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The Development of the Components of the Length Measurement Concept in the Procedure of Measurement Using a Ruler

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The research related to testing pupils' achievement in the field of Measurement and Measure in initial teaching of geometry points to an insufficient adoption of the basic components of the length measurement concept among pupils. In order to discover the cause, we looked at the basic components on which the procedure of measuring length using a ruler is based, highlighted the possibilities of introducing the procedure in measuring length, and determined pupils' achievement during the procedure of measuring length using a ruler. The research sample consisted of 145 pupils, out of which 72 were the 2nd grade pupils and 73 were the 4th grade pupils. A descriptive method was applied in the research. The technique we used was testing, and for the statistical data processing we used a χ^2 test. The results of the research show that, when drawing a straight line of a given length using a ruler, there is no statistical difference in achievement between the 2nd and 4th grade pupils, nor in the pupils' knowledge regarding drawing a ruler independently, while drawing a straight line of a given length using a "broken" ruler 4th grade pupils are statistically better. The results of the research indicate that pupils' achievement is better in doing standard tasks than in non-standard ones, given that the latter require conceptual knowledge. The components of the concept of length measurement using ruler have not been sufficiently developed yet, and these include: zero-point, partitioning a measured object in a series of consecutive measurement units and their iteration. We shed more light on the critical stage in the procedure of length measurement – the transition from non-standard to standard units and the formation of the length measurement scale. For further research, we propose to look at the formation of the concept of length measurement using the ruler through all its components and their inclusion in the mathematics curriculum, as well as examining the correlation of pupils' achievement in the procedure of measuring length with their achievement in measuring area (and volume).

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I. INTRODUCTION

The research examining pupils' procedural and conceptual knowledge of length measurement indicates that the basic components of this concept are underdeveloped in measuring length using either non-standard or standard units of measurement (Baroody, Feil, & Johnson, 2007). Procedural knowledge implies the knowledge of specific algorithmic procedures and their application, whereas conceptual knowledge implies understanding mathematical principles and processes which link pupils' current level of knowledge to the previously acquired knowledge (Skemp, 1993). The procedures aimed at developing conceptual knowledge imply a problem situation that requires that pupils think and correlate the concepts that are being formed with the previous ones (Henning, 2004). The research findings suggest the need to simultaneously build and link procedural and conceptual knowledge (Baroody et al., 2007; Henning, 2004; Schneider & Stern, 2010; Zeljić & Dabić, 2014; Đokić, 2013). As far as measuring by using standard units of measurement is concerned, pupils encounter the ruler as a basic measuring instrument at the beginning of their schooling. Despite the fact that a measuring scale is represented on the ruler (with iterated measurement units on it or, occasionally, with smaller and bigger measurement units), pupils rarely use it (Nunes, Light, & Mason, 1993). Given that the knowledge related to length measurement is fundamental in the field of measurement and measures and that it serves as a starting point for learning about area and volume measurement, building a solid foundation for acquiring this knowledge is of utmost importance. In addition, it has been observed that the number of pupils giving correct answers to the questions decreases as a new dimension is added to the tasks (length › area › volume) and that few pupils really understand how length measurement relates to area and volume measurement (Smith & Barrett, 2017; Tan Sisman & Aksu, 2012, 2015). We can conclude that the pupils measured the length of an object after they had read the measurement number on the ruler, but their approach raises several questions: whether they truly learnt the measurement procedure and whether they know how to apply the acquired knowledge in non-standard tasks. Given that our intention is to provide answers to these questions, we will look at the basic components on which the length measurement procedure using measurement tools is based, shed more light on the possibility of introducing the length measurement procedure, and explore the quality of pupils' knowledge in the procedure of length measurement.

II. THEORETICAL BACKGROUND

Length measurement involves using measurement tools, such as a metre or a ruler. When we read a number on a metre or ruler, we can say that the length of an object has been measured. The length of objects is something that children are encountered with on a daily basis and they need to know more about length and different ways of measuring it in order to carry out various activities. The measurement procedure consists of three basic steps:

1. selecting an attribute (characteristic) of the measured object;
2. selecting a measurement unit (including number 1) and
3. determining how many measurement units the measured value contains, e.g., by using a measurement instrument or in some similar way (Đokić, 2014b; Zöllner & Benz, 2013; Stephan & Clements, 2003).

According to Buys & De Moor (2008), when children start primary school, they are formally introduced to measures and measurement procedure, including the three basic ways of measuring:

- measuring by estimation and comparison,
- measuring by iteration (non-standard and standard) and
- measuring by using tools.

