convey the full complexity of teaching. To direct focus on particular aspects of teaching, approximations need to restrict what is being discussed; this study examines what help to highlight aspects of teaching for the purpose of learning teaching practices.

Rehearsals require designing approximations of practice that work to support preservice teachers' learning of the relational aspects of teaching. Rehearsals have

not been studied with secondary mathematics as the mathematical context, nor have they been studied with a focus on constituent components of the objects within the design of an approximation. This study contributes to the literature by decomposing some objects that are considered in planning approximations in the context of secondary mathematics. Thus, the current study investigates what would shape to design a rehearsal for secondary preservice teachers in teacher education.

Ⅲ. Data and Method

I took inductive thematic analysis to the data (Guest, MacQueen, & Namey, 2012; Saldaña, 2013). Its purpose is to identifying conceptual elements to create approximations of practice about secondary mathematics teaching practice. In the recorded interviews and classes and other resources, I particularly examined what is considered in planning and enacting rehearsals with secondary preservice teachers. These were grounded moments and episodes in the recordings. While these interviews and classes might not involve the typical instructional demands as teacher education instruction, they would offer beneficial resources to examine how to organize and enact rehearsals to teach practice of teaching in order to have stable elements that could work well for a pedagogical goal of teacher education. I also used Grossman and her colleagues' (2009) demonstration about approximations of practice, which are opportunities for preservice teachers to engage in practices that are proximal to the practices of a mathematics teacher. This inductive process generated seven elements that can help teacher educators who intend to bring approximation of practice with a form of rehearsals for their preservice teachers. I revisited each recorded interview and classes and other resources again to elaborate each descriptive summary into an analytical explanation of each step. Ideas generated in the data analysis were further examined by repeating the analysis of the data through a constant and comparative process (Miles, Huberman, & Saldaña, 2014). Important episodes were identified and coded with relevant justification about the seven elements. The characterization of each element was crystallized. Thus, although grounded in empirical data, this study is primarily conceptual.

1. The Setting

This research study was completed at a large Midwestern state university in

the United States with twenty mathematics preservice teachers. The university offers a teacher certificate program to secondary teachers. The teacher education program emphasizes improving preservice teachers' capability to perform specific teaching practices. There were thirteen classes in this methods course, each three hours in length. Each class was planned by the teacher educator and with regular consultations about course activities with two former secondary teachers who were the field instructors, one inservice teacher, and occasional consultations with a group of secondary mathematics teacher educators from across the country. The overall and specific plan of this course was thus consisted by the product of synthesizing experiences and expertise.

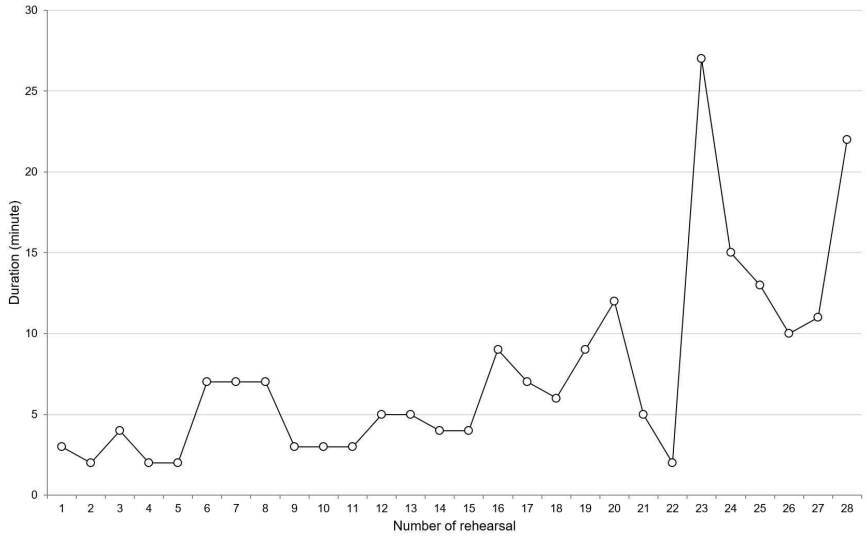
Preservice teachers were third-year undergraduates in middle and secondary school mathematics specialization program. In this semester, preservice teachers visited the secondary schools once in a week with the field instructors, who also participated in this course. There was no direct relationship between rehearsals and the field experience, although preservice teachers were encouraged to draw upon their field experiences in enacting and commenting on rehearsals.

