J Electr Eng Technol.2018; 13(6): 2521-2529
http://doi.org/10.5370/JEET.2018.13.6.2521

ISSN(Print) 1975-0102
ISSN(Online) 2093-7423

A Line-integral Fuzzy Lyapunov Functional Approach to Sampled-data Tracking Control of Takagi-Sugeno Fuzzy Systems

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Abstract – This paper deals with a sampled-data tracking control problem for the Takagi–Sugeno fuzzy system with external disturbances. We derive a stability condition guaranteeing both asymptotic stability and H-infinity tracking performance by employing a newly proposed time-dependent line-integral fuzzy Lyapunov–Krasovskii functional. A new integral inequality is also introduced, by which the proposed stability condition is formulated in terms of linear matrix inequalities. Finally, the effectiveness of the proposed method is demonstrated through a simulation example.

Keywords: Fuzzy control, Sampled-data control, Model reference tracking control, Time-dependent line-integral fuzzy Lyapunov–Krasovskii functional, Linear matrix inequality.

1. Introduction

Recently, linear matrix inequality (LMI)-based [1] stability analysis and controller synthesis for the Takagi-Sugeno (T–S) fuzzy system [2] have generated much interest within the control community. Using the sector nonlinearity concept [3], a wide class of nonlinear systems can be represented as a fixed structure, i.e. a fuzzy summation of linear sub systems. Thus, the stability of nonlinear control systems can be investigated in a systematic manner using the T–S fuzzy model. Accordingly, there are many applications such as disturbance attenuation control [4, 5], switching control [6], and decentralized control [7, 8].

The rapid development of digital computing technology has triggered a demand for sampled-data (SD) control, in which an analog model is controlled by a digital controller. Naturally, the SD control of a T–S fuzzy model has also been actively studied, and each study has generally been based on the direct discretization or input-delay approaches.

In the direct discretization method [8-10], the gain matrices are directly computed in the discrete-time domain using a discrete-time Lyapunov function. On the other hand, the input-delay approach [11-16] casts the SD control problem as an input-delay control problem, enabling the direct stability analysis of the SD control system under the continuous-time domain. In [16], the conservativeness of the conventional input-delay approach was relaxed by employing a time-dependent Lyapunov–Krasovskii functional (LKF). However, the conditions based on the study in [16] were still conservative because they were derived using a quadratic Lyapunov function (CQLF).

Therefore, a number of studies have attempted to relax the CQLF-based stability conditions by developing various types of non-quadratic Lyapunov functions. Among these, the fuzzy Lyapunov function (FLF)-based approaches [17-20] considerably reduced the conservativeness of conventional CQLF-based approaches. However, when using this function, the time derivative of the membership function naturally appears in the derived stability conditions, which requires the upper bounds assumption to convert stability conditions into LMIs [20]. To overcome this drawback, in [21], the line-integral FLF, which does not suffer from the time derivative problem, was proposed. The line-integral FLF has been successfully applied to various control areas, e.g. [22], but, to the best of our knowledge, there is no SD control method based on the function.

Despite the importance of tracking control, numerous studies have only considered the asymptotic stabilization of SD fuzzy-model-based control systems. Conventional studies, e.g. [23], provided results in terms of the bilinear matrix inequality form, which makes the control design difficult. Recently, an LMI-based fuzzy tracking control design method was proposed in [24], and in particular, an SD fuzzy tracking control problem was successfully dealt with using a descriptor model for the control system in [25]. However, there is still a lack of research on the SD fuzzy tracking control problem using a general parallel-distributed-compensation (PDC) law [3].

Therefore, this paper investigates the PDC-based SD fuzzy tracking control problem of a T–S fuzzy model with external disturbances. To simultaneously minimize both the tracking error and effects of the external disturbances on the system, we design a controller based on the H_∞ performance criterion. The stability of the control system is analyzed using a newly proposed time-dependent line-integral LKF. In addition, by introducing a novel integral inequality, we formulate the proposed stability conditions in terms of LMIs. Finally, the effectiveness of the proposed

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Received: April 30, 2018; Accepted: August 19, 2018

method is demonstrated through an example.

2. Problem Formulation

Consider the T–S fuzzy model described by the following IF–THEN rules [21]:

$$\mathcal{R}_i\text{: IF } x_1(t) \text{ is } M_1^{\alpha_{i1}} \text{ and } \cdots \text{ and } x_n(t) \text{ is } M_n^{\alpha_{in}}$$
$$\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t) + \varpi(t), \tag{1}$$

where $\mathcal{R}_i$ denotes the ith fuzzy rule, $i \in \mathcal{I}_r$ is a rule number, $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are the system and input vectors, respectively, $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are the system and input matrices, respectively, $\varpi(t) \in \mathbb{R}^n$ is the external disturbance, α_{ij} specifies which x_j-based fuzzy set is used in the ith fuzzy rule, and $M_j^{\alpha_{ij}}$, $(i,j) \in \mathcal{I}_r \times \mathcal{I}_n$ is the x_j-based fuzzy set in the ith fuzzy rule.

Applying the singleton fuzzifier, product inference engine, and center-average defuzzifier to (1), we have

$$\dot{x}(t) = \sum_{i=1}^{r} w_i\big(x(t)\big)\big(A_i x(t) + B_i u(t)\big) + \varpi(t), \tag{2}$$

where the fuzzy weighting function $w_i\big(x(t)\big)$ is defined as

$$w_i\big(x(t)\big) = \prod_{j=1}^{n} \mu_j^{\alpha_{ij}}\left(x_j(t)\right), \tag{3}$$

in which

$$\mu_j^{\alpha_{ij}}(x_j(t)) = \frac{\pi_j^{\alpha_{ij}}(x_j(t))}{\sum_{\alpha_{ij}=1}^{r_j} \pi_j^{\alpha_{ij}}(x_j(t))}, \quad \sum_{\alpha_{ij}=1}^{r_j} \mu_j^{\alpha_{ij}}(x_j(t)) = 1,$$

$\mu_j^{\alpha_{ij}}(x_j(t)) \in [0,1]$, $\alpha_{ij} \in [1, r_j]$, $\pi_j^{\alpha_{ij}}(x_j(t))$ is the membership function of $M_j^{\alpha_{ij}}$, and r_j is the number of x_j-based fuzzy sets.