As Battista (2006) points out, despite the fact that children first learn about measuring length without using numbers (they use visual estimation, direct comparisons, and correlations among the parts of the whole), this method is still used even after they have mastered the ability to measure length by iteration or using measurement tools. In fact, children understand length measurement only after they have learnt to use each of the three above-mentioned ways of measuring (Antić & Đokić, 2018; Battista, 2006; Buys & De Moor, 2008).

There are several important components of the length measurement concept on which learning process is based. They must be mentioned because they serve as a basis for drawing conclusions about the manner in which children think while they carry out measuring. The components include: 1) *partitioning*, 2) *unit iteration* 3) *transitivity reasoning*, 4) *conservation of distance*, 5) *accumulation of distance*, and 6) *unit-attribute relations* (Stephan & Clements, 2003). These components are fundamental for understanding the length measurement procedure. As their distribution and development in children is also a topic of discussion in the pertaining literature (Lehrer, 2003; Sarama & Clements, 2009; Stephan & Clements, 2003; Tan Sisman & Aksu, 2012), we assume that experience and education are two factors that play an important role. It is beyond any doubt that these components are the basis of the length measurement procedure and that they should be taken into account when working with children in this field of mathematics.

We shall focus now on the measurement procedure using measurement tools which is based on these fundamental components with some exhausted aspects specific for length measurement. A short description of the components is also provided (Antić & Đokić, 2018; Lehrer, 2003; Sarama & Clements, 2009; Stephan & Clements, 2003).

1. *Unit-attribute relations* – This is an ability to choose a suitable measurement unit to measure an object of a specific length. This component is based on the ability to estimate length and the ability to visualize the measured objects (Lehrer, 2003). Only then can the pupils decide if, for instance, it is more convenient to use a meter or centimeter as a unit of measurement to measure, for example, classroom length. The development of this component is also reflected in the correct expression of the measurement unit, as pupils sometimes mark length measurement units with the units for measuring area (or volume) (and vice versa).

2. *Partitioning* – It is a mental activity of splitting/dividing the length of an object into measurement units of the same length. This activity implies that pupils can perceive a whole as the sum of the parts whose lengths are iterated and, if necessary, even replaced, and the measuring numbers are added together. The mastery of this component can be tested by the task in which pupils have to make their own ruler. The pupils who mark the points on the ruler to represent the intervals of the same length have mastered the concept of partitioning (Lehrer, 2003; Stephan & Clements, 2003). Consequently, the pupils who have not mastered this component will not see the difference between the points on the ruler, i.e., they will not understand that the same length of the ruler can be divided into longer (1cm) or shorter (5mm) units and marked accordingly (Clements & Barett, 2003; Ryan & Williams, 2007; Sarama & Clements, 2009; Tan Sisman & Aksu, 2012).

3. *Unit iteration* – This component involves the ability to think about a unit of measurement as a part that constantly iterates along the whole length of an object which is being measured – without any overlap or leaving blank spaces. If a pupil, when measuring an object (e.g. a pencil), chooses another, shorter object or several objects of the same length (e.g. matches), the foundation for developing this component has been laid. The pupil can measure the length of the pencil by placing one match several times along the whole pencil, or by placing several matches next to one another along the pencil (Barrett, Jones, Thornton, & Dickson, 2003; Zöllner & Benz, 2016; Kamii & Clark, 1997). The research conducted among the six-year-olds (Ryan & Williams, 2007) indicates a typical mistake made in unit iteration. In the first task, when the units of measurement are placed from the beginning to the end of the object being measured, as many as 91% of the pupils provided a correct answer regarding the length of the object. The pupils were then given the second task where they had to measure the length of an

object which was positioned in such a manner that its beginning was not aligned with the first unit of measurement from which iteration normally starts. This task was performed correctly by 50% of the pupils.

If pupils do not learn the iteration procedure, they will have problems with measuring by using measurement tools. Namely, when performing the task of measuring the length of an object which is not aligned with the zero point on the ruler, pupils often provide incorrect answers (Bragg & Outhred, 2004; Kamii, 2006; Lehrer, 2003; Smith III, Males, Dietiker, Lee, & Mosier, 2013; Stephan & Clements, 2003). Understanding the metre and ruler as length measurement tools implies that children realise that these tools consist of identical measurement units, that the marks for measurement units – points – mark their beginning and end, and that each point can be used as the beginning (zero) for measuring length (Zöllner & Benz, 2016; Sarama & Clements, 2009; Smith III et al., 2013; Tan Sisman & Aksu, 2012).