2. Data

The data for this research are interviews of a teacher educator and observations of her mathematics methods course for twenty secondary preservice teachers. The data from the observation included whole course thirteen classes for one semester, and slides with planning notes created prior to each class and which contained the learning goals for the class. Following each class, the teacher educator was interviewed, prompting her to appraise the class and the preservice teachers' learning in terms of the goals for that class, and talk about any mathematical and pedagogical issues that arose during planning or teaching. Interviews also included the teacher educator's reflections relating to her thinking about why these events occurred and how to address them. All observations and interviews were audio recorded.

In total, 28 recorded rehearsals occurred over thirteen classes, as shown in Table 1. As the weeks progressed, the rehearsals generally increased in length, from under 5 minutes to over 25 minutes as shown in Figure 1. Preservice teachers worked in groups of four on the planning aspect of rehearsals. Groups were chosen by the teacher educator to present their rehearsal. If chosen, one group member enacted the plans. The rehearsals increased in complexity over time. As evidenced by Table 2, which summarizes the focal teaching practices, the

teaching practices of prior rehearsals were nested in the teaching practices of subsequent rehearsals.



[Figure 1] Duration of each rehearsal (in minutes)

Class	1	2	3	4	5	6	7	8	9	10	11	12	13	Total
Number of rehearsals	0	5	Not recorded	3	5	2	2	2	1	2	1	4	1	28

[Table 1] Number of Rehearsals in Each Class

Rehearsal	Class	Focal teaching practice for rehearsal (topics addressed)
1, 2, 3, 4, 5	2	Setting up a mathematical task (concavity and rate of change)
6, 7, 8	4	Launching and concluding a discussion (area of a region in square centimeters and then in square millimeters)
9, 10, 11, 12, 13	5	Making explicit connections between multiple representations (functions and discontinuities)
14, 15	6	Using an example to explain a procedure (graphs of functions and vertical shifts)
16, 17	7	Using an example to explain a procedure (extrema of quadratic functions)
18, 19	8	Using an example to explain a concept

		(power function, polynomial)
20	9	Diagnosing secondary student thinking (solving multi-step equations)
21, 22	10	Planning and using formative assessments (solving multi-step equations)
23	11	Using discourse moves and questioning to elicit secondary student thinking (algebraic expressions and equations)
24, 25, 26, 27	12	Leading a discussion (algebraic expressions and equations)
28	13	Leading a discussion (algebraic expressions and equations)

[Table 2] Focal Teaching Practices for Rehearsals

3. Vignette: Using an Example to Introduce a Mathematical Concept

To offer a taste of a rehearsal from the data in this study, this chapter brings a vignette of Class 8 whose focal teaching practice was using an example to introduce a mathematical concept. This vignette will be used again in the result section to demonstrate each element that the current study found.

In Class 8, rehearsals approximated planning for, doing, and reflecting on the focal teaching practice of using an example to introduce a mathematical concept. The approximations responded artifacts from local textbooks. The class began with a demonstration of the focal teaching practice by the teacher educator using a textbook representation of a ski lift, which is based on an example of a textbook (Bellman et al., 2009), to explain the concept of slope as a piecewise linear function with Figure 2.

The textbook representation was used to highlight a possible decomposition (Grossman et al., 2008) of this instance of explanation (e.g., Leinhardt, 2001), featuring the components of: motivating the idea with an example, which is possibly a story problem or real world example of a mathematical phenomenon, defining the concept precisely, and making explicit connections between the motivating example, the definition, any notation or expressions or equations used, and any terminology key to understanding the definition. In a rehearsal, skillful performance at these components was defined to involve mathematical talk by both preservice teacher, who played by a preservice teacher enacting an explanation, and secondary students, who played by other preservice teachers. The teacher educator described other examples related to slope and engaged the teacher educators in small and whole group discussions about how the features of the examples related to the concept of slope. She emphasized use of features to

Rehearsal: Introduce the concept of power function.

