

# A Coupled Recursive Total Least Squares-Based Online Parameter Estimation for PMSM

Yangding Wang\*, Shen Xu<sup>†</sup>, Hai Huang\*\*, Yiping Guo\* and Hai Jin\*\*

**Abstract** – A coupled recursive total least squares (CRTLS) algorithm is proposed for parameter estimation of permanent magnet synchronous machines (PMSMs). TLS considers the errors of both input variables and output ones, and thus achieves more accurate estimates than standard least squares method does. The proposed algorithm consists of two recursive total least squares (RTLS) algorithms for the  $d$ -axis subsystem and  $q$ -axis subsystem respectively. The incremental singular value decomposition (SVD) for the RTLS obtained by an approximate calculation with less computation. The performance of the CRTLS is demonstrated by simulation and experimental results.

**Keywords:** Online parameter estimation, Permanent magnet synchronous machine (PMSM), Total least squares (TLS), Singular value decomposition (SVD).

## 1. Introduction

Permanent magnet synchronous machines (PMSMs) are widely used in industrial servo drives, robotics, electric vehicles and wind power generators etc., due to their high efficiency, high power density, good control performance, simple structure and reliable operation [1]. It is important to obtain accurate knowledge of PMSM parameters for online condition monitoring, fault diagnosis and high-performance control system design.

Many methods have been presented to estimate PMSM parameters, which can be classified into offline methods and online ones. Some offline methods keep the machine at standstill state and inject signals of current or voltage. For example, a dc step voltage [2] was injected into the PMSM at standstill to estimate stator resistance and  $dq$  axis inductances. The standstill frequency-response (SSFR) test [3], which evaluates the responses with a sequence of sinusoidal signal injection, is an attractive method to estimate the machine parameters with the drawback of the requirement for special equipment. However, there are some drawbacks for the offline methods. These methods usually require additional measurement instruments and operations to keep the machine at standstill. More importantly, the machine parameters of PMSM vary with temperature change, magnetic saturation and demagnetization of rotors during the practical applications.

In order to overcome these shortcomings, the online methods update the estimated parameters in real time. Several online methods such as recursive least squares

(RLS) [4, 5], extended Kalman filter (EKF) [6], model reference adaptive system (MRAS) [7], neural network (NN) [8] and particle swarm optimization (PSO) [9] et al. have been proposed. RLS algorithm has been widely used to estimate PMSM parameters for its simplicity and universal application. But the results generated by RLS are easy to suffer from the noise interference. In [6],  $dq$  axis inductances estimated by EKF were used to improve the performance of maximum torque per ampere (MTPA) control. In the MRAS method [7], one parameter was estimated by setting the other parameters to be their nominal values. In [8], an adaline NN was used to estimate rotor flux linkage and stator resistance according to rotor position offsets. In [9], a PSO with learning strategy was proposed to estimate PMSM parameters. In addition, the parameters were estimated by two affine projection algorithms according to the parameter dynamics [10].

The method of Total least squares (TLS) is an improved standard least squares method by considering the errors of input variables and output variables at the same time. TLS has been employed to adaptive filtering [11] and parameter estimation for a number of different applications [12, 13]. The problem of TLS is usually solved by the singular value decomposition (SVD). The recursive TLS (RTLS) [14, 15] is required for the online application.

This paper proposes a coupled recursive total least squares (CRTLS) algorithm to estimate PMSM parameters online. The algorithm decomposes the problem into  $d$ -axis and  $q$ -axis subsystem, performs recursive TLS for each subsystem and couples the results of the two subsystems. In the recursive TLS, the incremental SVD is achieved by an approximate calculation with less computation [15]. Simulations and experiments are performed to verify the effectiveness of the proposed algorithm.

This paper is organized as follows. The model of PMSM and the method of RLS are described in Section 2. The

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CRTLS algorithm is proposed in Section 3. Simulation and experimental results are given in Section 4.

## 2. PMSM Model and RLS Algorithm for Parameter Estimation

### 2.1 PMSM model

Assuming that the PMSM has negligible cross-coupling magnetic saturation, structural asymmetry, iron losses, magnet eddy current loss, and rotor anisotropy, the model of PMSM in the synchronous rotating reference frame is shown as follows:

$$\begin{cases} u_d = R_s i_d + L_d \frac{di_d}{dt} - L_q i_q \omega_e \\ u_q = R_s i_q + L_q \frac{di_q}{dt} + L_d i_d \omega_e + \omega_e \lambda_m \end{cases} \quad (1)$$

where  $u_d$ ,  $u_q$  and  $i_d$ ,  $i_q$  are the  $d$ -axis and  $q$ -axis voltages and currents, respectively.  $R_s$  is the stator winding resistance.  $L_d$  and  $L_q$  are the equivalent  $d$ -axis and  $q$ -axis inductances, respectively.  $\lambda_m$  is the permanent magnet flux linkage.  $\omega_e$  is the electrical angular speed.