In this paper, the T–S fuzzy model (2) is driven to track the following reference model of the form:

$$\dot{x}_r(t) = A_r x_r(t) + r(t), \tag{4}$$

where $x_r(t) \in \mathbb{R}^n$ is the state vector of the reference model, $A_r \in \mathbb{R}^{n\times n}$ is a predefined asymptotically stable system matrix, and $r(t) \in \mathbb{R}^n$ is a bounded reference input vector.

The following SD fuzzy tracking controller sharing the same premise part with (1) is employed in this paper:

$$u(t) = u(t_k) = \sum_{j=1}^{r} w_j\big(x(t_k)\big)K_j e(t_k), t \in [t_k, t_{k+1}), \tag{5}$$

where $K_j \in \mathbb{R}^{m\times n}$ is the gain matrix to be determined, $e(t) := x(t) - x_r(t) \in \mathbb{R}^n$ is the tracking error vector, and t_k is the kth sampling time, and $h := t_{k+1} - t_k$ is a sampling period.

Combining (2) and (5), we have the following closed-loop dynamics of $x(t)$:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\{A_i x(t) + B_i K_j e(t_k)\} \\ &\quad +\varpi(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\{(A_i + B_i K_j)e(t) \\ &\quad +A_i x_r(t) - (t-t_k)B_i K_j \bar{e}(t)\} + \varpi(t),\end{aligned} \tag{6}$$

where $(t-t_k)\bar{e}(t) := e(t) - e(t_k)$.

Based on (4) and (6), we can write the dynamic behavior of the tracking error as follows:

$$\begin{aligned}\dot{e}(t) &= \dot{x}(t) - \dot{x}_r(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} w_i\big(x(t)\big)w_j\big(x(t_k)\big)\{(A_i + B_i K_j)e(t) \\ &\quad -(t-t_k)B_i K_j \bar{e}(t) + (A_i - A_r)x_r(t)\} \\ &\quad +\varpi(t) - r(t).\end{aligned} \tag{7}$$

By defining the augmented state vector for the global closed loop as $\chi(t) = col\{e(t), x_r(t)\} \in \mathbb{R}^{2n}$, and from (4) and (7), we have

$$\begin{aligned}\dot{\chi}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\{\Lambda_{ij}\chi(t) \\ &\quad -(t-t_k)\bar{\Lambda}_{ij}\bar{\chi}(t) + \mathcal{I}\phi(t)\},\end{aligned} \tag{8}$$

where $\chi(t) - \chi(t_k) := (t-t_k)\bar{\chi}(t)$, $\phi(t) = col\{\varpi(t), r(t)\}$, $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix,

$$\Lambda_{ij} = \begin{bmatrix} A_i + B_i K_j & A_i - A_r \\ \mathbf{0} & A_r \end{bmatrix}, \quad \bar{\Lambda}_{ij} = \begin{bmatrix} B_i K_j & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

and $\mathcal{I} = \begin{bmatrix} I_n & -I_n \\ \mathbf{0} & I_n \end{bmatrix}$.

The objective of this paper is to solve the following problem:

Problem 1 For a predefined constant sampling period h, find a gain matrix K_j with $j \in \mathcal{I}_r$ in (5) such that the equilibrium of the augmented system (8) is asymptotically stable when $\phi(t) = \mathbf{0}$; moreover, under the zero initial condition, the effect of $\phi(t)$ on the augmented state vector $\chi(t)$ is attenuated below a desired level, i.e.

$$\int_0^{t_f} \chi^T(t)\tilde{Q}\chi(t)dt \le \gamma^2 \int_0^{t_f} \phi^T(t)\phi(t)dt, \tag{9}$$

where $\tilde{Q} = diag\{Q, \mathbf{0}\} \in \mathbb{R}^{2n\times 2n}$, $0 \prec Q = Q^T \in \mathbb{R}^{n\times n}$ is the predefined positive definite matrix, and $\gamma > 0$ is the predefined attenuation level.

3. SD Tracking Control Design Strategy

The proposed time-dependent line-integral fuzzy LKF has the following form:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \text{ for } t \in [t_k, t_{k+1}), \quad (10)$$

where $V_i(t)$ with $i \in \mathcal{I}_3$ is a Lyapunov functional that is described in the following: First, $V_1(t) = V_{11}(t) + V_{12}(t) + V_{13}(t)$, which consists of the following Lyapunov functionals:

$$V_{11}(t) = 2\int_{C_1} g_1(\psi)\cdot d\psi, \quad V_{12}(t) = 2\int_{C_2} g_2(\psi)\cdot d\psi,$$
$$V_{13}(t) = \chi^T(t)P_3\chi(t),$$

where $0 \prec P_3 = P_3^T \in \mathbb{R}^{2n\times 2n}$ is a positive definite matrix to be determined, C_1 and C_2 are paths from the origin $\mathbf{0}$ to $e(t)$ and $x_r(t)$, respectively, $\psi \in \mathbb{R}^n$ is a dummy vector, $(\cdot)$ denotes the inner product, $d\psi \in \mathbb{R}^n$ is an infinitesimal displacement vector, and $g_1(\chi(t)) \in \mathbb{R}^n$ and $g_2(\chi(t)) \in \mathbb{R}^n$ are vector functions that shares the same premise parts as (1) and have the following forms:

$$g_1(\chi(t)) = \sum_{i=1}^{r} w_i(x(t))P_{1i}e(t), \quad (11)$$

$$g_2(\chi(t)) = \sum_{i=1}^{r} w_i(x(t))P_{2i}x_r(t), \quad (12)$$

in which

$$0 \prec P_{1i} = P_{1i}^T = P_{10} + D_{1i},$$
$$0 \prec P_{2i} = P_{2i}^T = P_{20} + D_{2i},$$
$$P_{10} = \begin{bmatrix} 0 & p_{12}^1 & \cdots & p_{1n}^1 \\ p_{21}^1 & 0 & \cdots & p_{2n}^1 \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}^1 & p_{n2}^1 & \cdots & p_{nn}^1 \end{bmatrix}, P_{20} = \begin{bmatrix} 0 & p_{12}^2 & \cdots & p_{1n}^2 \\ p_{21}^2 & 0 & \cdots & p_{2n}^2 \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1}^2 & p_{n2}^2 & \cdots & p_{nn}^2 \end{bmatrix},$$
$$D_{1i} = \mathrm{d}iag\{d_{11}^{\alpha_{i1}},\ldots,d_{1n}^{\alpha_{in}}\}, \quad D_{2i}\mathrm{d}iag\{d_{21}^{\alpha_{i1}},\ldots,d_{2n}^{\alpha_{in}}\},$$

$p_{ab}^1 = p_{ba}^1$ and $p_{ab}^2 = p_{ba}^2$ hold for $1 \le a, b \le n$, and, P_{10}, P_{20}, D_{1i}, and D_{2i} are to be determined.

The term $V_2(t)$ is defined as follows:

$$V_2(t) = (t_{k+1} - t)\int_{t_k}^{t}\sum_{i=1}^{r} w_i(x(s))\dot{\chi}^T(s)U_i\dot{\chi}(s)ds, \quad (13)$$

where $0 \prec U_i = U_i^T \in \mathbb{R}^{2n\times 2n}$ with $i \in \mathcal{I}_r$ are the positive definite matrices to be determined. Therefore, we can say that $V_2(t)$ is the fuzzy Lyapunov functional.

Finally, $V_3(t)$ is borrowed from [16], and has the following form:

$$V_3(t) = (t_{k+1} - t)\begin{bmatrix}\chi(t) \\ \chi(t_k)\end{bmatrix}^T \mathcal{H}\begin{bmatrix}\chi(t) \\ \chi(t_k)\end{bmatrix}, \quad (13)$$

where $\mathcal{H} = \mathcal{H}^T = \begin{bmatrix}\mathcal{H}_{11} & * \\ \mathcal{H}_{21} & \mathcal{H}_{22}\end{bmatrix}$,

$\mathcal{H}_{11} = 0.5(Z_1 + Z_1^T)$, $\mathcal{H}_{21} = -Z_1^T + Z_2^T$, $\mathcal{H}_{22} = -Z_2 - Z_2^T + 0.5(Z_1 + Z_1^T)$, and $2n \times 2n$ full rank matrices Z_1 and Z_2 are the arbitrary matrices to be determined.

On the other hand, the following lemma is required to formulate the proposed method in terms of LMIs.

Lemma 1 For a given positive definite matrix $U_i = U_i^T \in \mathbb{R}^{2n\times 2n}$ with $i \in \mathcal{I}_r$, if there exist matrices $Y_j \in \mathbb{R}^{10n\times 2n}$ and $S_j = S_j^T \in \mathbb{R}^{10n\times 10n}$ with $j \in \mathcal{I}_r$ such that the following inequality is satisfied:

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\begin{bmatrix} S_j & Y_j \\ Y_j^T & U_i\end{bmatrix} \succ 0, \quad (14)$$

then the following always holds:

$$-\int_{t_k}^{t}\sum_{i=1}^{r} w_i(x(s))\dot{\chi}^T(s)U_i\dot{\chi}(s)ds$$
$$\le (t - t_k)\sum_{j=1}^{r} w_j(x(t_k))\{2\eta^T(t)Y_j\bar{\chi}(t) + \eta^T(t)S_j\eta(t)\}, \quad (15)$$

where $Y_j = col\{Y_j^1, Y_j^2, Y_j^3, Y_j^4, Y_j^5\}$, $Y_j^a \in \mathbb{R}^{2n\times 2n}$ with $(a,j) \in \mathcal{I}_5 \times \mathcal{I}_r$ is a full rank matrix to be determined and $\eta(t) = col\{\chi(t), \dot{\chi}(t), \chi(t_k), \phi(t), \bar{\chi}(t)\} \in \mathbb{R}^{10n\times 1}$.

Proof. See Appendix A. ■

The solution to Problem 1 is summarized in the following theorem:

Theorem 1 The augmented system (8) satisfies the SD fuzzy tracking control problem given in Problem 1 if there exist positive definite matrices $\hat{P}_i$ and $\hat{U}_i$, symmetric matrices Ω_j^l and $\hat{S}_j$, full rank matrices N_1, N_2, $\hat{X}_1$, $\hat{X}_2$, $\hat{X}_3$, $\hat{Z}_1$, and $\hat{Z}_2$, and matrices $\bar{K}_j$ and $\hat{Y}_j$ such that the following LMIs hold:

$$\hat{\Phi}_{ij}^l + \hat{\Phi}_{ji}^l - \sum_{q=1}^{r}\lambda_q(\hat{\Phi}_{iq}^l + \Omega_i^l + \hat{\Phi}_{jq}^l + \Omega_j^l) \prec 0, \quad (16)$$