Materials to use: Textbook chapter on polynomials with the braking distance example.

Be sure to:

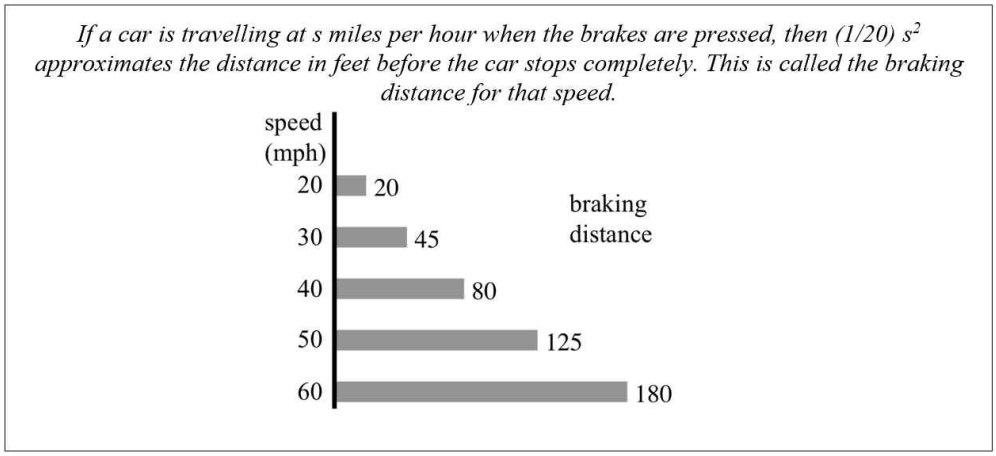
- **Motivate the concept with an opening example.** For this rehearsal, use the braking distance example from the homework.
- **Define the concept precisely.** Define any notation and equations you might use. Use vocabulary that might come up later in the chapter.
- **Relate the motivating example, definition, notation, and key vocabulary.** Make explicit connections.
  - Use the example to draw out key features of the concept
  - Relate the motivating example to the definition, notation or expressions or equations, and key vocabulary
  - In your connections, keep focus on what is most needed by the concept or how the concept will be used in the section of the textbook

[Figure 2] Instructions distributed to preservice teachers by the teacher educator in Class 8

help secondary students make sense of mathematics. She then introduced a rehearsal, which used an example about braking speed the preservice teachers had worked with on homework prior to the class. The preservice teachers worked in groups to plan such explanations. The braking distance example was as shown in Figure 3. The graph and text are based on that in the textbook. Prior to planning, the preservice teachers had worked on a warm-up featuring the definitions of power and polynomial functions extracted from local textbooks. The definition of power function used was "A power function is a function that can be expressed as $f(x) = cx^a$ where a and c are nonzero constants and x is a variable."

One preservice teacher, Kelly, volunteered and rehearsed her group's planned explanation. She started to describe a scenario with a brick wall 100 ft away from a moving car to her hypothetical secondary students, and walked through the textbook's example, saying that if the car were moving at 20 mph, the car would be able to stop before hitting the brick wall; but that if the car were moving at 50 mph, it would not "stop in time." Then she said, "So, what we see here is a function of a particular form. We're going to see several functions of similar forms. So what we're going to do is generalize. In this example we have some number here, in this case 1/20. Let's call that c." She wrote c on the board, then reviewed the terms coefficient and exponent by calling on her "secondary students" to define the terms. As she reviewed these terms, she wrote the

This preservice teacher's name is a pseudonym.



[Figure 3] The example that preservice teachers worked in groups

expression cx^a on the board in tandem. Finally, she summarized, "So a power function only has one term. That's very important because we'll see functions later with more than one term. A power function has a coefficient multiplied by a variable raised to some exponent. And the exponent and coefficient are real numbers."