The measured stator currents for machine control  $i_a$ ,  $i_b$  and  $i_c$  are phase variables with stationary reference frame.  $i_d$  and  $i_q$  can be computed by the following formulation:

$$\begin{bmatrix} i_d & i_q \end{bmatrix}^T = M^T \begin{bmatrix} i_a & i_b & i_c \end{bmatrix}^T \quad (2)$$

$$M = \frac{2}{3} \begin{bmatrix} \cos \theta_e & -\sin \theta_e \\ \cos(\theta_e - 2\pi/3) & -\sin(\theta_e - 2\pi/3) \\ \cos(\theta_e + 2\pi/3) & -\sin(\theta_e + 2\pi/3) \end{bmatrix} \quad (3)$$

The four parameters to be estimated are the stator resistance  $R_s$ , the PM flux linkage  $\lambda_m$ , and the model inductances  $L_d$  and  $L_q$ .

### 2.2 The method of least squares

The least squares (LS) algorithm has been widely applied into parameter estimation and system modeling due to its simplicity and universal application. A discrete-time system of PMSM can be modeled as

$$y(k) = h(k)\theta + e(k) \quad (4)$$

where

$$y(k) = \begin{bmatrix} u_d(k) & u_q(k) \end{bmatrix}^T \quad (5)$$

$$\theta = \begin{bmatrix} R_s & L_d & L_q & \lambda_m \end{bmatrix}^T \quad (6)$$

$$h(k) = \begin{bmatrix} i_d(k) & \dot{i}_d(k) & -\omega(k)i_q(k) & 0 \\ i_q(k) & \omega(k)i_d(k) & \dot{i}_q(k) & \omega(k) \end{bmatrix} \quad (7)$$

where the discrete-time  $k = 0, 1, 2, \dots$  and  $y(k)$  is system output.  $h(k)$  is the measured data vector, and  $\theta$  is the unknown model parameter vector.  $e(k)$  is a noise term.

For  $L$  samples, there are the following equations:

$$y_L = H\theta + e_L \quad (8)$$

where

$$y_L = [y(1), y(2), \dots, y(L)]^T \quad (9)$$

$$H = [h(1), h(2), \dots, h(L)]^T \quad (10)$$

$$e_L = [e(1), e(2), \dots, e(L)]^T \quad (11)$$

In most variants of least squares algorithms, the following criterion function is defined and minimized:

$$J := \sum_k (y(k) - h(k)\theta)^2 \quad (12)$$

The estimated vector  $\hat{\theta}$  is achieved by minimizing  $J$ .

$$\hat{\theta} = (H^T H)^{-1} H^T y_L \quad (13)$$

To identify the parameter vector  $\hat{\theta}$  online, a recursive instrumental variables algorithm is used. According to the recursive algorithm of least squares, we can also get recursive equations for the instrumental variables algorithm:

$$\hat{\theta}(k) = \hat{\theta}(k-1) + K(k)[y(k) - h(k)\hat{\theta}(k-1)] \quad (14)$$

$$K(k) = P(k-1)h^T(k)[h(k)P(k-1)h^T(k) + 1]^{-1} \quad (15)$$

$$P(k) = [I - K(k)h(k)]P(k-1) \quad (16)$$

where  $K(k)$  is the gain vector,  $P(k)$  is the covariance matrix.

In order to obtain unbiased estimates by the least squares method, the error  $e(k)$  should not be correlated. At the same time, LS assumes the data matrix  $H$  to be free of errors. These prerequisites are difficult to meet in practice. Hence LS is usually applied to systems with correlated errors and yields biased estimates. To achieve unbiased estimate, several modified LS methods have been proposed, such as generalized LS, extended LS, bias compensation LS, total LS, stochastic approximation and instrumental variables. In this paper, the method of total least squares (TLS) is adopted because of its attractive computation and ability to obtain consistent and asymptotically unbiased estimates for linear systems with misspecified noise model.

## 3. Coupled Recursive Total Least Squares Algorithm for Parameter Estimation

### 3.1 The errors of $dq$ -axis voltages and currents

To estimate the parameters of PMSM, it is necessary to measure  $d$ -axis and  $q$ -axis currents. The  $d$ -axis and  $q$ -axis

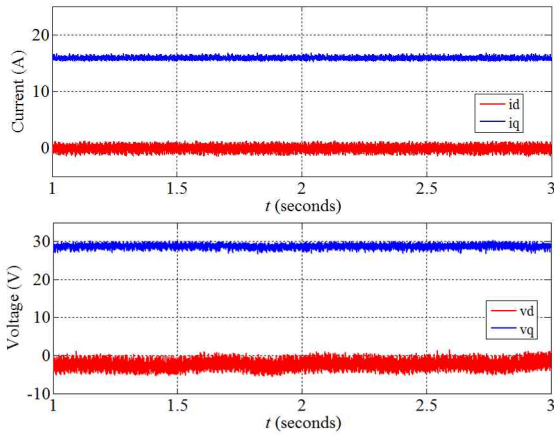


Fig. 1. Waveforms of  $dq$ -axis currents and voltages for a PMSM with vector control

current waveforms under constant load torque 10N.m which were obtained by later experiments are shown as Fig. 1. It is easy to see that the currents fluctuated within certain range. There are two reasons for this: one is the fluctuations of control signals yielded by the control algorithm of PMSM, the other is the external interference and measurement noise.

The  $d$ -axis and  $q$ -axis voltages are usually generated by a close-loop control for PMSM. Thus there is no need to consider the measurement noise. These two voltages cannot keep constant in most case. They fluctuate even for a constant load torque and rotating speed, as shown in Fig. 1.