$$\hat{\Phi}_{ij}^l + \Omega_i^l \prec 0, \quad (17)$$

$$\hat{\Phi}_{ij}^3 + \hat{\Phi}_{ji}^3 - \sum_{q=1}^{r}\lambda_q(\hat{\Phi}_{iq}^3 + \Omega_i^3 + \hat{\Phi}_{jq}^3 + \Omega_j^3) \succ 0, \quad (18)$$

$$\hat{\Phi}_{ij}^3 + \Omega_i^3 \succ 0, \quad (19)$$

for $(i,j,l) \in \mathcal{I}_r \times \mathcal{I}_r \times \mathcal{I}_2$, where the definitions of $\hat{\Phi}_{ij}^l$ for $l \in \mathcal{I}_2$ are given in (17), $\alpha > 0$, $\gamma > 0$, $h > 0$ are predefined scalar values, $\lambda_q > 0$ with $q \in \mathcal{I}_r$ are predefined scalars satisfying $w_q(x(t_k)) - w_q(x(t)) + \lambda_q \ge 0$ for all possible combinations of $x(t)$ and $x(t_k)$ for $t \in [t_k, t_{k+1})$ with $k \in \mathbb{R}_{>0}$, and

$$\hat{\Phi}_{ij}^{1}=\begin{bmatrix}\begin{bmatrix}\mathrm{He}(\Xi_{ij}-\hat{X}_1^T)-\hat{\mathcal{H}}_{11} & * & * & * \\ \hat{P}_i+\alpha\Xi_{ij}-N^T-\hat{X}_2^T+h\hat{\mathcal{H}}_{11}^T & h\hat{U}_i-\alpha\mathrm{He}(N) & * & * \\ \hat{X}_1-\hat{X}_3^T-\hat{\mathcal{H}}_{21} & \hat{X}_2+h\hat{\mathcal{H}}_{21} & \mathrm{He}(\hat{X}_3)-\hat{\mathcal{H}}_{22} & * \\ \mathcal{J}^T & \alpha\mathcal{J}^T & \mathbf{0} & -\gamma^2 I\end{bmatrix} & * \\ [[N_1\ \ \mathbf{0}]\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}] & -Q^{-1}\end{bmatrix},$$

$$\hat{\Phi}_{ij}^{2}=\begin{bmatrix}\begin{bmatrix}\mathrm{He}(\Xi_{ij}-\hat{X}_1^T)-\hat{\mathcal{H}}_{11} & * & * & * & * \\ \hat{P}_i+\alpha\Xi_{ij}-N^T-\hat{X}_2^T & -\alpha\mathrm{He}(N) & * & * & * \\ \hat{X}_1-\hat{X}_3^T-\hat{\mathcal{H}}_{21} & \hat{X}_2 & \mathrm{He}(\hat{X}_3)-\hat{\mathcal{H}}_{22} & * & * \\ \mathcal{J}^T & \alpha\mathcal{J}^T & \mathbf{0} & -\gamma^2 I & * \\ \hat{Y}_j^1+\hat{X}_1-\bar{\Xi}_{ij}^T & \hat{Y}_j^2+\hat{X}_2-\alpha\hat{\Xi}_{ij}^T & \hat{Y}_j^3+\hat{X}_3 & \hat{Y}_j^4 & \mathrm{He}(\hat{Y}_j^5)\end{bmatrix}+\hat{S}_j & * \\ [[N_1\ \ \mathbf{0}]\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}\ \ \mathbf{0}] & -Q^{-1}\end{bmatrix}, \quad (20)$$

where $\hat{S}_j=\begin{bmatrix}N&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&N&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&N&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&I&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&N\end{bmatrix}^T S_j\begin{bmatrix}N&\mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&N&\mathbf{0}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&N&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&I&\mathbf{0}\\ \mathbf{0}&\mathbf{0}&\mathbf{0}&\mathbf{0}&N\end{bmatrix}$, $\hat{Y}_j=col\{\hat{Y}_j^1,\hat{Y}_j^2,\hat{Y}_j^3,\hat{Y}_j^4,\hat{Y}_j^5\}$, $N=\begin{bmatrix}N_1&\mathbf{0}\\ \mathbf{0}&N_2\end{bmatrix}$,

$\Xi_{ij}=\Lambda_{ij}N=\begin{bmatrix}A_iN_1+B_i\bar{K}_j & (A_i-A_r)N_2\\ \mathbf{0} & A_rN_2\end{bmatrix}$, $\bar{\Xi}_{ij}=\bar{\Lambda}_{ij}N=\begin{bmatrix}B_i\bar{K}_j&\mathbf{0}\\ \mathbf{0}&\mathbf{0}\end{bmatrix}$, and $\bar{K}_j=K_jN_1$.

$$\hat{\Phi}_{ij}^{3}=\begin{bmatrix}\hat{S}_j & * \\ \hat{Y}_j^T & \hat{U}_i\end{bmatrix}. \quad (21)$$

Finally, the gain matrix can be obtained using the following equation:

$$K_j=\bar{K}_jN_1^{-1} \quad \text{for} \quad j\in\mathcal{I}_r. \quad (22)$$

Proof. The time derivatives of $V_1(t)$ for $t\in(t_k,t_{k+1})$ is as follows:

$$\dot{V}_1(t)=\dot{V}_{11}(t)+\dot{V}_{12}(t)+\dot{V}_{13}(t). \quad (23)$$

First, $\dot{V}_{11}(t)$ and $\dot{V}_{12}(t)$ are computed as follows:

$$\dot{V}_{11}(t)=\mathcal{L}_{f_1(t)}V_{11}(t)=2g_1^T(\chi(t))f_1(t), \quad (24)$$

$$\dot{V}_{12}(t)=\mathcal{L}_{f_2(t)}V_{12}(t)=2g_2^T(\chi(t))f_2(t), \quad (25)$$

where $\mathcal{L}_{f(t)}V(t)$ denotes the Lie derivative of $V(t)$ with respect to $f(t)$, $f_1(t)=\dot{e}(t)$, and $f_2(t)=\dot{x}_r(t)$.