After performing her rehearsal, Kelly first commented that she was not sure in a classroom, how secondary students "would take" the abstraction from an example to a formula. As she said, "I think students are going to see that example and think that they're supposed to find some value of c or something. But we're trying to get from this a general formula, and I am not sure how students would take that." Other preservice teachers also commented several times that Kelly related the context to the mathematical idea well. One preservice teacher commented that Kelly might have made an "explicit distinction" between power function and monomial, as both types would show up in the next section, the assignment for both functions can be expressed as a single term, but not all monomials are power functions. No one comments on Kelly's use of the idiomatic phrase "stop in time," which though colloquially valid, is mathematically inaccurate as the variables are speed and distance, not time.

Finally, the teacher educator commented on pressing further when making connections between a context and a mathematical expression. "When you're asking for understanding, you might make sure that your students know what the a and the c refer to. So you look at the general formula, and you go back to your example, and you say, what's the a? What's the c? What does it mean in terms of cars and braking?" The preservice teachers were then asked to plan an

explanation for polynomials using an example drawn from another textbook. A similar routine of planning, enactment, and reflection followed. This ends the vignette of one rehearsal in Class 8.

4. Data Analysis

Data analysis was based on multiple observations of all the recordings, the course syllabus, planning notes and the slides that the teacher educator used in each class. Using these resources strengthened the data interpretation and helped identify what the teacher educator used in designing and enacting rehearsals. Inductive thematic analysis to the data by Saldaña (2013) was used with the typical phases of data collection, note taking, coding, memoing, and sorting in order to analyze the data for identifying elements of interview statements and actions in the classrooms that could be construed to be informed by setting up and enacting activities for the class. These phases overlap and the process is repeated as categories emerge that describe the events. The theory is said to emerge as it is discovered in the data (Miles et al., 2014).

My initial sort of the data looked specifically at evidence related to emergent issues that the teacher educator considered in her planning and enacting for rehearsals. In this step, there emerged two different layers: teacher education and secondary mathematics classroom. Because the teacher educator needed to create an approximation in her teacher education classroom, these two layers coexisted. However, the secondary mathematics classroom was hypothetical in the actual teacher education classroom, and the context was created with the teacher educator's intention or plan. After this initial starting point, additional components emerged from the data through a process of constant comparisons.

As the data were sorted and resorted, common indicators of each layer, teacher education class and secondary mathematics class, emerged. It was common for each rehearsal to be designed and enacted. More interesting was the finding that it was not just the independent elements that used in planning and implementing rehearsals to teach teaching mathematics but also they are organically related one another to bring preservice teachers into the secondary mathematics classroom and to learn pedagogy to teach mathematics. Then, I began to reexamine the data again with the findings that I developed. This reexamination helped me confirm whether the findings function well. Two approaches were used in this reexamination. One was using the findings for each rehearsal, and the other was syntactically analyzed each component throughout the data. It guided me to

obviously recognize how the findings operate.

From a continued and repeated analysis of data, I was able to arrive at seven elements that can be used in designing rehearsals to teach teaching mathematics in a method course for secondary teachers, as shown in Table 3. These are in the framework that differentiate secondary mathematics classroom and teacher education classrooms according to teacher educator's purpose. As the final check, I returned to the literatures about teaching practice, approximations, representations, and decompositions of practice, and rehearsals to see if they had identified any critical issues that I had not accounted for in my framework. The inductive thematic analysis of data reduction, designed to effectively and efficiently represent the patterns in the entire data set, appears to have successfully reduced their seven elements of my framework. The final elements and their demonstration and episodes were reviewed by the researcher who is a teacher educator in mathematics education and had Ph.D. in mathematics in order in order to have reliability about the finding of the current study.

Ⅳ. Designing Approximations of Practice for Rehearsals

Seven elements can be conceptually functioned in designing approximations of practice for rehearsals as shown in Table 3. To offer the elaborated demonstration in each element, this chapter mainly uses episodes from the rehearsal and the interview related to Class 8, which was shown in the previous chapter.