Many authors (Barrett et al., 2003; Battista, 2006; Zöllner & Benz, 2013; Pijaže & Inhelder, 1996; Stephan & Clements, 2003) agree that understanding measurement units is vital for forming the length measurement concept. If pupils do not understand the correct iteration of measurement units, telling them that they have made a mistake while they are doing the task will not help them. Only when they fully comprehend that measurement units are a part of the whole, i.e., when they develop the iteration component, will they be able to carry out the procedure correctly (Kamii & Clark, 1997; Kamii, 2006). Otherwise, pupils will perceive iteration as a mere sequencing of parts (measurement units) along the entire object (Lehrer, 2003).

4. *Accumulation of distance* – During the iteration of measurement units, the length of an object is a distance between the beginning of the first and the end of the last measurement unit. Stephan, Cobb, Gravemeijer, & Estes (2001) conducted research among the first-graders. The pupils were given a task to measure the length of a carpet using their footsteps. While a pupil was measuring the length in this way, the examiner would stop him at one point and ask him/her to explain what, for example, number 6 represented. Some pupils said that 6 is the number of the last step, while the others said that it represented the total number of steps they had made, from the first step, up to the end of the sixth one. The latter pupils developed the component of the accumulation of distance. Lehrer (2003) points out the significance of this component along with the components of *partitioning* and *iteration.* Unless pupils understand length as a distance between the endpoints of an object, they will find it difficult to partition the length of an object into units of measurement, as well as to iterate them correctly, without an overlap or leaving a blank space.

5. *Conservation of distance* – This concept implies the realisation that the change of the quantitative attributes of an object is not caused only by the change of their appearance (Božin, 2014). In terms of length, conservation means that the length of an object does not change if that object is moved. Piaget's task is widely in use for testing conservation. Pupils are shown two strips of equal length, placed one above the other. When the pupils notice that the strips are of equal length, one strip is placed aside. After this, the pupils are asked again if the strips are of equal length. The pupils with underdeveloped conservation component will answer that the moved strip is longer (Stephan & Clements, 2003). Conservation, just like transitivity, is considered as a very important component of the length measurement concept (Battista, 2006; Zöllner & Benz, 2016; Kamii & Clark, 1997; Pijaže & Inhelder, 1996). The researchers have concluded that if pupils believe that moving an object will change its length, they will also conclude that the length of the unit of measurement changes in the procedure of iteration. On the other hand, some research have shown that a successful iteration does not depend on conservation. Hiebert (1981) observed that first-graders managed to measure the length of an object by iterating the units of measurement, although they still had not mastered the ability of conservation. Some research (Lehrer, 2003; Stephan & Clements, 2003) point out that recognising the inversive connection between the measurement number and the measurement unit is the only component requiring the ability of conservation.

6. *Transitivity reasoning* – It is understanding that: a) if the length of the 1st object equals the length of the 2nd object, and the 2nd object is of the same length as the 3rd object, the length of the 1st object equals the length of the 3rd one; b) if the length of the 1st object is bigger (smaller) than the length of the 2nd object, and the 2nd object longer (shorter) than the 3rd object, then the 1st object is longer (shorter) than the 3rd one. Developing transitivity component is very important and it implies an ability to use one object as an instrument for calculating and comparing the lengths of other objects. If pupils have mastered this component, they can use, say, a pencil as an instrument to determine whether two (stationary) objects are of the same length (Stephan & Clements, 2003). The significance of transitivity reasoning for the measurement procedure was recognised and experimentally proved by Piaget and his findings were confirmed by numerous studies of the later date (Kamii & Clark, 1997; Kamii, 2006).

7. *Additivity* − This component of the concept of measurement implies the realisation that the total distance between the endpoints of an object is equal to the summed distance of its parts (of arbitrary length). The development of the additivity component is based on the adoption of the length conservation component and these two components are interdependent. The adoption of this component also implies the realisation that the total length of an object remains the same after we have replaced its parts of an arbitrary length (Lehrer, 2003; Mitchell & Horne, 2011).

8. *Origin (zero point)*− Any point can be used as the starting one in measurement procedure because measurement scales on measurement tools are designed in such a manner that the length of an object is represented as a distance between two points on the length measurement scale. Pupils who have developed this component will consider the distance between 10 cm and 20 cm on the length measurement scale equal to the distance between, for example, 4 cm and 14 cm (Lehrer, 2003; Levine, Kwon, Huttenlocher, Ratliff, & Deitz, 2009).