By substituting (11) and (12) into (24) and (25), respectively, we obtain

$$\dot{V}_{11}(t)=2\sum_{i=1}^{r}w_i(x(t))e^T(t)P_{1i}\dot{e}(t) \quad \text{and}$$

$$\dot{V}_{12}(t)=2\sum_{i=1}^{r}w_i(x(t))x^T(t)P_{2i}\dot{x}_r(t), \quad (26)$$

which can be rewritten as

$$\dot{V}_{11}(t)+\dot{V}_{12}(t)=2\sum_{i=1}^{r}w_i(x(t))\begin{bmatrix}e(t)\\ x_r(t)\end{bmatrix}^T\begin{bmatrix}P_{1i}&\mathbf{0}\\ \mathbf{0}&P_{2i}\end{bmatrix}\begin{bmatrix}\dot{e}(t)\\ \dot{x}_r(t)\end{bmatrix}. \quad (27)$$

Now, from the trivial fact that $\dot{V}_{13}=2\chi^T(t)P_3\dot{\chi}(t)$, we can rewrite (27) as follows:

$$\dot{V}_1(t)=2\sum_{i=1}^{r}w_i(x(t))\begin{bmatrix}e(t)\\ x_r(t)\end{bmatrix}^T\left(\begin{bmatrix}P_{1i}&\mathbf{0}\\ \mathbf{0}&P_{2i}\end{bmatrix}+P_3\right)\begin{bmatrix}\dot{e}(t)\\ \dot{x}_r(t)\end{bmatrix}$$
$$=2\sum_{i=1}^{r}w_i(x(t))\chi^T(t)P_i\dot{\chi}(t), \quad (28)$$

where $\eta(t)=col\{\chi(t),\dot{\chi}(t),\chi(t_k),\phi(t),\bar{\chi}(t)\}$.

On the other hand, $\dot{V}_3(t)$ for $t\in(t_k,t_{k+1})$ is as follows:

$$\dot{V}_3(t)=2(t_{k+1}-t)\left(\chi^T(t)\mathcal{H}_{11}\dot{\chi}(t)+\chi^T(t_k)\mathcal{H}_{21}\dot{\chi}(t)\right)-\begin{bmatrix}\chi(t)\\ \chi(t_k)\end{bmatrix}^T\mathcal{H}\begin{bmatrix}\chi(t)\\ \chi(t_k)\end{bmatrix}. \quad (29)$$

Moreover, we have the following trivial null expressions:

$$0=2\sum_{i=1}^{r}\sum_{j=1}^{r}w_i(x(t))w_j(x(t_k))\{\chi^T(t)W^T+\alpha\dot{\chi}^T(t)W^T\}\times\{-\dot{\chi}(t)+\Lambda_{ij}\chi(t)-(t-t_k)\bar{\Lambda}_{ij}\bar{\chi}(t)+\mathcal{J}\phi(t)\}, \quad (30)$$

$$0=2\{\chi^T(t)X_1^T+\dot{\chi}^T(t)X_2^T+\chi^T(t_k)X_3^T\}\times\{-\chi(t)+\chi(t_k)+(t-t_k)\bar{\chi}(t)\}, \quad (31)$$

where $W = \begin{bmatrix} W_1 & \mathbf{0} \\ \mathbf{0} & W_2 \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$, W_1 and W_2 are $n\times n$ full rank matrices to be determined, X_1, X_2, and X_3 are $2n\times 2n$ full rank matrices to be determined, and α is a predefined positive scalar value.

Combining (28) and (29)-(31) yields

$$\begin{aligned}
&\dot{V}(t) + \chi^T(t)\tilde{Q}\chi(t) - \gamma^2\phi^T(t)\phi(t) \\
&\le \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\eta^T(t)\{\Theta_{ij}^0 + \tilde{I}Q\tilde{I}^T \\
&\quad +(t_{k+1}-t)\Theta_i^1 + (t-t_k)\Theta_{ij}^2\}\eta(t) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k)) \\
&\quad \times \eta^T(t)\{\frac{t_{k+1}-t}{h}(\Theta_{ij}^0 + h\Theta_i^1 + \tilde{I}Q\tilde{I}^T) \\
&\quad + \frac{t-t_k}{h}(\Theta_{ij}^0 + h\Theta_{ij}^2 + \tilde{I}Q\tilde{I}^T)\}\eta(t), \qquad (32)
\end{aligned}$$

where the definitions of Θ_{ij}^0, Θ_i^1, and Θ_{ij}^2 are given in (33)-(34), respectively, and $\tilde{I} = col\{[I_n \quad \mathbf{0}_{n\times n}], \mathbf{0}_{2n\times n}, \mathbf{0}_{2n\times n}, \mathbf{0}_{2n\times n}, \mathbf{0}_{2n\times n}\}$.

Thus, $\dot{V}(t) + \chi^T(t)\tilde{Q}\chi(t) - \gamma^2\phi^T(t)\phi(t) \le 0$ if and only if

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\{\Theta_{ij}^0 + h\Theta_i^1 + \tilde{I}Q\tilde{I}^T\} \prec 0, \qquad (35)$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\{\Theta_{ij}^0 + h\Theta_{ij}^2 + \tilde{I}Q\tilde{I}^T\} \prec 0. \qquad (36)$$

Applying the Schur complements on (35) and (36), we have

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\Phi_{ij}^l \prec 0 \qquad (37)$$

for $l \in \mathcal{I}_2$, where $\Phi_{ij}^1 = \begin{bmatrix} \Theta_{ij}^0 + h\Theta_i^1 & * \\ \tilde{I}^T & -Q^{-1} \end{bmatrix}$ and $\Phi_{ij}^2 = \begin{bmatrix} \Theta_{ij}^0 + h\Theta_{ij}^2 & * \\ \tilde{I}^T & -Q^{-1} \end{bmatrix}$.

