# SOME 4-TOTAL PRIME CORDIAL LABELING OF GRAPHS 

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#### Abstract

Let $G$ be a $(p, q)$ graph. Let $f: V(G) \rightarrow\{1,2, \ldots, k\}$ be a map where $k \in \mathbb{N}$ and $k>1$. For each edge $u v$, assign the label $\operatorname{gcd}(f(u), f(v))$. $f$ is called $k$-Total prime cordial labeling of $G$ if $\left|t_{f}(i)-t_{f}(j)\right| \leq 1, i, j \in$ $\{1,2, \cdots, k\}$ where $t_{f}(x)$ denotes the total number of vertices and the edges labelled with $x$. A graph with a $k$-total prime cordial labeling is called $k$ total prime cordial graph. In this paper we investigate the 4 -total prime cordial labeling of some graphs.

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## 1. Introduction

All Graphs in this paper are finite, simple and undirected. Let $G_{1}, G_{2}$ respectively be $\left(p_{1}, q_{1}\right),\left(p_{2}, q_{2}\right)$ graphs. The corona of $G_{1}$ with $G_{2}, G_{1} \odot G_{2}$ is the graph obtained by taking one copy of $G_{1}$ and $p_{1}$ copies of $G_{2}$ and joining the $i^{t h}$ vertex of $G_{1}$ with an edge to every vertex in the $i^{t h}$ copy of $G_{2}$. The cartesian product of two graphs $G_{1}$ and $G_{2}$ is the graph $G_{1} \times G_{2}$ with vertex set $V_{1} \times V_{2}$ and two vertices $u=\left(u_{1}, u_{2}\right)$ and $v=\left(v_{1}, v_{2}\right)$ are adjacent whenever [ $u_{1}=v_{1}$ and $u_{2}$ adj $v_{2}$ ] or [ $u_{2}=v_{2}$ and $u_{1}$ adj $v_{1}$ ]. The graph $L_{n}=P_{n} \times K_{2}$ is called ladder. A friendship graph $F_{n}$ is a graph which consists of n triangles with a common vertex. Ponraj et al. [4], has been introduced the concept of $k$ total prime cordial labeling and investigate the $k$-total cordial labeling of certain graphs. Also in $[4,5]$, we have investigated the 4 -total prime cordial labeling for path, cycle, star, bistar, flower graph, gear graph etc. In this paper we investigate the 4 -total prime cordiality of some graphs like comb, double comb, triangular snake, double triangular snake, ladder, friendship graph.

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## 2. Main results

Theorem 2.1. The comb $P_{n} \odot K_{1}$ is 4 -total prime cordial.
Proof. Let $P_{n}$ be the path $u_{1}, u_{2}, \ldots, u_{n}$. Let $v_{1}, v_{2}, \ldots, v_{n}$ be adjacent to the pendent vertices $u_{1}, u_{2}, \ldots, u_{n}$. Clearly $\left|V\left(P_{n} \odot K_{1}\right)\right|+\left|E\left(P_{n} \odot K_{1}\right)\right|=4 n-1$. Case 1. $n \equiv 0(\bmod 4)$.
Let $n=4 t, t \in \mathbb{N}$. Assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{t}$ and assign the label 3 to the vertices $u_{t+1}, u_{t+2}, \ldots, u_{2 t}$. Next we assign the label 2 to the vertices $u_{2 t+1}, u_{2 t+2}, \ldots, u_{3 t}$ and assign 1 to the vertices $u_{3 t+1}, u_{3 t+2}, \ldots, u_{n-1}$ respectively. Finally we assign the label 4 to the vertex $u_{n}$. Next we consider the vertices $v_{1}, v_{2}, \ldots, v_{n}$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{t}$ and assign the label 3 to the vertices $v_{t+1}, v_{t+2}, \ldots, v_{2 t}$ and 2 to the vertices $v_{2 t+1}, v_{2 t+2}, \ldots, v_{3 t}$. Next we assign 1 to the vertices $v_{3 t+1}, v_{3 t+2}, \ldots, v_{n-1}$ respectively. Finally we assign the label 3 to the vertex $v_{n}$. Clearly $t_{f}(1)=t_{f}(3)=$ $t_{f}(4)=4 t$ and $t_{f}(2)=4 t-1$.
Case 2. $n \equiv 1(\bmod 4)$.
Let $n=4 t+1, t \in \mathbb{N}$. In this case, assign the label to the vertices $u_{i}(1 \leq i \leq$ $n-1)$ and $v_{i}(1 \leq i \leq n-1)$ by in case 1 . Next assign the labels 2 and 3 to the vertices $u_{n}$ and $v_{n}$ respectively. Here $t_{f}(1)=t_{f}(2)=t_{f}(3)=4 t+1$ and $t_{f}(4)=4 t$.
Case 3. $n \equiv 2(\bmod 4)$.
Let $n=4 t+2, t \in \mathbb{N}$. As in case 2 , assign the label to the vertices $u_{i}(1 \leq i \leq$ $n-1)$ and $v_{i}(1 \leq i \leq n-1)$. Next we assign the labels 4 and 3 to the vertices $u_{n}$ and $v_{n}$ respectively. It is easy to verify that $t_{f}(1)=t_{f}(2)=t_{f}(3)=4 t+2$ and $t_{f}(4)=4 t+1$.
Case 4. $n \equiv 3(\bmod 4)$.
Let $n=4 t+3, t \in \mathbb{N}$. Assign the label to the vertices $u_{i}(1 \leq i \leq n-3)$ and $v_{i}$ $(1 \leq i \leq n-3)$ as in case 1 . Finally we assign the labels $4,2,3$ to the vertices $u_{n-2}, u_{n-1}$ and $u_{n}$ and $3,4,2$ to the vertices $v_{n-2}, v_{n-1}$ and $v_{n}$ respectively. Here $t_{f}(1)=t_{f}(2)=t_{f}(4)=4 t+3$ and $t_{f}(3)=4 t+2$.