9. *Relation between number and measurement* − This component implies the realisation of the dependence of the number on the selection of the measurement unit, i.e., its size. Iteration of units of measurement can be viewed as a quantitative procedure of counting their total number (Zöllner & Benz, 2016; Sarama & Clements, 2009). Expressing the results of the measurement procedure, e.g. a notebook is long 17 lengths of a stick, or a wall is 2 meters long, contains both a measuring number and a unit of measurement. The number depends on the unit of measurement, and their relation is proportional (Zöllner & Benz, 2016; Lehrer, 2003). Carpenter & Lewis (1976) found that when comparing two measurement results, children focus on the measurement number. Accordingly, if the units of measurement are the same, they give the correct answer, and if the units of measurement are different, they answer incorrectly, saying that the longer object is the one whose number is bigger, e.g. 17cm> 5m.

Length measurement procedure is based on connecting the idea of number with geometry, along with the mastery of length conservation and transitivity reasoning. The research of the length measurement procedure can be divided into two groups, taking into consideration that there are two measurement procedures – measuring using standard and measuring using non-standard measurement units. Measuring using non-standard measurement units is an activity that is frequently organised in preschool and early primary education. For instance, the real objects (paper clips, strips, sticks, straws, etc.) are used as measurement units. The researchers who are in favour of this approach consider it necessary for children to realise the significance of and the need for using standard measurement units, as well as to develop some of the crucial components of the length measurement concept (Antić & Đokić, 2018; Carpenter & Lewis, 1976; Kamii & Clark, 1997; Lehrer, 2003; Stephan & Clements, 2003; Van de Walle et al., 2018).

Although this approach – from non-standard to standard measurement units – produced good effects, research shows that a strict adherence to it is unnecessary. Using standard measurement units and a ruler as a measurement instrument turned out to be less demanding and more entertaining activity for children than measuring using non-standard units (Nunes et al., 1993). Another reason for favouring the standard measurement units is that using them enables pupils to get prepared for everyday life as soon as possible

(Gomezescobar, Fernandez-Cezar, & Guerrero, 2018). However, many pupils know how to perform measurement procedure using a ruler, but there is an insufficient understanding of the relations among length measurement units, as well as of the length measurement scale (Nunes et al., 1993; Nührenbörger, 2001). This claim has been supported by various research projects dealing with the mistakes occurring when a ruler is used for measuring (Bragg & Outhred, 2000; Bragg & Outhred, 2004; Gomezescobar et al., 2018; Kamii & Clark, 1997; Levine et al., 2009). The analysis of the research on the procedural and conceptual knowledge of pupils points to the following errors:

1. pupils begin measuring from point 1;
2. pupils count the points or numbers, not the intervals between them on the measuring scale;
3. pupils focus on the point on the right side of the object being measured, not paying attention if the beginning of the object is aligned with the point 0 or not;
4. confusing length measurement units with other measurement units;
5. pupils align the measurement scale incorrectly with the object being measured (Tan Sisman & Aksu, 2015).

The conclusion is that pupils perceive the two procedures as two entirely different and separate activities (Bragg & Outhred, 2004):

1. they perceive measuring with non-standard units as counting the objects used for measuring,
2. whereas in measuring using a ruler their focus is on aligning the ruler with the object being measured and reading the number on the points of the scale.

The best recommendation for avoiding this error in children's understanding of measurement is to establish a clear connection between the iterated non-standard measurement units and the intervals between the points on the scale for measuring using standard measurement units.

Our focus in tracing the developmental perspective of length measurement was on the pupils of the second and fourth grades of primary school and our aim was to gain a deeper insight into the situation in the first cycle of primary education in the Republic of Serbia (pertaining by-laws, only in Serbian language): *Pravilnik o programu nastave i učenja za drugi razred osnovnog obrazovanja i vaspitanja*, 2018; *Pravilnik o nastavnom planu i programu za četvrti razred osnovnog obrazovanja i vaspitanja*, 2006). Our sample consisted of the second-grade pupils experienced in length measurement, given that they had already become familiar with the measurement procedure and a ruler as a measuring instrument at the end of the first grade, as well as on the fourth-grade pupils who were expected to have mastered the measurement procedure and to know how to use it. Let us mention that the first cycle of primary education in Serbia lasts four years (with additional four years of the second cycle which, together with one year of preschool education, makes a total of nine years of compulsory primary education).

III. METHODOLOGY

The main purpose of this study was to examine pupils' level of knowledge of the procedure of length measurement using a ruler in a specific pedagogical situation. During the school year 2018/2019, the primary education curricula were revised - a new curruculum was developed for the second grade, while the revision of the fourth-grade curriculum is under way - so we have chosen pupils of the second and fourth grades to determine the level of pupils' mastery of measurement procedure in the first cycle of primary education, as well as their knowledge of the developmental components on which it is based. We pursued three *research questions*:

1. How pupils draw a line of a specific length using a ruler and whether there are differences in the level of knowledge between the second-graders and the fourth-graders?
2. How pupils draw a line of a specific length using a 'broken' ruler (a ruler that does not begin from zero point) and whether there are differences in the level of knowledge between the second-graders and the fourth-graders?
3. How pupils draw a ruler independently if they have been given a measurement unit line and whether there are differences in the level of knowledge between the second-graders and the fourth-graders?

