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Buyer's Price and Inventory Policy with Price Dependent Demand for Decaying Items Day terms Supplier Credit in a Two-stage Supply Chain

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Abstract

In deriving the economic order quantity (EOQ) formula, it is tacitly assumed that the buyer has to pay product price while receiving the product from the supplier. However, as a marketing policy, some suppliers permit a delay in payments to the buyers to increase demand for the product they made. Credit transactions would have a positive effect on both suppliers and buyers. For a supplier who offers trade credit, it is an effective means of price differentiation to increase the demand for the product. Availability of opportunity to delay the payment in buyer effectively reduces the cost of holding stocks and therefore, the buyer has a lot of price options to choose his sales price for a customer. Since the buyer's order is affected by the customer's demand, the problems of determining the sales price and EOQ are interdependent and must be solved simultaneously. From this perspective, this paper evaluates the problem of determining the optimal sales price and EOQ for the buyer at the same time when the supplier allows a delay in payments for the product whose demand is represented as a function that decreases linearly with the sales price. For the analysis, it is also assumed that inventory is exhausted not only by customer's but also by decay.

Keywords: *Credit period, Pricing, Linear decreasing demand function, EOQ, Decay*

1. Introduction

In this paper, we are trying to extend the model presented by Shinn [1] to the case of decaying products. We also consider the supply chain consisting of the customer, the buyer and the supplier. As stated by Shinn [1], some suppliers allow a delay in payments of the product to increase the demand for the product they produce. In this consideration, many research papers appeared which deal with the problem of determining the inventory policy under trade credit, Goyal [2] and Teng et al. [3] examined the mathematical model for obtaining an economic order quantity (EOQ) when the supplier allows a fixed delay in payments of the product. Kreng and Tan [4] analyzed the inventory model under two steps of credit policy depending on the size of order.

However, the availability of opportunity to delay the payment from the supplier becomes the effective means to reduce inventory carrying costs for buyer. And therefore, the buyer may choose the sales price from wider range of the price option expect to increase the customer's demand. Since the buyer's order size is affected by the customer's demand, the problems of determining the sales price and order quantity for buyer should be

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solved at the same time not independent. In this regard, a number of research works have dealt with the problem determining the price and order quantity simultaneously under the assumption that the demand for the customer is a function of the sales price set by the buyer. Chang et al. [5], Dye and Ouyang [6], and Ouyang et al. [7] introduced the problem determining the price and lot-size jointly under permissible delay in payments when the customer's demand rate is expressed by a constant price elasticity function of sales price. Avinadav et al. [8] and Shi et al. [9] presented the problem of determining the price and lot-size jointly without trade credit assuming that the demand rate is expressed by a linear decreasing function of sales price. Also, Shinn [1] introduced the problem determining the price and lot-size simultaneously under trade credit in the case of the linear decreasing demand function of sales price.

All of the research papers mentioned above implicitly assumed that inventory is exhausted by the customer's demand alone. This assumption is considered quite valid for inventory items that do not perish or decay. However, there were numerous types of inventory whose utility is not constant over time. In this case, inventory is exhausted not only by the customer's demand but also by decay. From this point of view, Ghare and Schrader [10] assuming exponential decay of the inventory under constant demand, derived a revised form of EOQ. Cohen [11], and Chen and Chang [12] analyzed the joint price and order quantity determination problem for an exponentially decaying product. Recently, Mahata and Goswami [13] introduced the inventory model to the case of decaying products under trade credit. And Tsao and Sheen [14] also presented the problem determining the price and lot-size at the same time for an exponentially decaying product under trade credit.

From this point of view, we try to deal with the problem determining the sales price and order quantity simultaneously when the supplier permits a delay in payments and the demand of the product is a linear decreasing function of sales price. Also, considering that there are a lot of types of inventory item whose utility is not constant over time, we also assume that inventory is exhausted not only by the customer's demand but also by decaying. In the next section, we formulate an appropriate mathematical model. The characteristics of the model are analyzed and its solution algorithm is presented in Section 3. A numerical example is solved in Section 4, which is followed by conclusions.

2. Development of the Model

2.1 Assumptions and Notations

In deriving the mathematical model, it is essentially the same model that Shinn [1] mentioned except for the condition of decay. Following assumptions and notations are used.