As a result, both  $d$ -axis and  $q$ -axis voltages and currents are subjected to fluctuations. The exists of these fluctuations degrade the performance of the parameter estimation methods for PMSM. These undesired fluctuations can be treated as the errors of the signal. According to the theory of system identification, the parameter estimation problem which takes the errors in both input variables and output ones into account is modeled as error-in-variables (EIV) [16]. The parameters in EIV model can be estimated by several methods such as TLS, instrumental variables, bias-eliminating least squares, et al [16].

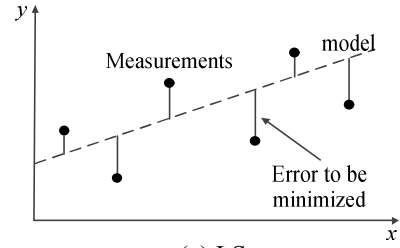
### 3.2 The method of total least squares

The difference between LS and TLS is shown in Fig. 2. LS corrects the data vertically with the consideration of only the error of output variable  $y$ . While TLS performs perpendicular data corrections with the consideration of both the errors of input variable  $x$  and output variable  $y$ .

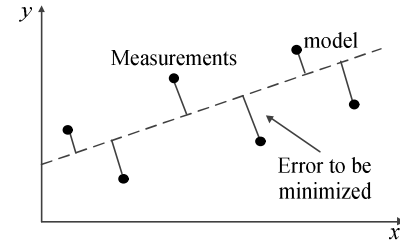
The model of TLS is given as

$$y + e = (H + F)\hat{\theta} \quad (17)$$

Assuming that the output vector  $y$  is subjected to error  $e$  and the data matrix  $H$  is subjected to error  $F$ . This model can be rewritten as



(a) LS



(b) TLS

Fig. 2. The difference between LS and TLS

$$(C + \Delta C) \begin{pmatrix} \hat{\theta} \\ -1 \end{pmatrix} = 0 \quad (18)$$

where  $C = [H \ y]$  and  $\Delta C = [F \ e]$ .

The goal of TLS is to minimize the error matrix  $\Delta C$ , which is performed in the sense of the Frobenius norm:

$$\|\Delta C\|_F^2 = \sum_i \sum_j \Delta C_{ij}^2 \quad (19)$$

Without loss of generality, the singular value decomposition (SVD) of  $C$  is

$$C = U \Sigma V^T \quad (20)$$

where  $U$  and  $V$  are the orthonormal matrixes, and

$$\Sigma = \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_{n+1}) \quad (21)$$

$\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_{n+1}$  are the singular values of  $C$ .

Define the partitioning of  $V$

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \quad (22)$$

A TLS solution exists if and only if  $V_{22}$  is nonsingular. The parameter estimates can be determined as

$$\hat{\theta}_{\text{TLS}} = -V_{22}^{-1}V_{12} \quad (23)$$

This batch algorithm is not applicable to online parameter estimations due to the update of SVD in each iteration.

### 3.3 Recursive total least squares

To identify the parameter vector  $\hat{\theta}$  online, an efficient

incremental SVD is needed.

Let  $C(k)$  be the matrix at  $k$ -th iteration,  $k=1, 2, \dots$ . When  $C(k)$  is replaced by its SVD (20), we get

$$C^T(k)C(k) = (U \Sigma V^T)^T (U \Sigma V^T) = V \Sigma_{m+1} V^T \quad (24)$$

where  $U^T U = I$  and  $\Sigma_{m+1} = \Sigma^T \Sigma = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_{m+1}^2)$ .

Define the matrix

$$Q(k) = [C^T(k)C(k)]^{-1} = (V \Sigma_{m+1} V^T)^{-1} = V \Sigma_{m+1}' V^T \quad (25)$$

where  $\Sigma_{m+1}' = \text{diag}(\sigma_1^{-2}, \sigma_2^{-2}, \dots, \sigma_{m+1}^{-2})$ .

The recursive form of  $Q(k)$  is

$$\begin{aligned} Q(k) &= [C^T(k)C(k)]^{-1} \\ &= [(C^T(k-1) \quad d(k))(C^T(k-1) \quad d(k))^T]^{-1} \\ &= [C^T(k-1)C(k-1) + d(k)d(k)^T]^{-1} \\ &= [Q^{-1}(k-1) + d(k)d(k)^T]^{-1} \end{aligned} \quad (26)$$

where  $d(k)$  is the  $k$ -th column of  $C^T(k)$ .  $d(k)$  is also the measured vector at  $k$ -th iteration.

According to the matrix inversion, the recursive matrix of  $Q(k)$  is given as

$$Q(k) = [I - \frac{Q(k-1)d(k)d(k)^T}{1 + d(k)^T Q(k-1)d(k)}]Q(k-1) \quad (27)$$

$V(k)$  is an orthogonal matrix and its column vectors  $v_1(k), v_2(k), \dots, v_{m+1}(k)$  are eigenvectors of matrix  $C^T(k)C(k)$ . These column vectors can be considered to be a set of normal orthogonal basis in  $m+1$  dimensional space, where  $m$  is the number of parameter in  $\theta$ . Thus vector  $v_{m+1}(k-1)$  can be express by the linear combination of eigenvectors  $v_1(k), v_2(k), \dots, v_{m+1}(k)$ . That is

$$v_{m+1}(k-1) = s_1(k)v_1(k) + \dots + s_{m+1}(k)v_{m+1}(k) \quad (28)$$

where  $s_i(k)$  is the coefficient of normal orthogonal basis.