A random sample consisted of 145 pupils, 72 (49.65%) second-graders (mean age = 8.0 years) and 73 (50.35%) fourth-graders (mean age = 10.0 years) from one urban primary school in Belgrade, the Republic of Serbia. Our research is *descriptive* (Coolican, 1999). The results are shown in numerical form and the data were quantitative. Testing is the *research technique,* and a test containing measuring tasks was designed for this purpose (*Appendix*). The test helped us to determine in one pedagogical situation how the pupils were using the ruler to measure length. The obtained data are presented in terms of frequency (and percentage) of pupils' answers, and the χ^2 test of homogeneity is used for determining the statistical differences among the pupils. All data are presented in the form of tables. When presenting our results, we have chosen to use frequency (and percentages) (Coolican, 1999) when examining pupils' understanding of measurement concept. Many studies in measurement apply this approach (Bragg & Outhred, 2004; Gomezescobar et al., 2018; Mitchell & Horne, 2011; Nunes et al., 1993; Tan Sisman & Aksu, 2015).

Procedure. All pupils were given a measurement test. The pupils were supposed to do the knowledge test (*Appendix*) individually and using a paper and a pencil. The test consisted of three tasks and it lasted 15 minutes. Two test items consisted of a picture of a ruler (the second one of a 'broken' ruler) that varied in length which appeared on the ruler; the first one beginning with 0 point, and the second one in misaligned positions. These items do not start and end with whole centimetres. The third (and last) task did not contain the picture of ruler. The pupils were given paper clips as measurement units instead of ruler. The tasks involved: 1) drawing a line of a specific length using a ruler, 2) drawing a line of a specific length using a 'broken' ruler, and 3) drawing a ruler individually. Pupils' tasks were classified into two categories: correct and incorrect (marked as 1 or 0). The incorrect tasks were analysed according to types of pupils' errors.

IV. RESULTS

1. DRAWING A LINE OF A SPECIFIC LENGTH USING A RULER

The frequency and the percentages of their answers are shown in Table 1.

Table 1. Frequency and percentages of pupils' responses when drawing a line of a specific length using a ruler.

	2^{nd} grade pupils		4^{th} grade pupils	
	f	%	*f*	%
Pupils who drew a line of a specific length using a ruler	59	81.94	64	87.67
Pupils who did not draw a line of a specific length using a ruler	13	18.06	9	12.33
Total number of pupils	72	100	73	100

The χ^2 test of homogeneity confirmed that there were no statistically significant differences in the knowledge between the second-graders and the fourth-graders ($\chi^2(1, 145)=0.924$, p=0.364, p>0,00). This result gives rise to concern, given that the fourth-graders' level of mastery of drawing a line of a specific length using a ruler is not statistically much different than that of the second-graders, despite the fact that the majority of the pupils of both grades completed the task successfully.

Interestingly, out of all pupils who did the task correctly, only one fourth-grader decided to draw a line of a specific length starting from the point on the ruler marked with number 2. The other pupils started to draw from the zero point. The pupils who did not do the task correctly had made some expected, typical mistakes – they would start to draw a line from the point 1 or they would draw the line by counting the points, not the measurement units. Such mistakes indicate that the following components of the length measurement concept have not been developed: partitioning, iteration of measurement units and the starting point, i.e., the zero point. Although a small number of the pupils made mistakes in performing the task, the prescribed standards of pupils' knowledge consider this task to be the key one (*Standardi postignuća – obrazovni standardi za kraj prvog ciklusa obaveznog obrazovanja i vaspitanja*, 2011) in the field of Measurement and Measure. For this reason, there is indeed plenty of room for concern.

2. DRAWING A LINE OF A SPECIFIC LENGTH USING A 'BROKEN' RULER

The frequency and percentages of pupils' responses are shown in Table 2.

Table 2. Frequency and percentages of pupils' responses when drawing a line of a specific length using a 'broken' ruler.