Assumptions:

(1) Replenishment is instantaneous with a known and constant lead time.

(2) The demand rate for the customer is a linear function of the buyer's sales price.

(3) No shortages are allowed.

(4) The inventory system involves only one item.

(5) The supplier permits a certain credit period and sales revenue generated during the period is deposited in an interest with rate I. At the end of the period, the product price is settled and the buyer begins paying the capital opportunity cost for the products in inventory with rate R ($R \geq I$).

(6) Inventory is exhausted not only by customer's demand but also by decay. Decay follows exponential distribution with parameter λ.

Notations:

S = ordering cost.

C = purchase cost per unit.

tc = credit period set by the supplier.

H = inventory holding cost without the capital opportunity cost.
I = earned interest rate (as a percentage).
R = capital opportunity cost (as a percentage).
Q = order quantity.
T = replenishment cycle time.
D = the customer's annual demand rate, as a function of the buyer's selling price, $D = a - bP$ where a and b are positive constants.
P = the buyer's sales price, $P < a/b$.
λ = a positive number representing the inventory decaying rate.
$q(t)$ = inventory level at time t.

2.2 Proposed model development

For the case of exponential decay, as stated by Ghare and Schrader [10], the rate at which inventory decay will be proportional to inventory level at time t, $q(t)$. Thus, at any time t, the depletion rate of inventory is

$$\frac{dq(t)}{dt} = -\lambda q(t) - D \ . \tag{1}$$

And its solution is

$$q(t) = q(0)e^{-\lambda t} - \frac{D}{\lambda}(1 - e^{-\lambda t}) \ . \tag{2}$$

Equation (2) represents the inventory level at time t as the combined effects of the customer's demand and decay over time.

Now, we present the inventory loss caused by decay. Let $q^0(t)$ be the inventory level without decay at time t. Then, the inventory loss caused by decay becomes

$$q^0(t) - q(t) = (q(0) - Dt) - \left(q(0)e^{-\lambda t} - \frac{D}{\lambda}\left(1 - e^{-\lambda t}\right)\right) \tag{3}$$

$$= q(t)\left(e^{\lambda t} - 1\right) - Dt + \frac{D}{\lambda}(e^{\lambda t} - 1) \ . \tag{4}$$

Hence, the order quantity per cycle becomes

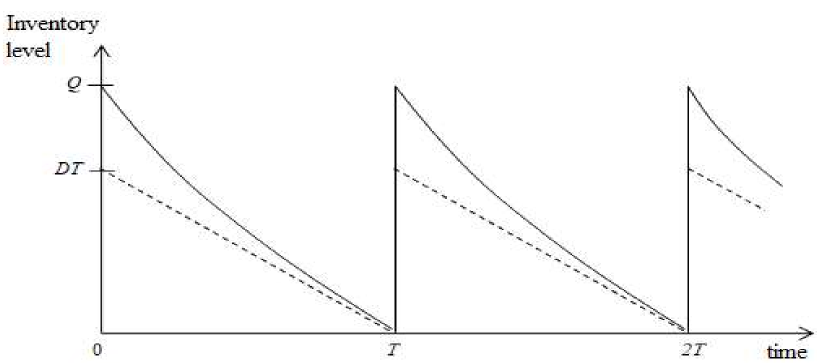


Figure 1. Inventory Level ($q(t)$) vs. Time (t)

$$Q = (q^0(T) - q(T)) + DT. \tag{5}$$

Note that because of the inventory carrying costs, it is obviously better off to have the inventory level reach zero just before reordering, i.e., $q(T) = 0$. Figure 1 explains the inventory levels over time. Demand rate, D appeared as the slope of the dashed line. With $q(T) = 0$, we have

$$Q = \frac{D}{\lambda}(e^{\lambda T} - 1) \quad . \tag{6}$$

Also, by $q(0) = Q$, the inventory level at time t is

$$q(t) = \frac{D}{\lambda}(e^{\lambda(T-t)} - 1) \ , \ 0 \le t \le T. \tag{7}$$

The purpose of this paper is to find the joint price and replenishment cycle time which maximizes the buyer's annual net profit $\Pi(P,T)$ and $\Pi(P,T)$ consists of the following five elements.