Define vector

$$g(k) = Q(k)v_{m+1}(k-1) \quad (29)$$

Then

$$\begin{aligned} g(k) &= Q(k)v_{m+1}(k-1) \\ &= (\sum_{i=1}^{m+1} \frac{1}{\sigma_i^2} v_i(k)v_i^T(k)) (\sum_{j=1}^{m+1} s_j(k)v_j(k)) = \sum_{i=1}^{m+1} \frac{s_i(k)}{\sigma_i^2} v_i(k) \end{aligned} \quad (30)$$

According to (27), there is a highly correlation between matrix  $Q(k)$  and  $Q(k-1)$ . Then vector  $v_{m+1}(k)$  and  $v_{m+1}(k-1)$  are highly correlated because they are eigenvectors of the two matrixes. Thus

$$s_{m+1}(k) \gg s_i(k) \quad (i=1, 2, \dots, m) \quad (31)$$

$$\frac{s_{m+1}(k)}{\sigma_{m+1}^2(k)} \gg \frac{s_i(k)}{\sigma_i^2(k)} \quad (i=1, 2, \dots, m) \quad (32)$$

Vector  $g(k)$  can be approximated as

$$g(k) \approx \frac{s_{m+1}(k)}{\sigma_{m+1}^2(k)} v_{m+1}(k) \quad (33)$$

The estimated vector  $v_{m+1}(k)$  can be obtained by normalization

$$\hat{v}_{m+1}(k) = g(k) / \|g(k)\| \quad (34)$$

According to (23), the recursive parameter estimates are

$$\hat{\theta}_{\text{RTLS}} = -\frac{1}{v_{m+1,m+1}} [v_{1,m+1}, \dots, v_{m,m+1}]^T \quad (35)$$

where  $v_{i,m+1}$  is the  $i$ -th element of the estimated vector  $\hat{v}_{m+1}(k)$ ,  $i=1, 2, \dots, m+1$ .

In general, RTLS updates matrix  $Q(k)$  from (27) based on the measured vector  $d(k)$ , then calculates the vector  $v_{m+1}(k)$  from (29) and (34). Finally, the estimated parameter  $\hat{\theta}_{\text{RTLS}}(k)$  can be obtained from (35) based on the vector  $v_{m+1}(k)$ .

### 3.4 Coupled recursive total least squares

When RTLS is used to estimate the parameters of PMSM directly, the measured vector  $d(k)$  becomes a matrix according to the PMSM model (1). Then the recursive matrix  $Q(k)$  needs to calculate the matrix inversion at each recursion, which results in heavy computational burden and should usually be avoided for online algorithm. A coupled RTLS (CRTLS) is proposed to reduce computation load for each iteration. The coupled identification is an effective method for multi-input multi-output (MIMO) system [17].

The schematic diagram of the CRTLS for PMSM is

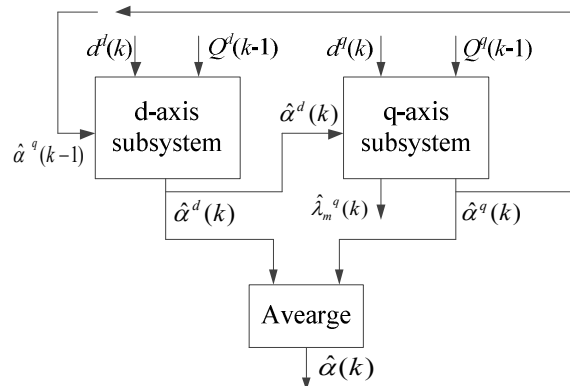


Fig. 3. Schematic diagram of CRTLS

shown in Fig. 3. The variables denote a vector or matrix in  $d$ -axis subsystem for superscript “ $d$ ” or  $q$ -axis subsystem for superscript “ $q$ ”. The PMSM model is decomposed into two subsystems ( $d$ -axis subsystem and  $q$ -axis subsystem). Each subsystem is estimated by RTLS. The common parameters for  $dq$  axis subsystems are  $R_s$ ,  $L_d$  and  $L_q$ . Define the common parameter vector

$$\alpha(k) = [R_s(k) \quad L_d(k) \quad L_q(k)]$$

For  $k$ -th estimation cycle,  $d$ -axis subsystem replaces  $\hat{\alpha}^d(k-1)$  with  $\hat{\alpha}^q(k-1)$ , and  $q$ -axis subsystem replaces  $\hat{\alpha}^q(k-1)$  with  $\hat{\alpha}^d(k)$ . In this way the latest estimated common parameter vector  $\hat{\alpha}$  is always used for the next iteration of another subsystem. Such operations are based on the theory of Jacobi and Gauss-Seidel iterations [17]. Finally, CRTLS sets the average of common parameters estimated by each subsystem as the estimated result, while the rest of estimated parameters keep themselves.