	2nd grade pupils		4th grade pupils	
	f	%	*f*	%
Pupils who drew a line using a 'broken' ruler	22	30.56	43	58.90
Pupils who did not draw a line using a 'broken' ruler	50	69.44	30	41.10
Total number of pupils	72	100	73	100

In the second task, the second-graders demonstrated a considerably lower level of knowledge than the fourth-graders ($\chi^2(1, 145)=11.778$, $p=0.001$, $p<0,00$). The task followed the previous one and the aim was to check if the pupils were able to use their knolwedge in solving a non-standard task. As *Table 2* shows, the second-graders were less successful in solving Task 2 than in solving Task 1. Namely, 22 (15.20%) pupils did Task 1 incorrectly and 80 (55.20%) pupils provided incorrect answers for Task 2, whereas 123 (84.80%) pupils did Task 1 correctly and 65 (44.80%) pupils did Task 2 correctly. These results were also confirmed when the χ^2 test of homogeneity was applied (χ^2 (1, 290)=50.874, $p=0.000$, $p<0,00$), taking into consideration the knowledge of all pupils, of both the second and fourth grades. The conclusion is that all pupils underperformed on Task 2.

The mistakes made in solving this task were expected. The most common mistake, made by 14 second-graders and one fourth-grader, involved drawing a line starting from the point marked with 3 and ending at the point marked with 6. In this way, the pupils confirmed a typical mistake of misunderstanding the zero point as the starting point on the length measurement scale. In addition, some pupils drew a line at a distance from the point 3 to point 4 (1cm total length) and claimed that it was 4 cm long, although the line ended at point 4. There was yet one response that deserves our comment. Namely, as many as 16 pupils (12 second-graders and 4 fourth-graders) answered that it was impossible to draw a 4 cm long line because the ruler was broken and it started from the point 3. This response was also observed among the seventh-grade pupils from schools in Ankara (Turkey), where nearly a half of pupils claimed that a broken ruler could not be used for measurement (Tan Sisman & Aksu, 2015). The obtained results indicate that pupils demonstrate better knowledge in standard tasks, while in the non-standard ones, requiring conceptual knowledge, they tend to underperform.

3. DRAWING A RULER INDEPENDENTLY

The frequencies and percentages of pupils' responses are shown in Table 3.

Table 3. Frequency and percentages of pupils' responses when drawing a ruler independently.

	2^{nd} grade pupils		4^{th} grade pupils	
	f	%	*f*	%
Pupils who drew a ruler	24	33.33	34	46.58
Pupils who did not draw a ruler	48	66.67	39	53.42
Total number of pupils	72	100	73	100

In the third task, the χ^2 test of homogeneity showed that there were no statistically significant differences between the second-graders and fourth-graders ($\chi^2(1, 145)=2.648$, $p=0.128$, $p> 0.00$). This result worried us just as the result in the first task did. As Table 3 shows, the pupils were less successful in solving Task 3 than in solving Task 1. Namely, 22 (15.20%) pupils did Task 1 incorrectly and 87 (60%) pupils provided incorrect answers for Task 3, whereas 123 (84.80%) pupils did Task 1 correctly and 58 (40%) pupils did Task 3 correctly. These results were also confirmed when the χ^2 test of homogeneity was applied ($\chi^2(1, 290)=50.874$, $p=0.000$, $p<0,00$), taking into consideration the knowledge of all pupils, of both the second and fourth grades. Therefore, the pupils performed better in Task 1 than in Task 3. In addition, Table 3 shows that the pupils were equally successful in doing Tasks 3 and 2. In terms of numbers, 80 (55.20%) pupils did task 2 incorrectly, 87 (60%) pupils gave incorrect answers in task 3, whereas 65 (44.80%) pupils were successful at task 2 and 58 (40%) pupils at task 3. These results were also confirmed when the χ^2 test of homogeneity was applied (χ^2 (1, 290)=0.692, $p=0.406$, $p>0,00$), taking into consideration the knowledge of all pupils, of both the second and fourth grades. There is no statistically significant difference in pupils' achievement in Task 3 and Task 2.

The pupils who did Task 3 correctly were using two strategies. First, they would put one clip on the paper, then they would measure the point whose length was equal to the length of the clip, and then they would repeat the procedure several times by iterating the line. A variation of this strategy was also used, when pupils would place the clip vertically and use its narrower part to draw the measurement unit. The second strategy involved iterating two clips which the pupils would place next to each other, mark the point at the end of the second clip, then mark the point between them, and then repeat the same procedure. The pupils who did Task 3 incorrectly used the clips only as an aid while drawing straight lines. As they did not take into account the distance between the points, it was obvious that they had not developed the partitioning component and the iteration of measurement units.