(1) Sales revenue per year, AR

$AR = PD$

(2) Purchasing cost per year, AP

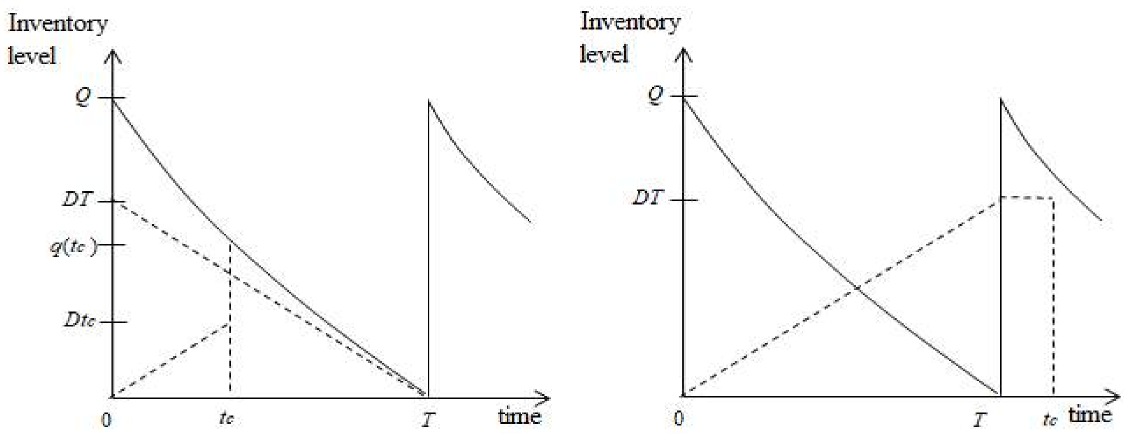
$AP = CQ/T = \frac{CD(e^{\lambda T}-1)}{\lambda T}$

(3) Ordering cost per year, AO

$AO = S/T$

(4) Inventory holding cost per year, AH

$AH = \frac{H}{T}\int_0^T q(t)dt = \frac{HD(e^{\lambda T}-\lambda T-1)}{\lambda^2 T}$



(a) $tc \le T$ **(b)** $tc > T$

Figure 2. Credit period (*tc*) vs. Replenishment cycle time (*T*)

(5) Capital opportunity cost per year, AC

(i) Case 1($tc \le T$): (see Fig. 2(a)) When the products are sold, the sales revenue is used as a means of earning interest with annual rate I during the credit period tc. The average number of products in inventory earning interest during time (0, tc) is $Dtc/2$ and the interest earned for order becomes $(\frac{Dtc}{2})(tcCI)$). When the credit is settled, the products still in inventory must be financed with annual rate R. Since the average number of products during time (tc, T) becomes $\frac{1}{(T-tc)}\int_{tc}^{T} q(t)dt$, the interest payable per order can be expressed as $CR\int_{tc}^{T} q(t)dt$. Therefore,

$$AC = \frac{1}{T}\left(CR\int_{tc}^{T} q(t)dt - \frac{CDtc^2}{2}\right) = \frac{CRD\left(e^{\lambda(T-tc)}-\lambda(T-tc)-1\right)}{\lambda^2 T} - \frac{CDtc^2}{2T}.$$

(ii) Case 2($tc > T$): (see Fig. 2(b)) For the case of $tc > T$, all the sales revenue is used as a means of earning interest with annual rate I during the credit period tc. The average number of products in inventory earning interest during time (0, T) and (T, tc) become $DT/2$and DT, respectively. Therefore,

$$AC = -\frac{1}{T}\left(\frac{DT}{2}TCI + DT(tc - T)CI\right) = \frac{CDT}{2} - CDtc.$$

The buyer's annual net profit can be expressed as

$$\Pi(P,T) = AR - AP - AO - AH - AC.$$

Depending on the relative size of tc to T, $\Pi(P,T)$ has two different expression as follows:

(1) Case 1($tc \le T$)

$$\Pi_1(P,T) = PD - \frac{CD\left(e^{\lambda T}-1\right)}{\lambda T} - \frac{S}{T} - \frac{HD\left(e^{\lambda T}-\lambda T-1\right)}{\lambda^2 T} - \left(\frac{CRD\left(e^{\lambda(T-tc)}-\lambda(T-tc)-1\right)}{\lambda^2 T} - \frac{CDtc^2}{2T}\right), \tag{8}$$