1. Purposes of rehearsals in the context of teacher education
Pedagogical context
2. Topics
3. Teaching purpose
4. Records of practice
5. Student background
Assessment
6. Inherent assessment
7. Discursive assessment

[Table 3] What Would Shape to Design Approximations of Practice for Rehearsals

1. Purposes of Rehearsals in the Context of Teacher Education

The first one is purposes of rehearsals in the context of teacher education. Teaching is purposeful (Hiebert & Grouws, 2007; Sleep, 2012). That is, instructional goals shape what instruction a teacher educator designs, how a teacher educator steers instruction, and what preservice teachers can learn. Therefore, the purposes of rehearsal offer the overall direction and purposes of what a teacher educator intends by a planned rehearsal, and these purposes exist in the layer of teacher education.

The purposes of rehearsals in Class 8 were shown in the planning notes: "For teachers to understand how to decompose the use of an opening example that involves talking about the concept's key features, related vocabulary, and algebraic notation via that example"; "For (preservice) teachers to practice helping future students see math as 'reason'-able, by using multiple representations while introducing a concept and using these representations to make sense of how vocabulary, notation, equations, and the concept fit together." These purposes appeared to organize the teacher educator's instruction, from the prompts for discussion following the teacher educator's demonstration, to the scope of the approximation. The prompts for the discussion included eliciting from teachers the concept introduced, the definition, key features of the definition, and vocabulary and notation to emphasize; and then how these features related to features of subsequent examples presented about slope, and how these examples could be used to help secondary students make sense of slope. The planning notes for the demonstration also include the mathematical demonstration about the use of variables x_1, x_2, y_1, and y_2 and their interpretation as mathematical objects, as objects within the scope of the ski lift example, and how these interpretations relate to each other and the expression $(y_2 - y_1)/(x_2 - x_1)$ from the "secondary students" of the demonstration class. Elaborations of the purposes embedded in discussion about the demonstration as well as the approximation instructions provided language for the commentary following the rehearsal.

2. Pedagogical Context

From the second one to fifth one are on the virtual or pseudo situation that a teacher educator has set out as teaching practice. In other words, they are created in the teacher education classroom, but it is a virtual space that is assumed as secondary mathematics classroom. Such contexts in teacher education with a focus

on practice and its disciplinary demands offer sites that teachers as learners practice tasks of teaching that demand content knowledge for teaching. Although there are various elements of real mathematics class, the current study particularly points out the four elements, which are *topic, teaching purpose, records of practice*, and *student background*, and call them as pedagogical context.

Topic includes a topical name of a course or unit (e.g., polynomials and their operations), a more focused topic (e.g., power functions), or a specific issue (e.g., distinguishing power functions from monomials). To select a topic for a rehearsal in teacher education, perspectives of mathematics, usability, curriculum, and teaching practice were considered. In Class 8, power functions were topic as a scaffold for working with the topic of polynomials. When asked why she chose to focus on polynomials, the teacher educator explained that polynomials are a primary class of examples in calculus. In the interview, the teacher educator said, "Calculus is a keystone of the college curriculum, and polynomials are a keystone of high school curriculum." Mathematical rationale seemed to be used for selecting polynomials as a focal topic. Moreover, teaching practice perspective was considered as well: "[Polynomials] are very difficult to explain well. ... $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ ... There is so much notation and vocabulary going on there." To provide a scaffold for explaining polynomials, the teacher educator used the topic of power function, saying, "We can talk about that [power functions] before doing the larger thing of explaining a polynomial. Because the definition of power function is much less complicated than the definition of polynomial, so that was a way of scaffolding up to polynomial." The teacher educator seemed to focus on the ways that teaching the concept of polynomial is not easy because it requires knowledge about language, in particular notation and terminology for concepts important to related topics such as factoring.