Theorem 2.2. The double comb $P_{n} \odot 2 K_{1}$ is 4 -total prime cordial.
Proof. Let $P_{n}$ be the path $u_{1} u_{2} \ldots u_{n}$. Let $x_{i}, y_{i}(1 \leq i \leq n)$ be the pendent vertices adjacent to $u_{i}(1 \leq i \leq n)$. Obviously $\left|V\left(P_{n} \odot 2 K_{1}\right)\right|+\left|E\left(P_{n} \odot 2 K_{1}\right)\right|=$ $6 n-1$.
Case 1. $n \equiv 0(\bmod 4)$.
Let $n=4 t, t \in \mathbb{N}$. Consider the vertices of path $P_{n}$. Assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{t}$ and assign the label 3 to the vertices $u_{t+1}, u_{t+2}, \ldots, u_{2 t}$. Next we assign the label 2 to the vertices $u_{2 t+1}, u_{2 t+2}, \ldots, u_{3 t}$ then assign 1 to the vertices $u_{3 t+1}, u_{3 t+2}, \ldots, u_{n-1}$. Finally we assign the label 3 to the vertex $u_{n}$. Next we move to the pendent vertices. Assign the label to the vertices $x_{i}, y_{i}$ $(1 \leq i \leq n-1)$ as in $u_{i}(1 \leq i \leq n-1)$. Now we assign the labels 2 to the vertex $x_{n}$ and 4 to the vertex $y_{n}$. Clearly $t_{f}(2)=t_{f}(3)=t_{f}(4)=6 t$ and $t_{f}(1)=6 t-1$.
Case 2. $n \equiv 1(\bmod 4)$.

Let $n=4 t+1, t \in \mathbb{N}$. As in case 1 , assign the label to the vertices $u_{i}, x_{i}, y_{i}$ $(1 \leq i \leq n-1)$. Now we assign the labels $3,2,4$ to the vertices $u_{n}, x_{n}$ and $y_{n}$ respectively. Here $t_{f}(1)=t_{f}(2)=t_{f}(4)=6 t+1$ and $t_{f}(3)=6 t+2$.
Case 3. $n \equiv 2(\bmod 4)$.
Let $n=4 t+2, t \in \mathbb{N}$. Assign the label to the vertices $u_{i}, x_{i}, y_{i}(1 \leq i \leq n-1)$. Next we assign the labels $4,2,3$ respectively to the vertices $x_{n}, u_{n}$ and $y_{n}$. It is easy to verify that $t_{f}(1)=t_{f}(2)=t_{f}(3)=6 t+3$ and $t_{f}(4)=6 t+2$.
Case 4. $n \equiv 3(\bmod 4)$.
Let $n=4 t+3, t \in \mathbb{N}$. As in case 1 , assign the label to the vertices $u_{i}, x_{i}, y_{i}$ $(1 \leq i \leq n-3)$. Now we assign the labels $4,3,2,4,3,2,4,3,2$ to the vertices $x_{n-2}, x_{n-1}, x_{n}, u_{n-2}, u_{n-1}, u_{n}, y_{n-2}, y_{n-1}$ and $y_{n}$ respectively. Here $t_{f}(2)=t_{f}(3)=t_{f}(4)=6 t+4$ and $t_{f}(1)=6 t+5$.

Theorem 2.3. The graph $C_{n} \odot 2 K_{1}$ is 4 -total prime cordial.
Proof. Let $C_{n}$ be the cycle $u_{1} u_{2} \ldots u_{n} u_{1}$. Let $x_{i}, y_{i}(1 \leq i \leq n)$ be the pendent vertices adjacent to $u_{i}(1 \leq i \leq n)$. It is easy to verify that $\left|V\left(C_{n} \odot 2 K_{1}\right)\right|+$ $\left|E\left(C_{n} \odot 2 K_{1}\right)\right|=6 n$.
Case 1. $n \equiv 0,1,3(\bmod 4)$.
Let $n=4 t, n=4 t+1, n=4 t+3, t \in \mathbb{N}$. The vertex labelled in Theorem 2.2 is also a 4 -total prime cordial of $C_{n} \odot 2 K_{1}$.
Case 2. $n \equiv 3(\bmod 4)$.
Let $n=4 t+2, t \in \mathbb{N}$. Assign the label to $u_{i}, x_{i}, y_{i}(1 \leq i \leq n)$ as in Theorem 2.2. Finally interchange the labels of $u_{n}$ and $x_{n}$. Obviously this induced vertex labels is a 4-total prime cordial of $C_{n} \odot 2 K_{1}$.