Now, define $W^{-1} = N = \begin{bmatrix} W_1^{-1} & \mathbf{0} \\ \mathbf{0} & W_2^{-1} \end{bmatrix} = \begin{bmatrix} N_1 & \mathbf{0} \\ \mathbf{0} & N_2 \end{bmatrix}$ and for a matrix M with $\in \{P_i, U_i, X_1, X_2, X_3, Y_j^1, Y_j^2, Y_j^3, Y_j^4, Y_j^5, Z_1, Z_2, \mathcal{H}_{11}, \mathcal{H}_{21}, \mathcal{H}_{22}\}$, let $\widehat{M} = N^TMN$. Then, applying the congruence transformation to (37) with $diag\{N, N, N, I, I\}$ for $l = 1$ and $diag\{N, N, N, I, N, I\}$ for $l = 2$, respectively, we have

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\widehat{\Phi}_{ij}^l \prec 0, \qquad (38)$$

where $l \in \mathcal{I}_2$ and the definition of $\widehat{\Phi}_{ij}^l$ is given in (20). Thus, we can say that if (38) holds, $\dot{V} + \chi^T(t)\tilde{Q}\chi(t) - \gamma^2\phi^T(t)\phi(t) \le 0$ for $t \in (t_k, t_{k+1})$. In what follows, we handle the mismatched fuzzy weighting functions $w_i(x(t))$ and $w_j(x(t_k))$ presented in (38). The following procedure is motivated by [15]. First, by rewriting (38), we have

$$\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\widehat{\Phi}_{ij}^l \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))\{w_j(x(t_k)) + w_j(x(t)) \\
&\quad - w_j(x(t)) + \lambda_j - \lambda_j\}\widehat{\Phi}_{ij}^l \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t))\left\{\widehat{\Phi}_{ij}^l - \sum_{q=1}^{r}\lambda_q\widehat{\Phi}_{iq}^l\right\} \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))\big(w_j(x(t_k)) - w_j(x(t)) + \lambda_j\big)\widehat{\Phi}_{ij}^l \\
&\quad \prec 0 \qquad (39)
\end{aligned}$$

where λ_j with $j \in \mathcal{I}_r$ are predefined scalar values satisfying $w_j(x(t_k)) - w_j(x(t)) + \lambda_j \ge 0$ for all possible combinations of $j \in \mathcal{I}_r$, $x(t)$, and $x(t_k)$.

On the other hand, for any symmetric matrices Ω_i^l with $(i, l) \in \mathcal{I}_r \times \mathcal{I}_2$, the following is obvious:

$$\Theta_{ij}^0 = \begin{bmatrix} \mathrm{He}(W^T\Lambda_{ij} - X_1^T) - \mathcal{H}_{11} & * & * & * & * \\ P_i + \alpha W^T\Lambda_{ij} - W - X_2^T & -\alpha\mathrm{He}(W) & * & * & * \\ X_1 - X_3^T - \mathcal{H}_{21} & X_2 & \mathrm{He}(X_3) - \mathcal{H}_{22} & * & * \\ \mathcal{I}^TW & \alpha\mathcal{I}^TW & \mathbf{0} & -\gamma^2 I & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \qquad (33)$$

$$\Theta_i^1 = \begin{bmatrix} \mathbf{0} & * & * & * & * \\ \mathcal{H}_{11}^T & U_i & * & * & * \\ \mathbf{0} & \mathcal{H}_{21} & \mathbf{0} & * & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \Theta_{ij}^2 = \begin{bmatrix} \mathbf{0} & * & * & * & * \\ \mathbf{0} & \mathbf{0} & * & * & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & * & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & * \\ Y_j^1 + X_1 - \overline{\Lambda}_{ij}^TW & Y_j^2 + X_2 - \alpha\overline{\Lambda}_{ij}^TW & Y_j^3 + X_3 & Y_j^4 & \mathrm{He}(Y_j^5) \end{bmatrix} + S_j. \qquad (34)$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))\big(w_j(x(t_k))-w_j(x(t))+\lambda_j-\lambda_j\big)\Omega_i^l = \mathbf{0}. \tag{40}$$

Adding (40) into (39), we have

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t))\left\{\widehat{\Phi}_{ij}^l-\sum_{q=1}^{r}\lambda_q\widehat{\Phi}_{iq}^l\right\}\\&+\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))\big(w_j(x(t_k))-w_j(x(t))+\lambda_j\big)\widehat{\Phi}_{ij}^l\\&+\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))\big(w_j(x(t_k))-w_j(x(t))+\lambda_j-\lambda_j\big)\Omega_i^l\\&=\frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t))\{\widehat{\Phi}_{ij}^l+\widehat{\Phi}_{ji}^l\\&-\sum_{q=1}^{r}\lambda_q(\widehat{\Phi}_{iq}^l+\Omega_i^l+\widehat{\Phi}_{jq}^l+\Omega_j^l)\}\\&+\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))\big(w_j(x(t_k))-w_j(x(t))+\lambda_j\big)\\&\times\left(\widehat{\Phi}_{ij}^l+\Omega_i^l\right)\prec 0,\end{aligned} \tag{41}$$

from which we know that (38) holds if and only if LMIs (16) and (17) are satisfied for $(i,j,l)\in\mathcal{I}_r\times\mathcal{I}_r\times\mathcal{I}_2$.

Moreover, it was assumed that (14) holds; thus, once again applying the congruence transformation to (14) with $diag\{N,N,N,I,N,N\}$, we can obtain

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(t))w_j(x(t_k))\widehat{\Phi}_{ij}^3 \succ 0, \tag{42}$$

where the definition of $\widehat{\Phi}_{ij}^3$ is located in (21).