Teaching purpose is about the instructional situation that preservice teachers are to enact in a rehearsal. While the contextual teaching purpose is a purpose that preservice teachers need to achieve with hypothetical secondary students in a rehearsal, the purposes of rehearsals in the context of teacher education illustrated previously are purposes that the teacher educator for the rehearsal as a whole, and may have as much to do with disposition of the preservice teachers as the actions taken by the preservice teachers within a rehearsal. Examples of contextual teaching purposes for rehearsals included from the data, "Work with your students on relating different representations of graph discontinuities to each other, explaining the graph, how it relates to an input/output table, and how the

graph relates to the equation" (Class 5), and "Teach a procedure for graphing functions of the form $f(x)=ax^2+k$, using this situation to build students' intuition about how graphs of quadratics are related to each other in general" (Class 6). In these teaching purposes specification is tightly wound with the topic, where the purpose transfers across topics, from concepts about lines to concepts about polynomials. The topic may suggest teaching purposes specific to the topic, but teaching purposes can also span topics. It looks very clear that in Class 8 preservice teachers were expected to intend in the rehearsal was to "Introduce the concept of a power function." Implicit in this purpose was to introduce the concept so that it is sensible to secondary students.

Record of practice includes excerpts from textbooks, example problems, secondary student work, board work by either a teacher or secondary students, manipulatives or instructional representations, secondary student explanations, classroom dialogue, or other records of practice (Ball & Cohen, 1999). It offers important chances to plan and perform a rehearsal with concrete referents. For example, sample secondary student work using different representations, such as graphical, tabular, symbolic was offered in Class 5, and copies of textbooks to show several tables of values for quadratic functions were used in Class 6. For Class 8, several pages of two textbooks with definitions, properties, and problems were used for the topics of polynomial and power functions. Reading through these textbook excerpts was asked to the preservice teachers prior to the class.

Student background identifies information about secondary students' prior knowledge, performance, disposition, or other characteristics. Student background was provided by the teacher educator with concrete statements, when the purpose was more closely tied to responding to secondary students' prior knowledge. Or, the teacher educator asked a volunteer preservice teacher who performed a rehearsal what he or she assumed as his or her hypothetical secondary students' background. In general, a way in which secondary student background was considered was in planning the "secondary student" roles with common secondary student conceptions to raise in the form of "secondary student" responses or questions to the preservice teacher enacting a rehearsal. In Class 8, the volunteer preservice teacher was asked to explicate her hypothetical secondary students' background and assumed that her secondary students understood the concepts of coefficient, variable, and function, which were considered in the planning of the rehearsal by the teacher educator.

3. Assessment

Pedagogical context is a space that preservice teachers actually perform their rehearsals and a teacher educator can observe them in that space. Pedagogical context is, therefore, a learning space that preservice teachers practice tasks of teaching embedded in the teaching practice under the teacher educator 's and other preservice teachers' assessment. Assessment, in the context of rehearsals, exists in the space of teacher education and is a social practice. In Jordan and Putz (2004) terms, preservice teachers and teacher educators engaged in implicit *inherent assessment* and shared explicit *discursive assessment*.

Inherent assessment occurs implicitly and inferences or actions taken as a result of the assessment tend to be individual actions. These assessments occur in real time. Jordan and Putz (2004) identified the example of a children licking muffin batter after a mother's doing so as an inherent assessment by the child that a small taste of the muffin batter is okay to do. Inherent assessment is important to consider in teacher education because inferences – about what is acceptable or permissible or not – may well be based on the representations that approximations respond to. Another example of inherent assessment given by Jordan and Putz (2004) is that of a teacher glancing at a secondary student's work and judging that the student understands the assignment. Another form of inherent assessment may have taken place in the lack of commentary on the phrase "stop in time." A teacher educator must make decisions about how to spend limited time well, and she may have decided to focus on making connections between representations at the expense of discussing the role of mathematical precision. The teacher educator hypothesized a potential lack of facility in the "relating" work by the preservice teachers in general. This assessment of the class shaped the subsequent instruction, including the design of the homework assigned following the rehearsal. The decision about the homework most appropriate to assign, and in revising the existing planned homework to incorporate this aspect of making connections between symbolic and contextual, had been made during the rehearsals by the teacher educator without a formal announcement to the class. A teacher educator's inherent assessment concerns the instruction of the course and influences the direction of the course.