V. DISCUSSION

We based our research on the components of the concept of length measurement using a ruler and explored the development of the components, as well as how pupils can draw a ruler and its scale when given a unit of measurement. The research results show that, as far as drawing a line of a given length using of a ruler is concerned, the second-graders and the fourth-graders demonstrate no difference in their knowledge. The same conclusion can be drawn regarding their knowledge of drawing a ruler independently, while the fourth-graders are better at drawing a line of a specific length using a 'broken' ruler. All pupils, of both grades, demonstrated better knowledge of drawing a line of a specific length using a ruler than at doing the same using a 'broken' ruler and drawing the ruler independently. All pupils demonstrated the same level of knowledge when it comes to drawing a line of a specific length using a 'broken' ruler and drawing the ruler independently.

The rule book regulating the second-grade curriculum, entitled *Pravilnik o programu nastave i učenja za drugi razred osnovnog obrazovanja i vaspitanja* (2018), specifies that pupils should measure the length of objects using standard measurement units, mainly a metre (as a basic unit), decimetre and centimetre (as smaller units), and that they should become aware of the relations between them. The recommendation is that pupils should learn about the units of measurement through different practical activities. The importance of making estimations in the measurement procedure is emphasised, and an example is provided in terms of a process that begins with estimating the length/distance, checking the estimation by measuring, and then analysing the possible pupils' errors. The rules formulated in this way in the new second-grade curriculum entitled *Pravilnik o programu nastave i učenja za drugi razred osnovnog obrazovanja i vaspitanja* (2018) provide greater support to pupils than the ones in the previous rule book for the second grade of primary school entitled *Pravilnik o nastavnom planu i programu za prvi i drugi razred osnovnog obrazovanja i vaspitanja* (2004). Despite this fact, the interpretation of introducing the measurement procedure using the ruler as a measurement tool is still missing.

With regard to the field of Measurement and Measures, no differences were identified between the second-grade curriculum**,** *Pravilnik o programu nastave i učenja za drugi razred osnovnog obrazovanja i vaspitanja* (2018), and the curriculum for the fourth grade of primary school, *Pravilnik o nastavnom planu i programu za četvrti razred osnovnog obrazovanja i vaspitanja* (2006). However, there is a difference in the way of introducing the units for measuring the area and calculating the area of geometric figures (optional and the volume of geometric solid). It remains to be seen what changes will the new curriculum for the fourth-grade of primary school in Serbia entail.

Educational standards for the end of the first cycle of primary education (*Standardi postignuća – obrazovni standardi za kraj prvog ciklusa obaveznog obrazovanja i vaspitanja*, 2011) give us an in-depth insight into measures and measurement and enable us to determine the developmental perspective by looking at the requirements set for every level of pupils' knowledge. At basic level, pupils are expected to know length measurement units and correlations among them, as well as to use the procedure for measuring the length of an object presented in an image and using the given measurement unit. At intermediate level, pupils are expected to be able to turn the smaller measurement units into the bigger ones, and vice versa. The authors of the *Standardi postignuća – obrazovni standardi za kraj prvog ciklusa obaveznog obrazovanja i vaspitanja* (2011) were aware that area (and volume) measurement derived from the length measurement. This fact is manifested in the achievement standards set for an advanced level where pupils are expected to be able to complete the tasks involving these measurements (area and scope, i.e., volume).

VI. CONCLUSION AND IMPLICATIONS FOR TEACHING LENGTH MEASUREMENT USING A RULER

The research results indicate that pupils demonstrate better knowledge in standard tasks than in non-standard tasks that require conceptual knowledge. Based on everything discussed in the above text, we can conclude that there is a low level of mastery of the basic components of the concept of length measurement using a ruler, such as the zero point on the length measurement scale, partitioning an object into length measurement units, and their iteration. Also, we observed that there is a poor understanding of the structure of the ruler as a tool for measuring length.

Our suggestion is that the basic components of the concept of length measurement should be included explicitly in the written curriculum, as well as the information about common pupils' misconceptions. That would be supportive for teachers, and it would contribute to a better development of the concept of length measurement.

1. PEDAGOGICAL IMPLICATIONS

The recommendations in the form of pedagogical implications include designing non-standard tasks and their inclusion in textbooks, interpretations of the mathematics curricula, and identification of pupils' strategies when doing length measurement. Bragg & Outhred (2000) propose three steps relevant for measuring length using non-standard measurement units and these steps should be taken into consideration:

- identifying the part of an object whose length we want to measure and taking 0 as a starting point in measurement,
- iteration of measurement units in the length measurement procedure and
- counting the units.