For  $d$ -axis subsystem, the model is the first expression in (1) and can be rewritten as the following for  $k$ -th iteration:

$$u_d(k) = [i_d(k) \quad i_d'(k) \quad -\omega(k)i_q(k)] [R_s \quad L_d \quad L_q]^T$$

Based on the model, the measured vector  $d^d(k)$  and measured data matrix  $C^d(k)$  are

$$d^d(k) = [i_d(k) \quad i_d'(k) \quad -\omega(k)i_q(k) \quad u_d(k)] \quad (36)$$

$$C^d(k) = \begin{bmatrix} i_d(1) & i_d'(1) & -\omega(1)i_q(1) & u_d(1) \\ i_d(2) & i_d'(2) & -\omega(2)i_q(2) & u_d(2) \\ \vdots & \vdots & \vdots & \vdots \\ i_d(k) & i_d'(k) & -\omega(k)i_q(k) & u_d(k) \end{bmatrix} \quad (37)$$

It is clear that matrix  $C^d(k)$  is composed of  $k$  measured vectors  $d^d(1)$ ,  $d^d(2)$ , ...,  $d^d(k)$ .

For  $k$ -th estimation cycle, RTLS for  $d$ -axis subsystem has input data including the measured vector  $d^d(k)$ , recursive matrix  $Q^d(k-1)$  and the common parameter vector  $\hat{\alpha}^q(k-1)$ , and outputs the new recursive matrix  $Q^d(k)$  and the estimated common parameter vector  $\hat{\alpha}^d(k)$ . Matrix  $Q^d(k)$  can be updated according to (27). The vector  $v_{m+1}^d(k-1)$  in (29) corresponding to  $\hat{\alpha}^q(k-1)$  can be calculated as:

$$v_{m+1}^d(k-1) = \frac{[-\hat{\alpha}^q(k-1) \quad 1]}{\|[-\hat{\alpha}^q(k-1) \quad 1]\|} \quad (38)$$

(38) obtains the vector  $v_{m+1}^d(k-1)$  by the estimated parameter vector, whose function is opposite to (35). Then vector  $v_{m+1}^d(k)$  can be calculated according to (29) and (33). At last,  $\hat{\alpha}^d(k)$  can be obtained according to (35). The procedure of  $d$ -axis subsystem for  $k$ -th estimation cycle is listed in the following:

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**RTLS for  $d$ -axis subsystem:**

Input:  $d^d(k)$ ,  $Q^d(k-1)$ ,  $\hat{\alpha}^q(k-1)$

Output:  $Q^d(k)$ ,  $\hat{\alpha}^d(k)$

- Update  $Q^d(k)$  from (27)
  - Get  $v_{m+1}^d(k-1)$  from (38)
  - Get  $g^d(k)$  from (29)
  - Get  $v_{m+1}^d(k)$  from (34)
  - Get  $\hat{\alpha}^d(k)$  from (35)
- 

**Table 1.** Nominal parameters of a prototype PMSM

|                      |         |
|----------------------|---------|
| Stator Resistance    | 0.032Ω  |
| $d$ -axis Inductance | 0.71mH  |
| $q$ -axis Inductance | 1.33mH  |
| Rotor Flux Linkage   | 0.108Wb |
| Rated Power          | 20kW    |
| Rated Phase Current  | 70A     |
| Rated Speed          | 3000rpm |
| Number of Poles      | 8       |

For  $q$ -axis subsystem, the model is the second expression in (1) and can be rewritten as the following for  $k$ -th iteration:

$$u_q(k) = [i_q(k) \quad \omega(k)i_d(k) \quad i_q'(k) \quad \omega(k)] [R_s \quad L_d \quad L_q \quad \lambda_m]^T$$

Based on this model, the measured vector  $d^q(k)$  and measured data matrix  $C^q(k)$  are

$$d^q(k) = [i_q(k) \quad \omega(k)i_d(k) \quad i_q'(k) \quad \omega(k) \quad u_q(k)] \quad (39)$$

$$C^q(k) = \begin{bmatrix} i_q(1) & \omega(1)i_d(1) & i_q'(1) & \omega(1) & u_q(1) \\ i_q(2) & \omega(2)i_d(2) & i_q'(2) & \omega(2) & u_q(2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ i_q(k) & \omega(k)i_d(k) & i_q'(k) & \omega(k) & u_q(k) \end{bmatrix} \quad (40)$$

The  $q$ -axis subsystem has the same process as  $d$ -axis subsystem. The vector  $v_{m+1}^q(k-1)$  in (29) corresponding to  $\hat{\alpha}^d(k)$  can be calculated as:

$$v_{m+1}^q(k-1) = \frac{[\hat{\alpha}^d(k) \quad \hat{\lambda}_m(k-1) \quad -1]}{\|[\hat{\alpha}^d(k) \quad \hat{\lambda}_m(k-1) \quad -1]\|} \quad (41)$$

Matrix  $Q^q(k)$ , vector  $v_{m+1}^q(k)$ , the estimated common parameter vector  $\hat{\alpha}^q(k)$  and the estimated flux linkage  $\hat{\lambda}_m(k)$  also can be calculated as  $d$ -axis subsystem does. The procedure of  $q$ -axis subsystem for  $k$ -th estimation cycle is listed in the following:

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**RTLS for  $q$ -axis subsystem**