In Class 8, a result of inherent assessment in the rehearsal seemed to occur, for example, in Kelly's writing the expression cx^a on the board in tandem with

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Jordan and Putz (2004) contrast these informal assessments with *documentary assessment,* formal assessments resulting in a standard evaluation statement or a number.

calling on "students" to recall the meanings of coefficient and exponent. In the interview following Class 8, the teacher educator in this study discussed her assessment of the rehearsals that class:

> I think it's very important for helping students understand how to even read a definition and modeling when you look at notation (in this situation) … This is something that all the preservice teachers need to work on, knowing how to press for understanding, and what questions to ask.

The mathematical issues in this situation involved linking algebraic expressions to their meaning both as mathematical objects and to the context in which arise. The work of relating notation to meaning included explicit discussion of notation, such as c, x, and a, and the expressions containing them, such as cx^a. This assessment of the class appeared to shape the subsequent instruction, including the design of the homework assigned following this rehearsal that focused on relating "atoms" (e.g., c, x, a), "molecules", and "compounds" of algebraic expressions to their meaning in a story problem.

In contrast to implicit inherent assessment, *discursive assessments* are explicit and shared. They publically problematize and enforce issues concerning quality, support innovation, and encourage thinking and learning (Jordan & Putz, 2004). In the context of the current study, discursive assessment occurred after each rehearsal, when the teacher educator and preservice teachers reflected about the rehearsal as a group. The ability for preservice teachers to extrapolate potential cause-effect relationships is related to a teacher's mathematical and pedagogical content knowledge (Kersting, 2008), and may help them learn from practice (Hiebert, Morris, Berk, & Jansen, 2007). In orchestrating discursive assessments, a teacher educator may need to consider, as Hiebert et al. (2007) proposed, the learning goals, observations about the approximations, potential cause-effect relationships, and potential alternatives. Each applies to two contexts: the actual teacher education classroom and its community of preservice teachers and the teacher educator, and the approximated secondary mathematics classroom and its hypothetical community. The application to the first context is influenced by the purposes of the rehearsal. The application to the second context may influence the construction of pedagogical contexts.

In Class 8, when Kelly commented that she was not sure how secondary students "would take" the abstraction from an example to a formula, she was making public a question that intertwines mathematical and pedagogical

perspectives. She furthermore hypothesized a cause-effect relationship, that because secondary students may have in mind finding a specific value, they may not be receptive to the use of the example in constructing a general formula. When the preservice teachers commented several times that Kelly related the context to the mathematical idea, they may have enforced the norm that relating mathematical ideas and context is valued – even if the teacher educator assessed their facility at relating ideas and context to need improvement.

My analysis identified what shaped the ways that practice was brought into a teacher education course that used rehearsals as a way of approximating practice. I emphasize here that the seven are distinct, yet, as illustrated above, interact with each other.

Ⅴ. Discussion: Dual Contexts with Dual Perspectives in Approximations of Practice

Rehearsals in teacher education can offer preservice teachers opportunities for deliberate practice within instructional situations (Ericsson, 2002). The current study identified what can be considered to design approximations for mathematics teaching practice and showed how those functioned in one teacher educator's practice. Based on the analysis, the current study makes two claims and talks implication to mathematics teacher education for South Korea.

The first is the coordination of dual contexts in designing approximations for rehearsals. The first context is that of teacher education, and the second is secondary mathematics teaching. The latter is nested into the former, and these two settings work together organically. The context of teacher education consists of purposes of rehearsals, pedagogical context within the rehearsal, and assessment frames; and purposes of rehearsals shape the pedagogical context and assessment. The dual contexts require teacher educators to switch gears appropriately and spontaneously for both settings. Because rehearsals are hypothetical situations that are intentionally created by teacher educators who are in charge of them, teacher educators' decisions determine the quality of teaching that preservice teachers practice and learn in and from approximations of practice.