Applying a procedure similar to that for (39)-(41) to (42), we have LMIs (18) and (19) guaranteeing (42). Thus, we conclude that $\dot{V}(t)+\chi^T(t)\tilde{Q}\chi(t)-\gamma^2\phi^T(t)\phi(t)\leq 0$ for $t\in(t_k,t_{k+1})$ is guaranteed by the LMIs of (16)-(19). Finally, we have

$$\begin{aligned}&\lim_{\delta\to 0}\sum_{k=0}^{t_f}\int_{t_k+\delta}^{t_{k+1}-\delta}\left(\dot{V}(t)+\chi^T(t)\tilde{Q}\chi(t)-\gamma^2\phi^T(t)\phi(t)\right)dt\\&=\lim_{\delta\to 0}\sum_{k=0}^{t_f}\{V(t_{k+1}-\delta)-V(t_k+\delta)\\&+\int_{t_k+\delta}^{t_{k+1}-\delta}(\chi^T(t)\tilde{Q}\chi(t)-\gamma^2\phi^T(t)\phi(t))dt\}\\&=V(t_f)-V(0)+\int_0^{t_f}\left(\chi^T(t)\tilde{Q}\chi(t)-\gamma^2\phi^T(t)\phi(t)\right)dt\\&\leq 0\end{aligned}$$

$$\begin{aligned}&\Rightarrow\int_0^{t_f}\chi^T(t)\tilde{Q}\chi(t)dt\leq V(0)-V(t_f)\\&+\gamma^2\int_0^{t_f}\phi^T(t)\phi(t)dt.\end{aligned} \tag{43}$$

Under the zero initial condition, it is proved that $V(0)=0$and $V(t_f)\geq 0$, from which we conclude that the augmented system (8) satisfies the H_∞ performance criterion given in (9). This concludes the proof. ■

4. Numerical Example

To demonstrate the effectiveness of the proposed SD tracking control method, we conducted a simulation using YALMIP [29] running on MATLAB 2018a. Consider an inverted pendulum on a cart whose dynamic behavior is represented by the following nonlinear dynamic equation [30]:

$$\begin{aligned}\ddot{\theta}(t)=&\frac{g\sin(\theta(t))-am_pL\dot{\theta}^2(t)\sin(2\theta(t))/2}{4L/3-am_pL\cos^2(\theta(t))}\\&+\frac{-a\cos(\theta(t))u(t)}{4L/3-am_pL\cos^2(\theta(t))}+\omega(t),\end{aligned} \tag{45}$$

where $\theta(t)$ is the angular position of the pole, $g=9.8m/s^2$ is the acceleration of gravity, $m_p=0.2kg$ is the mass of the pole, $M_c=0.8kg$ is the mass of the cart, $a=1/(m_P+M_c)$, $L=0.5m$ is the length of the pole, $\omega(t)$ is the time-varying external disturbance, and $u(t)$ is the control input.

The original nonlinear dynamic model (45) can be modeled as the following T–S fuzzy model [30]:

$$\dot{x}(t)=\sum_{i=1}^{2} w_i(x(t))\big(A_ix(t)+B_iu(t)+\varpi(t)\big), \tag{46}$$

where $x(t)=col\{x_1(t),x_2(t)\}=col\{\theta(t),\dot{\theta}(t)\}$,

$\varpi(t)=col\{\mathbf{0},\omega(t)\}$,

$$A_1=\begin{bmatrix}0 & 1\\ g/(4L/3-am_PL) & 0\end{bmatrix},$$
$$A_2=\begin{bmatrix}0 & 1\\ 2g/\{\pi(4L/3-am_PLb^2)\} & 0\end{bmatrix},$$
$$B_1=\begin{bmatrix}0\\ -a/(4L/3-am_PL)\end{bmatrix},$$
$$B_2=\begin{bmatrix}0\\ -ab/(4L/3-am_PLb^2)\end{bmatrix},$$

$b=\cos(88°)$, $\alpha_{11}=1$, $\alpha_{21}=2$, and $\omega(t)=\cos(10t)$,

$$w_1(x(t))=\begin{cases}1-\frac{2}{\pi}x_1(t), & \text{if } 0\leq x_1(t)\leq\frac{\pi}{2}\\ 1+\frac{2}{\pi}x_1(t), & \text{if } -\frac{\pi}{2}\leq x_1(t)\leq 0\end{cases} \tag{47}$$

and $w_2(x(t)) = 1 - w_1(x(t))$.

In this example, we demonstrate that state variables of (47) are successfully driven to the following reference model even if there is an external disturbance:

$$\dot{x}_r(t) = A_r x_r(t) + r(t), \tag{48}$$

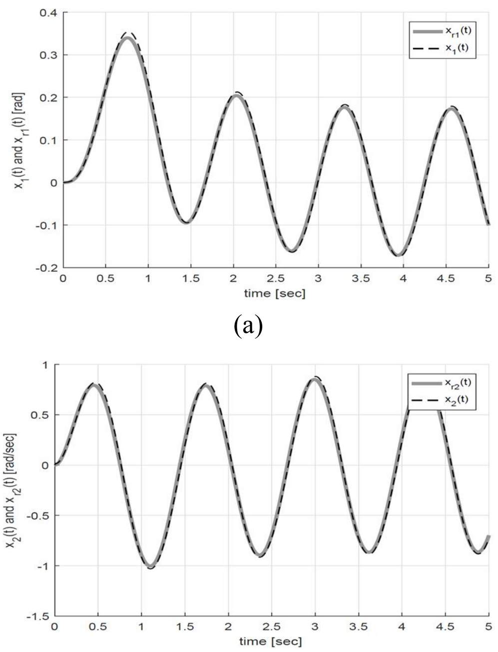
where $A_r = \begin{bmatrix} 0 & 1 \\ -4 & -4 \end{bmatrix}$ and $r(t) = \begin{bmatrix} 0 \\ 5\sin(5t) \end{bmatrix}$.