(2) Case 2($tc > T$)

$$\Pi_2(P,T) = PD - \frac{CD\left(e^{\lambda T}-1\right)}{\lambda T} - \frac{S}{T} - \frac{HD\left(e^{\lambda T}-\lambda T-1\right)}{\lambda^2 T} - \left(\frac{CDT}{2} - CDtc\right). \tag{9}$$

3. Determination of Buyer's Pricing and Lot sizing Policy

The problem is to find an optimal sales price $P^*$and an optimal replenishment cycle time T^* which maximize $\Pi(P,T)$. Once P^* and T^* are found, an optimal order quantity Q^* can be obtained by equation (6). Although the objective function is differentiable, the resulting equation is difficult to handle mathematically; that is, it is impossible to find an optimal solution in explicit form. Thus, we have to solve the model approximately through a truncated Taylor series expansion on the exponential term, i.e.,

$$e^{\lambda T} \approx 1 + \lambda T + \frac{1}{2}\lambda^2 T^2, \tag{10}$$

which is a valid approximation for smaller values of λT. With the above approximation, the annual net profit can be written as

$$\Pi_1(P,T) = PD - CD - \frac{S}{T} - \frac{(H+C\lambda)DT}{2} - \left(\frac{C(R-1)Dt_c^{\,2}}{2T} + \frac{CRDT}{2} - CRDt_c\right), \tag{11}$$

$$\Pi_2(P,T) = PD - CD - \frac{S}{T} - \frac{(H+C\lambda)DT}{2} - \left(\frac{CIDT}{2} - CIDt_c\right). \tag{12}$$

Note that (10) is exact when $\lambda = 0$ so that equations (8) and (9) reduce to the exact formulas equations (11) and (12) for non-decay product. Also, equations (11) and (12) are the same formulas as stated by Shinn [1] referred to except for replacing H with $H + C\lambda$.

For the first and second order conditions with respect to T, we have

$$\frac{\partial \Pi_1(P,T)}{\partial T} = \frac{S}{T^2} - \frac{(H+C\lambda+CR)D}{2} + \frac{C(R-I)Dt_c^{\,2}}{2T^2}, \tag{13}$$

$$\frac{\partial \Pi_2(P,T)}{\partial T} = \frac{S}{T^2} - \frac{(H+C\lambda+CI)D}{2}, \tag{14}$$

$$\frac{\partial^2 \Pi_1(P,T)}{\partial T^2} = -\frac{2S}{T^3} - \frac{C(R-I)Dt_c^{\,2}}{T^3}, \tag{15}$$

$$\frac{\partial^2 \Pi_2(P,T)}{\partial T^2} = -\frac{2S}{T^3}. \tag{16}$$

For a fixed P, $\Pi(P,T)$ is a concave function of T, and there exist a unique value T_i, which maximizes $\Pi_i(P,T)$ as follows:

$$T_1 = \sqrt{\frac{2S_1}{H_1 D}}, \text{ where } S_1 = S + \frac{C(R-I)Dt_c^{\,2}}{2} \text{ and } H_1 = H + C\lambda + CR, \tag{17}$$

$$T_2 = \sqrt{\frac{2S}{H_2 D}}, \text{ where } H_{12} = H + C\lambda + CI. \tag{18}$$

Note that since the demand rate D is a function of P, each T_i can be expressed as a function of P; that is $T_i = T_i(P)$.

Now, from equations (17) and (18), we can see the following useful property for a fixed P.

Property 1. $T_1(P) \geq t_c$ if and only if $T_2(P) \geq t_c$. If $T_1(P) \geq t_c$, then $\Pi_2(P,T)$ is increasing in T over $T < t_c$. If $T_2(P) < t_c$, then $\Pi_1(P,T)$ is decreasing in T over $T \geq t_c$.

Proof.