Input:  $d^q(k)$ ,  $Q^q(k-1)$ ,  $\hat{\alpha}^d(k)$ ,  $\hat{\lambda}_m(k-1)$

Output:  $Q^q(k)$ ,  $\hat{\alpha}^q(k)$ ,  $\hat{\lambda}_m(k)$

- Update  $Q^q(k)$  from (27)
  - Get  $v_{m+1}^q(k-1)$  from (41)
  - Get  $g^q(k)$  from (29)
  - Get  $v_{m+1}^q(k)$  from (34)
  - Get  $\hat{\alpha}^q(k)$  and  $\hat{\lambda}_m(k)$  from (35)
-

For the common parameters in the two subsystems, CRTLS takes their average as the estimations. For the other parameters in the subsystems, CRTLS directly uses the estimations obtained by the subsystems. Then the parameter vector estimated by CRTLS can be expressed as

$$\hat{\theta}_{\text{CRTLS}}(k) = [\hat{\alpha}(k) \quad \hat{\lambda}_m(k)] \quad (42)$$

where  $\hat{\alpha}(k)$  is set as following:

$$\hat{\alpha}(k) = (\hat{\alpha}^d(k) + \hat{\alpha}^q(k)) / 2 \quad (43)$$

### 3.5 Consideration of saturation and cross-coupling effects

The proposed algorithm is based on the assumption that  $d$ -axis and  $q$ -axis are independent of each other. But there exist some effects for the practical PMSM such as magnetic saturation and cross-coupling, which lead to a coupled relation between  $d$ -axis and  $q$ -axis. To include the influence of the two effects, the  $d$ -axis inductance  $L_d$  and  $q$ -axis inductance  $L_q$  in the PMSM model (1) can be modeled as a relation between  $d$ -axis and  $q$ -axis. To include the influence of the two effects, the  $d$ -axis inductance  $L_d$  and  $q$ -axis inductance  $L_q$  in the PMSM model (1) can be modeled as a function of both the  $dq$ -axis currents and are denoted by  $L_d(i_d, i_q)$  and  $L_q(i_d, i_q)$  respectively. And these functions are usually approximated as the exponential function [18] or polynomial expressions [19]. When  $L_d$  and  $L_q$  in the PMSM model (1) are replaced by the approximation functions, the  $d$ -axis and  $q$ -axis could be regarded as a decoupled relationship. The proposed CRTLS algorithm will be still applicable to the modified model with some changes.

In this paper, the PMSM model (1) is still used because this model is accurate enough for most PMSM applications. The influence of the two effects is in small measure up to the rated current levels because of the large air gap and the magnet's equivalent air-like gap [20].

## 4. Simulation and Experimental Results

### 4.1 Simulation results

The proposed CRTLS algorithm was demonstrated by Matlab/Simulink. A vector control was simulated to generate PMSM running data. The parameters of a prototype PMSM

used in the simulation are listed in Table 1. The simulation was calculated by a fix-step solver with step size  $5\mu\text{s}$ . SVPWM switching frequency and stator current sample period are 5kHz and  $200\mu\text{s}$  respectively.

For comparison, CRTLS, TLS and RLS were applied to estimate the parameters of PMSM with a sudden load torque change. The load torque of PMSM was set to 10N.m at the startup and step to 20N.m after a second at a constant rotor speed 300rpm. Two subsystems of CRTLS algorithm have the following initial values:  $\hat{\alpha}^d(0) = [0 \ 0 \ 0]^T$ ,  $\hat{\lambda}_m(0) = 0$ ,  $v_{m+1}^d(0) = [0 \ 0 \ 0 \ 1]^T$ ,  $v_{m+1}^q(0) = [0 \ 0 \ 0 \ 0 \ 1]^T$ , Matrix  $Q^d(0) = Q^q(0) = 10^5 I$  where  $I \in \mathbb{R}^{5 \times 5}$  is an identity matrix.

Table 2 shows the parameters estimated by different algorithms at time 2.5s with such load torque. It is clear that the four parameters estimated by CRTLS have less error (maximum error is 3.75%, minimal error is 1.20%) than the ones estimated by RLS. CRTLS performs better due to considering both the errors of input variables and output variables at the same time.

According to Table 2, the parameters estimated by CRTLS are very close to the ones estimated by TLS. This indicates that the approximation of RTLS and coupled implementation of RTLS are feasible for TLS to estimate parameters of PMSM. CRTLS is generally preferred over TLS in view of the computational burden and computer memory.

To confirm the performance with noise interference for the proposed CRTLS algorithm, we injected Gauss noise to  $d$ -axis and  $q$ -axis currents respectively. The estimated processes for CRTLS and RLS are shown in Fig. 4. It can be seen that both the proposed CRTLS and standard RLS can converge to the true values and the former has higher accuracy.