Now, setting $\lambda_q = 0.35$, $\alpha = 0.1$, $Q = I$, and $h = 0.01$, and applying Theorem 1 to (50), we have the following gain matrices:

$$K_1 = [249.25 \quad 81.18], K_2 = [294.66 \quad 96.67]. \tag{49}$$

Under the zero initial condition $\chi(0) = [0 \quad 0 \quad 0 \quad 0]^T$, the time responses of the state variables are shown in Fig. 1, from which we can see that the system states are successfully driven to follow the reference system.

W compared the performance indexes of the proposed method with the conventional method [25]. We chose the parameters $\sigma = 1$, $\zeta = 2$, and $\rho = 1.35$ as in [25]. The computed performance indexes are listed in Table 1.



(a)

(b)

Fig. 1. The time responses of state variables of the example system (thin solid line) and reference model (thick solid line)

Table 1. Comparison of performance indexes of method proposed in [25] and this paper

Sampling period	H. K. Lam [25]	Proposed
$h = 0.001$	0.0116	0.0060
$h = 0.010$	0.0124	0.0092
$h = 0.020$	Unstable	0.0146

5. Conclusion

This paper has investigated the SD tracking control problem for a T-S fuzzy model with external disturbances. The condition guaranteeing both the asymptotic stability and H_∞ tracking performance was derived in terms of LMIs based on the newly proposed time-dependent line-integral fuzzy LKF. In addition, we proposed the method for manipulating the integral term presented in the proposed condition using the novel integral inequality. Finally, a simulation example illustrated the effectiveness of the proposed method.

6. Appendix

6.1 Proof of Lemma 1

Proof. The following is obvious:

$$\begin{aligned} 0 &= 2\sum_{j=1}^{r} w_j(x(t_k))\eta^T(t)Y_j\left(\chi(t) - \chi(t_k) - \int_{t_k}^{t}\dot{\chi}(s)ds\right) \\ &= 2\sum_{j=1}^{r} w_j(x(t_k))\eta^T(t)Y_j(\chi(t) - \chi(t_k)) \\ &\quad - \int_{t_k}^{t} 2\sum_{j=1}^{r} w_j(x(t_k))\eta^T(t)Y_j\dot{\chi}(s)ds. \end{aligned} \tag{50}$$

From [14], it is clear that the following matrix inequality,

$$-2E^T R \le E^T F^{-1} E + R^T F R \tag{51}$$

holds for appropriate dimensional matrices E, F, and R.

Now, letting $E = Y_j^T\eta(t)$, $R = \dot{\chi}(t)$, and $F = U_i$, we get

$$\begin{aligned} &-2\eta^T(t)Y_j\dot{\chi}(s) \le \eta^T(t)Y_jU_i^{-1}Y_j^T\eta(t) + \dot{\chi}^T(s)U_i\dot{\chi}(s) \\ &\Rightarrow -\int_{t_k}^{t} 2\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(s))w_j(x(t_k))\eta^T(t)Y_j\dot{\chi}(s)ds \\ &\le \int_{t_k}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(s))w_j(x(t_k))\{\eta^T(t)Y_jU_i^{-1}Y_j^T\eta(t) \\ &\quad + \dot{\chi}^T(s)U_i\dot{\chi}(s)\}ds. \end{aligned} \tag{52}$$

Substituting (52) into (50) yields

$$\begin{aligned} 0 &\le 2\sum_{j=1}^{r} w_j(x(t_k))\eta^T(t)Y_j(\chi(t) - \chi(t_k)) \\ &\quad + \int_{t_k}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(s))w_j(x(t_k))\eta^T(t)Y_jU_i^{-1}Y_j^T\eta(t)ds \\ &\quad + \int_{t_k}^{t}\sum_{i=1}^{r} w_i(x(s))\dot{\chi}^T(s)U_i\dot{\chi}(s)ds. \end{aligned} \tag{53}$$

Suppose that (14) holds. Then, applying the Schur complements on (14) yields

$$\sum_{i=1}^{r}\sum_{j=1}^{r} w_i\big(x(s)\big)w_j\big(x(t_k)\big)\big\{S_j - Y_j U_i^{-1} Y_j^T\big\} \succ 0. \quad (54)$$

Now, pre- and post-multiplying (55) by $\eta^T(t)$ and $\eta(t)$, respectively, and integrating it with respect to s from t_k to t, we have the following inequality:

$$0 \le \int_{t_k}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(s))w_j(x(t_k)) \times \eta^T(t)\{S_j - Y_j U_i^{-1} Y_j^T\}\eta(t)ds \quad (56)$$

which implies that

$$\int_{t_k}^{t}\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(x(s))w_j(x(t_k))\eta^T(t)Y_j U_i^{-1} Y_j^T \eta(t)ds \le (t-t_k)\sum_{j=1}^{r} w_j(x(t_k))\eta^T(t)S_j\eta(t) \quad (57)$$

Finally, we can further majorize (57) as follows:

$$0 \le 2\sum_{j=1}^{r} w_j(x(t_k))\eta^T(t)Y_j(\chi(t)-\chi(t_k)) + (t-t_k)\sum_{j=1}^{r} w_j(x(t_k))\eta^T(t)S_j\eta(t) + \int_{t_k}^{t}\sum_{i=1}^{r} w_i(x(s))\dot{\chi}^T(s)U_i\dot{\chi}(s)ds,$$

from which we conclude that (15) always hold if (14) is satisfied. ■

Acknowledgements

This work was supported by Korea Electric Power Corporation (Grant number: R18XA04) and by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2016R1A6A1A03013567, NRF-2018R1 A2A2A14023632)

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