From equation (17), $T_1(P) \geq t_c$ can be rewritten as

$$\sqrt{\frac{2S + C(R-I)Dt_c^{\,2}}{(H+C\lambda+CR)D}} \geq t_c. \tag{19}$$

Squaring both side of equation (19) and rearranging,

$$2S + C(R-I)Dt_c^{\,2} \geq (H + C\lambda + CR)Dt_c^{\,2}, \tag{20}$$

$$\sqrt{\frac{2S}{(H+C\lambda+CI)D}} \geq t_c. \tag{21}$$

Equation (21) implies that $T_2(P) \geq tc$ and $T_2(P)$ is a maximum point of $\Pi_2(P,T)$, which is a concave function. Thus $\Pi_2(P,T)$ is increasing in T over $T < tc$. Also, from equations (19) and (21), $T_2(P) < tc$ implies that $T_1(P) < tc$. Likewise, $T_1(P)$ is a maximum point of $\Pi_1(P,T)$ and so $\Pi_1(P,T)$ is decreasing in *T* over $T \geq tc$. Q.E.D.

Property 1 refers that for a fixed *P*, the optimal replenishment cycle time $T^*(P)$ which maximizes $\Pi(P,T)$ is known to be either $T_1(P)$ or $T_2(P)$ because $\Pi(P,T)$ is continuous at $T = tc$. Moreover, if $T_1(P) \geq tc$, then $T_1(P)$ becomes $T^*(P)$. And if $T_1(P) < tc$, then $T_2(P) < tc$ and $T_2(P)$ becomes $T^*(P)$.

Now, let us consider $T_1(P) \geq tc$. Since the demand rate *D* is also a function of *P*, the inequality can be rewritten as

$$T_1(P) = \sqrt{\frac{2S + C(R-I)Dtc^2}{(H + C\lambda + CR)D}} \geq tc \quad . \tag{22}$$

Rearranging equation (22),

$$P \geq \frac{a}{b} - \frac{2S}{b(H + C\lambda + CI)tc^2} \quad . \tag{23}$$

Let

$$P_0 = \frac{a}{b} - \frac{2S}{b(H + C\lambda + CI)tc^2} \quad . \tag{24}$$

It is self evident that for any $P \geq P_0$, the inequality $T_1(P) \geq tc$ holds. So, we conclude that $T_1(P)$ determined by *P* value which satisfies the inequality (23) becomes an optimal replenishment cycle time $T^*(P)$. Similarly, $T_2(P)$ becomes an optimal replenishment cycle time $T^*(P)$ only if $P < P_0$. Consequently, substituting *T* with $T_i(P)$ in $\Pi_i(P,T)$, we have a problem of maximizing $\Pi_i(P, T_i(P)$ that is a single variable function of *P*. With $\Pi_1^0(P) = \Pi_1(P, T_1(P))$ for $P \geq P_0$ and $\Pi_2^0(P) = \Pi_2(P, T_2(P))$ for $P < P_0$, the following single variable objective function is obtained.

$$\Pi_1^0(P) = \left(P - C(1 - Rtc)\right)D - \sqrt{2S_1H_1D} \text{ for } P \geq P_0, \tag{25}$$

$$\Pi_2^0(P) = \left(P - C(1 - Itc)\right)D - \sqrt{2SH_2D} \text{ for } P < P_0. \tag{26}$$

Therefore, an optimal solution (P^*, T^*) which maximizes $\Pi(P,T)$ is found by searching over $\Pi_i^0(P)$. Considering the characteristics of $\Pi_i^0(P)$, we can find the following property related to the shape of $\Pi_i^0(P)$.

Property 2. $\Pi_1^0(P)$ is a concave-convex-concave function of *P* and $\Pi_2^0(P)$ is a concave-convex function of *P*.

Proof.