To quantitatively evaluate the enhancement brought by the proposed algorithm, the relative error vector and the mean square deviation (MSD) are defined as follow:

$$e_k = \begin{bmatrix} \frac{\Delta R_s(k)}{R_s(k)} & \frac{\Delta L_d(k)}{L_d(k)} & \frac{\Delta L_q(k)}{L_q(k)} & \frac{\Delta \lambda_m(k)}{\lambda_m(k)} \end{bmatrix}^T \quad (44)$$

$$\text{MSD} = 10 \times \log_{10}(E(\|e_k\|_F^2)) \quad (45)$$

where  $R_s(k)$ ,  $L_d(k)$ ,  $L_q(k)$ , and  $\lambda_m(k)$  are the estimated parameters and  $\Delta R_s(k)$ ,  $\Delta L_d(k)$ ,  $\Delta L_q(k)$ , and  $\Delta \lambda_m(k)$  are the identification absolute error at the  $k$ -th estimation cycle. The MSD with the unit of dB is used as the evaluation index. The MSDs for different algorithms are shown in Fig. 4. It is shown that the proposed algorithm

**Table 2.** The estimated results for the simulations

| Parameters         | True Value | RLS             |        | CRTLS           |        | TLS             |        |
|--------------------|------------|-----------------|--------|-----------------|--------|-----------------|--------|
|                    |            | Estimated Value | Error  | Estimated Value | Error  | Estimated Value | Error  |
| $R_s$ [ $\Omega$ ] | 0.032      | 0.0358          | 11.87% | 0.0332          | 3.75%  | 0.0330          | 3.12%  |
| $L_d$ [mH]         | 0.71       | 0.653           | -8.03% | 0.688           | -3.10% | 0.689           | -2.96% |
| $L_q$ [mH]         | 1.33       | 1.273           | -4.29% | 1.292           | -2.86% | 1.293           | -2.78% |
| $\lambda_m$ [Wb]   | 0.108      | 0.1101          | 1.94%  | 0.1093          | 1.20%  | 0.1092          | 1.11%  |

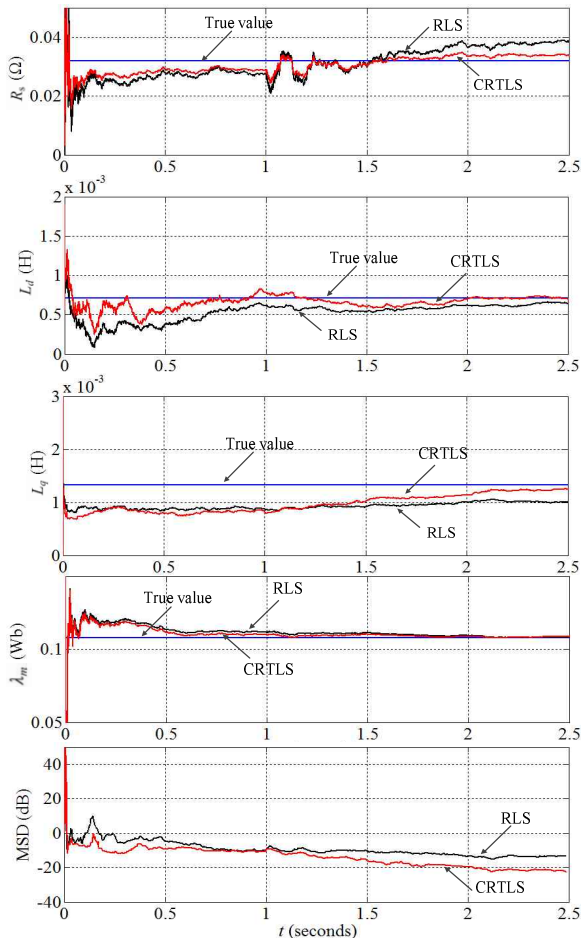


Fig. 4. Simulation results under a sudden change in load torque at a constant rotor speed

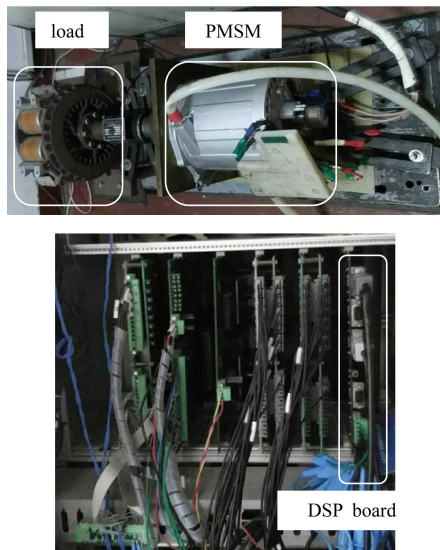


Fig. 5. Experimental setup of PMSM drive system

generates lower MSDs than the standard RLS algorithm. The proposed algorithm yields a MSD reduction of 8.98 dB compared to the RLS algorithm at the end of the

simulation, which suggests an improved accuracy for the estimated parameters.

### 4.2 Experimental Results

In order to verify the proposed CRTLs algorithm, experiments were carried out. The hardware platform of the DSP-based vector control system is shown in Fig. 5. DSP TMS320F28335 with a clock frequency of 150MHz is used to execute the control algorithm. The inverter switching frequency and sample period are 5kHz and 200us, respectively. The parameters of PMSM are listed in Table 1. This machine was designed for electric vehicle drives. The load is an eddy current retarder.

Two operating conditions are adopted to verify the feasibility of the proposed algorithm. Condition 1 has a sudden load torque change from 10N.m to 20N.m at constant rotor speed 300rpm. Condition 2 performs a sudden rotor speed change from 300rpm to 600rpm at constant load torque 10N.m.