Primary school teachers play a significant role in this process, given that they have to teach different strategies for counting measurement units and discuss them with their pupils. Bearing in mind that the measurement procedure involves counting the units of measurement, it is important that pupils understand what exactly they have to count. Understanding this component of measurement contributes to their understanding of the measurement procedure. As we noticed that pupils do not realise the correlation between measuring with non-standard and measuring with standard units of measurement, as well as that it is difficult for them to create their own ruler, which was also confirmed in our research, the activities that may contribute to the conceptualisation of pupils' knowledge involve, according to Smith & Barrett (2017), giving examples with 'broken' rulers such as the following ones:

- the scale on the ruler does not begin at 0 point,
- the distance between the points is not identical (the points marked for different measurement units),
- the numbers next to the points on the scale are 'mixed up', etc.

While pupils perform these tasks, teachers should discuss with them the appearance of the rulers and give them an opportunity to express their observations and conclusions about the length measurement procedure.

Levine et al. (2009) show us that in grade 2 in the USA many pupils are unable to identify the unit of measurement for the length on a ruler, while Bragg & Outhred (2004) identify the same problem in grades 5 and 6 in Australia. In both cases, a particularly relevant fact was the pupils' belief that the unit markers were the measures and not a feature marking the end of each unit. Pupils should learn that length is a linear entity and the point of origin can therefore be identified and written as zero. Therefore**,** the space between the endpoints of an object with a line should be called the 'units of measure' and pupils would consider these objects as they are *counting of discrete objects*. The continuous nature of measures calls for a counting action that does not use a 'point-count' action, but rather a counting action reflecting the essential movement from the point of origin. This must be recognised as an important component in the new curriculum in the Republic of Serbia for understanding the area of measurement.

The rationale behind this study was to contribute to the body of research impacting the changes of the fourth-grade curriculum in the Republic of Serbia and leading to its final version that will mark the end of the curricular changes in the first cycle of mathematics education. We follow the lengths measured rectilinearly in a manner of Hans Freudenthal (2002). In order to determine the distance along a 'broken' path, the lengths of the 'pieces' are added. The first spontaneous measuring acts of children observed by Freudenthal included pacing and spanning (between a thumb and a forefinger), or by means of the palm, or with parallel fingers in sand, or parallel hands at chest height, thereby not using any instrument that would suggest rectilinearity. Nevertheless, rectilinearity plays a part even here: the steps taken are straightforward, the spans are prolongations of each other. The straight line is present mentally, rather than instrumentally, in such measuring acts. This idea of Freudenthal should be incorporated in the first cycle of mathematics education in the Republic of Serbia.

2. ADVANTAGES AND LIMITATIONS OF THE RESEARCH

Some limitations of the conducted research should be looked at here as well. The research sample might appear to be insufficiently representative, but it still offers an insight into the situations for learning in the Republic of Serbia and can be considered a small-scale research that precedes a bigger one, e.g. in the bigger one we should include more than one school, as well as some schools from rural parts of Serbia. Limitation was made in the assessment of the test. There was not inter-rater reliability checking, more generally, we did not use any methods for checking the reliability and validity of the test; reliability as consistency within a test or between repeated uses of it in the same circumstances, and validity indicating whether a test measures what it was created to measure.

On the other hand, the advantages of this research are as follows: pointing to pupils' mistakes in measurement using a ruler, shedding more light on the crucial moment of moving from non-standard to standard measurement units, as well as the formation of the measurement scale.

The questions that deserve an answer in some future research include exploring the formation of the concept of length measurement using a ruler through all its components and their inclusion in the mathematics curriculum, as well as determining the development of the components of the length measurement concept which influence conceptual understanding of the concept in measuring the area (volume) at the end of the first and the beginning of the second cycle of the mathematics education in Serbia.

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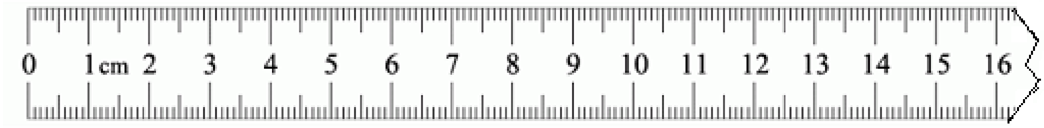
Appendix - *Tasks*

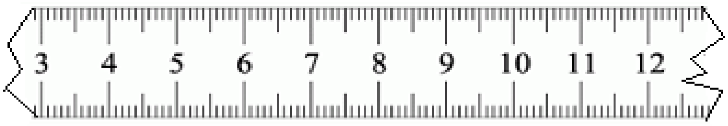
Task 1.

The image below represents a ruler. Draw a 5 cm-long line.



Task 2.

The image below represents a broken ruler. Draw a 4 cm-long line, if possible. If not, write down why it is not possible.



Task 3.

You have two paper clips. Draw a ruler using the clips as measurement units.