For Case 1, the shape of $\Pi_1^0(P)$ can be studied by examining its first and second derivative with respect to *P*.

$$\Pi_1^0(P)' = \frac{d\Pi_1^0(P)}{dp} = D + D'\big(P - C(1 - Rtc\ )\big) - D'(S + C(R - I)Dtc\ ^2)\sqrt{\frac{H_1}{2S_1D}}\ , \tag{27}$$

$$\Pi_1^0(P)'' = \frac{d^2\Pi_1^0(P)}{dp^2} = 2D' + \frac{b^2S^2}{2S_1D}\sqrt{\frac{H_1}{2S_1D}}\ . \tag{28}$$

Then, (28) can be set to zero so that

$$\Pi_1^0(P)'' = 0. \tag{29}$$

Substituting D with $a - bP$ in (28), we have the following quadratic equation of P;

$$f(P) = Kb^2P^2 - 2(Ka + S)bP + a(Ka + 2S) - \left(\tfrac{1}{4}b^2S^4H_1\right)^{1/3},\ K = Ctc\ ^2(R - I). \tag{30}$$

Because $Kb^2 > 0$ and the discriminant of $f(P)$ is positive, $f(P)$ have two real roots with $\widehat{P_{11}}$ and $\widehat{P_{12}}$. By the quadratic formula, we have the following two roots;

$$\widehat{P_{11}} = \frac{2(Ka+S)b - \sqrt{4b^2(S^2 + K(\frac{b^2S^4H_1}{4})^{1/3}}}{2Kb^2}\ , \tag{31}$$

$$\widehat{P_{12}} = \frac{2(Ka+S)b + \sqrt{4b^2(S^2 + K(\frac{b^2S^4H_1}{4})^{1/3}}}{2Kb^2}\ . \tag{32}$$

Therefore,

$$f(P) > 0 \ \text{ for } < \widehat{P_{11}}\ , \tag{33}$$

$$f(P) < 0 \ \text{ for } \widehat{P_{11}} < P < \widehat{P_{12}}\ , \tag{34}$$

$$f(P) > 0 \ \text{ for } \widehat{P_{12}} < P\ . \tag{35}$$

Note that $f(P) > 0$ implies $\Pi_1^0(P)'' < 0$, which means the concave function and $f(P) < 0$ implies $\Pi_1^0(P)'' > 0$, which means the convex function. So, $\Pi_1^0(P)$ is a concave-convex-concave function of P.

For Case 2, the shape of $\Pi_2^0(P)$ can be studied by examining its first and second derivative with respect to P.

$$\Pi_2^0(P)' = \frac{d\Pi_2^0(P)}{dp} = D + D'\big(P - C(1 - lc\ )\big) - \frac{b^2}{2D^2}\sqrt{\frac{H_2SD}{2}}\ , \tag{36}$$

$$\Pi_2^0(P)'' = \frac{d^2\Pi_2^0(P)}{dp^2} = 2D' + \frac{b^2}{2D^2}\sqrt{\frac{H_2SD}{2}}\ . \tag{37}$$

Let $\widehat{P_2}$ be the solution of

$$\Pi_2^0(P)'' = 0 \tag{38}$$

and we have

$$\widehat{P_2} = \frac{a}{b} - \frac{1}{b}\left(\frac{b}{4\sqrt{2}}\sqrt{H_2 S}\right)^{2/3} \quad . \tag{39}$$

Then,

$$\Pi_2^0(P)'' < 0 \ \text{ for } < \widehat{P_2} \ , \tag{40}$$
$$\Pi_2^0(P)'' > 0 \ \text{ for } \widehat{P_2} < P. \tag{41}$$

Therefore, we conclude that $\Pi_2^0(P)$ is a concave-convex function of *P*. Q.E.D.

Now, to find the extreme points of $\Pi_i^0(P)$, let us consider the first order condition for $\Pi_i^0(P)$ with respect to *P* as follows;

$$\Pi_1^0(P)' = \frac{d\Pi_1^0(P)}{dp} = 0 \ , \tag{42}$$
$$\Pi_2^0(P)' = \frac{d\Pi_2^0(P)}{dp} = 0 \ . \tag{43}$$

From the results of Property 2, equation (42) has three roots with P_{11}^*, P_{12}^* and P_{13}^*, and equation (43) has two roots with P_{21}^* and P_{22}^*. Although $\Pi_i^0(P)$ can be differentiated, the resulting equation is mathematically intractable; that is, it is impossible to find P_{ij}^* in explicit form. Thus, we can find P_{ij}^* by numerical search method. Consequently, only the elements in set $\Omega = \{P_{21}^{,*}, P_{11}^*, P_{13}^*, P_0,\ a/b - \varepsilon$, where ε is very small positive number} become candidates for *P* because $\Pi_1^0(P) = \Pi_2^0(P)$ at $P = P_0$.