Table 3 shows the experimental results for the two conditions. For Condition 1, the estimated stator resistance has the maximum error of -6.87% and rotor flux linkage

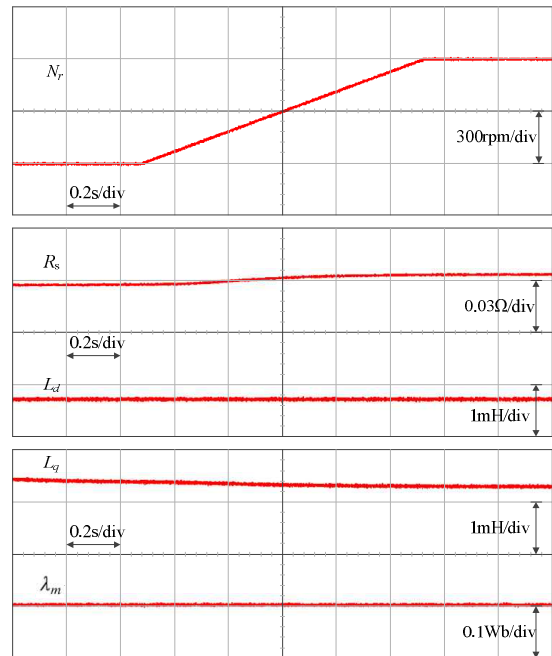


Fig. 6. Experimental results under a change in rotor speed at a constant load torque

Table 3. The estimated results for the experiments

| Parameters         | True value | Condition 1     |        | Condition 2     |        |
|--------------------|------------|-----------------|--------|-----------------|--------|
|                    |            | Estimated value | Error  | Estimated value | Error  |
| $R_s$ [ $\Omega$ ] | 0.032      | 0.0298          | -6.87% | 0.0305          | -4.69% |
| $L_d$ [mH]         | 0.71       | 0.755           | 6.34%  | 0.749           | 5.49%  |
| $L_q$ [mH]         | 1.33       | 1.392           | 4.66%  | 1.381           | 3.83%  |
| $\lambda_m$ [Wb]   | 0.108      | 0.1055          | -2.31% | 0.1061          | -1.76% |

has the minimum error of -2.31%. Condition 2 has less error than Condition 1 does. The changes of the estimated parameters for Condition 2 are shown in Fig. 6. The estimated stator resistance and  $q$ -axis inductance approach to the true values a little during the acceleration. While the estimated  $d$ -axis inductance and rotor flux linkage are not affected.

Comparing the experimental results with the simulation results for Condition 1, the former has more error. There are several reasons for this. Firstly, there are some differences between the model of PMSM and the real motor. The effects such as magnetic saturation, motor loss, et al, are not included in the model. Secondly, the measured currents and rotating speed may have measurement error due to noise, electromagnetic interference or the measuring devices. This condition can be ignored for the simulations. Thirdly, the true values of PMSM parameters may vary with operation temperature and stator currents in the practical application.

Another experiment is conducted to further verify the performance of the proposed algorithm on the PMSM with vector control. The operations of  $i_d=0$  and different  $i_q$  are performed on the experimental machine. The influence of saturation and cross-couple effects is considered. The  $dq$ -axis inductances are measured by an offline method based on the PMSM model (1), which performs the operations at a locked-rotor position. The measured and estimated  $dq$ -axis inductances are shown in Fig. 7. It can be seen that  $d$ -axis inductance almost keep a constant for different  $i_q$ . The measured  $q$ -axis inductance has little changes when  $i_q$  is less than 50A, and then becomes less for a bigger  $i_q$ , which is resulted by the effect of saturation. The results of  $d$ -axis inductance estimation have an average error of 5.21%

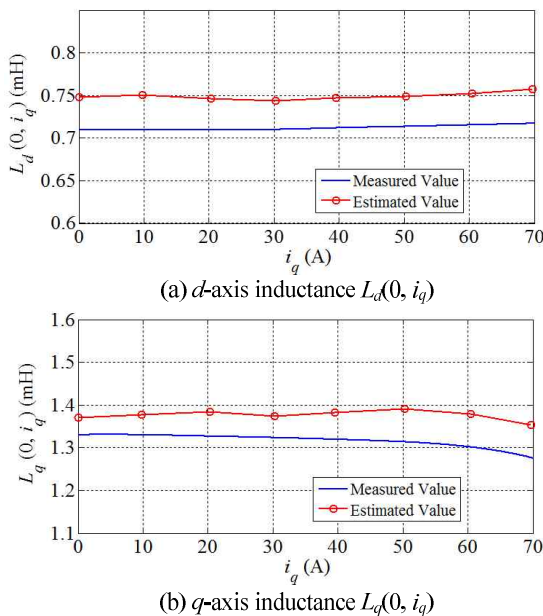


Fig. 7. Experimental results of  $dq$ -axis inductance estimations with  $i_d=0$  and different  $i_q$  currents

and a maximum error of 5.70%. The average error and maximum error for the  $q$ -axis inductance estimation are 4.65% and 6.08% respectively. It is also observed that the estimated  $q$ -axis inductance can follow the variation of the measured  $q$ -axis inductance due to saturation.

## 5. Conclusion

According to the coupled identification and the total least squares identification technique, this paper proposes the online parameter estimation method using the CRTLS algorithm for PMSM. The algorithm can simultaneously estimate stator resistance, rotor flux linkage,  $d$ -axis inductance and  $q$ -axis inductance. The effectiveness of the proposed algorithm has been demonstrated through the Simulink-based simulation results and the PMSM experimental results. The presented algorithm is applicable to interior and surface-mount PMSM.

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