The use of pedagogical context here bears similarities with that found by Lai, Jacobson, and Thames (2013, April) who analyzed teaching contexts embedded in assessment items of mathematical knowledge for teaching. They found that items' depicted teaching purposes, records of practice, secondary student background, and classroom organization (e.g., whether the secondary students are engaged in whole

group discussion or in small group work) shaped item doers' reasoning. Furthermore, they found that competent performance on the items depended on reasoning that used features of the pedagogical context in integral ways. However, they did not find general knowledge about a topic (e.g., "polynomials") to be as critical to the reasoning as how the topic was contextualized in records of practice (e.g., a specific problem using polynomials). In the current study, I found that the topic did play a role in *planning* rehearsals, even if general knowledge about the topic was not prominent within rehearsals. I also did not find classroom organization to be prominent, but this may have been because of the uniformity of classroom organization across the rehearsals.

The second is synchronizing mathematical and pedagogical perspectives in and out of rehearsals. All seven draw from the two perspectives. For example, when the teacher educator of this study selected a mathematical topic for a rehearsal, she also considered difficulties of performing a certain task of teaching, in particular, explaining a concept and its importance in disciplinary mathematics. To assess rehearsals, both mathematical and pedagogical soundness must be considered. Mathematical and pedagogical perspectives prevail throughout rehearsals for skilled teaching of mathematics. Dual contexts and dual perspectives are prominent in designing and enacting rehearsals for preservice teachers. Designing approximations of practice for rehearsals involves clear and conscious distinctions between dual contexts and perspectives and, at the same time, demands smooth transition between them.

The analysis and the claims of the current study would offer implication for mathematics teacher education for South Korea. Teacher education program should bring practice into the classroom of teacher education because teaching must be learned in practice (Feiman-Nemser & Remillard, 1996). Ball and Cohen (1999) identified how teachers need to learn this knowledge. First, they must learn how to size up a situation from moment to moment, such as how to investigate what students are doing and thinking, and how instruction has been understood. Second, teachers have to learn to use such knowledge to improve their practice, such as how to use what teachers learn about students' work and ideas to inform and improve teaching. Third, teachers would need to learn how to operate experimentally in response to students and situations, such as how to frame, guide, and revise tasks and to pose and reformulate questions. In South Korea, field experience is one independent course, which seems to be understood as opportunities to "experience" or "taste" teaching practice. It, however, could be hard that preservice teachers acquire professional and specific guidance with

professional knowledge and performance, and they might make themselves be accustomed to what might be tainted by so-called traditional ways in mathematics instruction. Being in a real mathematics classrooms or simply observing mathematics classes is not enough to train preservice teachers to be ready to teach mathematics (Darling-Hammond, 2006b). In this sense, rehearsals offer alternative ways to help teacher educators create approximations of practice in their teacher education classrooms and support preservice teachers to practice focal teaching practice as shown in Table 2 under teacher educator's guidance. Developing lesson plans and implementing them in teacher education programs in South Korea often depends on curriculum materials, such as textbooks. However, it does not consider records of practice and student background which are critical to constructing pedagogical context. Because teaching mathematics lays on mathematical topics as well as interactions among a teacher and students, the findings of the current study would provide specific approximations of practice in teacher education.

Ⅵ. Conclusions

The reported research analyzed recordings of a secondary mathematics methods course for preservice teachers and interviews of the teacher educator instructing the course. This study identified what shaped approximations and planning for their enactment in instruction in a teacher education course. Although the data is limited to one course by one teacher educator, the findings are still valuable as a grounded hypothesis for carrying out approximations of practice for teacher education of mathematics. The parallel analysis of observations of instruction and interviews of the teacher educator provides potential ways to clarify how the seven elements align enactment of instruction with rationales for instructional decisions. Moreover, the findings of this research may contribute to identification of what is salient in instructional decision making by teacher educators. Whatever resources teacher educators consider for their courses, they have rationales and ways of thinking about what they use or not, how they use them, what they offer to preservice teachers as learners, and how resources are used and adapted. The point is that what would shape to design a rehearsal reported here are central tools to make decisions for those questions. In other words, they ultimately serve the purpose of preservice teachers′ learning in ways that function in practice.

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