For P_{1j}^*, $j = 1, 3$ to be a candidate of P^* in Case 1, each P_{1j}^* must lie on $[P_0, a/b)$ and also for $P_{21}^{,*}$ to be a candidate of P^* in Case 2, $P_{21}^{,*}$ must lie on $(0, P_0)$. For P_0 to be a candidate of P^* in Case 1, $\Pi_1^0(P)$ must be decreasing at $P = P_0$. Also, for $a/b - \varepsilon$ to be a candidate of P^* in Case 1, $\Pi_1^0(P)$ must be increasing at $P = a/b - \varepsilon$. Note that depending on the relative size of $\widehat{P_{11}}$, $\widehat{P_{12}}$, $\widehat{P_2}$, P_0 and *a*/*b*, each element in Ω can be a candidate value or not. And therefore, some elements in Ω can be dropped from consideration in search of P^*.

Based on the above properties for the candidates of P^*, we have developed the following solution algorithm to determine the buyer's optimal sales price and replenishment cycle time.

Solution Algorithm

Step 1. This step computes $\Pi_2^0(P)$for the candidate values in set Ω.

1.1. Compute $\Pi_2^0(P)'$ at $P = P_0$ by equation (36).

1.2. If $\Pi_2^0(P_0)' < 0$, then compute $\Pi_2^0(P)$ at $P = P_{21}^{,*}$ and go to Step 1.4.

Otherwise, go to Step 1.3.

1.3. If $\widehat{P_2} < P_0$, then compute $\Pi_2^0(P)$ at $P = P_{21}^{,*}$ and go to Step 1.4.

Otherwise, go to Step 1.4.

1.4. Determine $T_2(P)$ by (18) and go to Step 2.

Step 2. This step computes $\Pi_1^0(P)$ for the candidate values in set Ω.

2.1. Compute $\Pi_1^0(P)'$ at $P = P_0$ by equation (27).
2.2. If $\Pi_1^0(P_0)' \geq 0$, then go to Step 2.3.
Otherwise, go to Step 2.4.
2.3. Compute $\Pi_1^0(P)'$ at $P = \frac{a}{b} - \varepsilon$ by equation (27).
2.3.1. If $\Pi_1^0(a/b - \varepsilon)' \geq 0$, then go to Step 2.3.2.
Otherwise, go to Step 2.3.4.
2.3.2. If $/b < \widehat{P_{11}}$, then compute $\Pi_1^0(P)$ at $P = a/b - \varepsilon$ and go to Step 2.5.
Otherwise, go to Step 2.3.3.
2.3.3. If $P_0 < \widehat{P_{11}}$, then compute $\Pi_1^0(P)$ at $P = P_{11}^*,\ a/b - \varepsilon$ and go to Step 2.5.
Otherwise, compute $\Pi_1^0(P)$ at $P = a/b - \varepsilon$ and go to Step 2.5.
2.3.4. If $\widehat{P_{11}} \leq P_0$, then compute $\Pi_1^0(P)$ at $P = P_{13}^*$ and go to Step 2.5.
Otherwise, go to Step 2.3.5.
2.3.5. If $/b < \widehat{P_{12}}$, then compute $\Pi_1^0(P)$ at $P = P_{11}^*$ and go to Step 2.5.
Otherwise, compute $\Pi_1^0(P)$ at $P = P_{11}^*, P_{13}^*$ and go to Step 2.5.
2.4. Compute $\Pi_1^0(P)'$ at $P = a/b - \varepsilon$.
2.4.1. If $\Pi_1^0(a/b - \varepsilon)' \geq 0$, then compute $\Pi_1^0(P)$ at $P = P_0,\ a/b - \varepsilon$ and go to Step 2.5.
Otherwise, go to Step. 2.4.2.
2.4.2. If $\widehat{P_{12}} < P_0$, then compute $\Pi_1^0(P)$ at $P = P_0$ and go to Step 2.5.
Otherwise, go to Step 2.4.3.
2.4.3. If $/b < \widehat{P_{12}}$, then compute $\Pi_1^0(P)$ at $P = P_0$ and go to Step 2.5.
Otherwise, compute $\Pi_1^0(P)$ at $P = P_0, P_{13}^*$ and go to Step 2.5.
2.5. Determine $T_1(P)$ by (17) and go to Step 3.
Step 3. Select the optimal sales price(P^*) and replenishment cycle time(T^*) which gives the maximum annual net profit among those obtained in steps 1 and 2.

4. Numerical Example

As a marketing policy, some suppliers offer credit periods to the buyers for the stimulation of product demand. The availability of the credit period from the supplier effectively reduces the cost of holding stocks for the buyer and therefore, the buyer can discount his sales price expecting to increase the customer′s demand. For the analysis, we considered the situation that the customer′s demand rate is a downward slopping linear function of the buyer′s sales price. Also, in consideration of the situation that the utility of inventory does not remain constant over time, we dealt with the problem under the assumption of decay. The following example will be used to illustrate the solution algorithm.

Let the customer′s demand rate be the linear function (let a = 10,000 and b = 1,250). That is, D =10,000 - 1,250P. Also, let C = \$3, S = \$50, tc = 0.3, H = \$0.1, R = 0.15(= 15%), I = 0.1(= 10%) and $\lambda = 0.3$. The solution procedure generates the buyer′s optimal sales price(P^*) and replenishment cycle time(T^*) for the approximate model through the following steps.

Step 1.1. From equation (24), $P_0 = 7.32$. Then, compute $\Pi_2^0(P)'$ at P = 7.32 by equation (36).
Step 1.2. Since $\Pi_2^0(P_0)' = -4.831 < 0$, compute $\Pi_2^0(P)$ at $P = P_{21}^*$.

Solving equation (43) numerically on the price interval, $< P_0$, we obtain $P_{21}^{*}(= 5.46)$ and compute $\Pi_2^0(P)$ at $P = 5.46$.

Step 1.4. Compute $T_2(P)$ at P = 5.46 by equation (18) and go to Step 2.

Step 2.1. Compute $\Pi_1^0(P)'$at P = 7.32 by equation (27).

Step 2.2. Since $\Pi_1^0(P_0)' = -4{,}409 < 0$, go to Step 2.4.

Step 2.4. Compute $\Pi_1^0(P)'$ at $P = a/b - \varepsilon\ (= 7.99)$ by equation (27).

Step 2.4.1. Since $\Pi_1^0(7.99)' = 14{,}869 > 0$, compute $\Pi_1^0(P)$ at P = 7.32, 7.99 and go to Step 2.5.

Step 2.5. Compute $T_1(P)$ at P = 7.32, 7.99 by equation (17) and go to Step 3.

Step 3. Since $\Pi_2^0(5.46)$=7,453.21, $\Pi_1^0(7.32) = 3{,}432.68$ and $\Pi_1^0(7.99) = -3.62$, an optimal solution $(P^*, T^*$) becomes (5.46, 0.1555) with its maximum annual net profit $7,453.21.

To determine the optimal solution P^* and T^* for the approximate model, the availability of the credit period from the supplier enables the buyer to select the sales price within a wider range of selling price ($P < a/b$ = 8). According to the solution algorithm, we just need to consider P = 5.46, 7.32, 7.99 for finding the buyer′s optimal decision. From the results obtained by comparing the profit values with P = 5.46, 7.32, 7.99, the buyer′s optimal sales price(P^*) becomes 5.46 and replenishment cycle time(T^*) becomes 0.1555 with its maximum annual net profit $7,453.21. And the buyer's optimal order quantity(Q^*) can be obtained by equation (6).

5. Conclusion

In this paper, we analyzed the problem determining the buyer's price and order size simultaneously for an exponentially decaying product when the supplier permits a delay in payments. Recognizing that the major reason for the supplier to offer a credit period to the buyers is to stimulate the order of the product and the buyer's order size is affected by the customer's demand, we expressed the customer's demand rate of the product with the downward slopping linear function. After modeling the appropriate mathematical model, we examined the characteristics of the buyer′s annual net profit function and suggested the solution algorithm to determine the buyer's optimal sales price and replenishment cycle time. Although the model has a very complex structure, we can easily find the optimal solution using the numerical search method. As a result, we have shown that the availability of the credit period from the supplier enables the buyer to select the sales price within a wider range of selling price expecting an additional demand for customer.

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