Theorem 2.4. The ladder $L_{n}$ is 4 -total prime cordial.
Proof. Let $V\left(L_{n}\right)=\left\{u_{i}, v_{i}: 1 \leq i \leq n\right\}$ and $E\left(L_{n}\right)=\left\{u_{i} u_{i+1}, v_{i} v_{i+1}: 1 \leq i \leq\right.$ $n-1\} \cup\left\{u_{i} v_{i}: 1 \leq i \leq n\right\}$. Clearly $\left|V\left(L_{n}\right)\right|+\left|E\left(L_{n}\right)\right|=5 n-2$.
Case 1. $n \equiv 0(\bmod 4)$.
Let $n=4 t, t \in \mathbb{N}$. Assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{t}$ and assign the label 2 to the vertices $u_{t+1}, u_{t+2}, \ldots, u_{2 t}$. Next we assign the label 3 to the vertices $u_{2 t+1}, u_{2 t+2}, \ldots, u_{3 t}$ then assign the label 1 to the vertices $u_{3 t+1}, u_{3 t+2}, \ldots, u_{n-1}$. Finally we assign the label 4 to the vertex $u_{n}$. Next we consider the vertices $v_{i}(1 \leq i \leq n)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{t}$ and assign the label 2 to the vertices $v_{t+1}, v_{t+2}, \ldots, v_{2 t}$. Next we assign the label 3 to the vertices $v_{2 t+1}, v_{2 t+2}, \ldots, v_{3 t}$ then assign the label 1 to the vertices $v_{3 t+1}, v_{3 t+2}, \ldots, v_{n-1}$. Finally we assign the label 3 to the vertex $v_{n}$. Clearly $t_{f}(1)=t_{f}(2)=5 t$ and $t_{f}(3)=t_{f}(4)=5 t-1$.
Case 2. $n \equiv 1(\bmod 4)$.
Let $n=4 t+1, t \in \mathbb{N}$. As in case 1 , assign the label to the vertices $u_{i}, v_{i}$ $(1 \leq i \leq n-1)$. Next we assign the label 4,3 respectively to the vertices $u_{n}$ and $v_{n}$. Here $t_{f}(1)=t_{f}(3)=t_{f}(4)=5 t+1$ and $t_{f}(2)=5 t$.
Case 3. $n \equiv 2(\bmod 4)$.

Let $n=4 t+2, t \in \mathbb{N}$. Assign the label to the vertices $u_{i}, v_{i}(1 \leq i \leq n-4)$ by in case 1 . Now we assign the label $1,4,3,4$ to the vertices $u_{n-3}, u_{n-2}, u_{n-1}$ and $u_{n}$ respectively. Finally we assign $3,2,3,4$ respectively to the vertices $v_{n-3}, v_{n-2}$, $v_{n-1}$ and $v_{n}$. It is easy to verify that $t_{f}(1)=t_{f}(2)=t_{f}(3)=t_{f}(4)=5 t+2$.
Case 4. $n \equiv 3(\bmod 4)$.
Let $n=4 t+3, t \in \mathbb{N}$. In this case, assign the label to the vertices $u_{i}, v_{i}$ $(1 \leq i \leq n-6)$ by in case 1 . Then we assign the labels $4,4,3,2,1,1$ to the vertices $u_{n-5}, u_{n-4}, u_{n-3}, u_{n-2}, u_{n-1}$ and $u_{n}$ respectively. Finally we assign 4, $3,3,2,1,1$ respectively to the vertices $v_{n-3}, v_{n-3}, v_{n-3}, v_{n-2}, v_{n-1}$ and $v_{n}$. Here $t_{f}(2)=t_{f}(3)=t_{f}(4)=5 t+3$ and $t_{f}(1)=5 t+4$.

Theorem 2.5. The triangular snake $T_{n}$ is 4 -total prime cordial.
Proof. Let $P_{n}$ be the path $u_{1} u_{2} \ldots u_{n}$. Let $v_{1}, v_{2}, \ldots, v_{n-1}$ be the vertices such that $v_{i}$ is adjacent to both $u_{i}$ and $u_{i+1}(1 \leq i \leq n-1)$. It is easy to verift that $\left|V\left(T_{n}\right)\right|+\left|E\left(T_{n}\right)\right|=5 n-4$.
Case 1. $n \equiv 0(\bmod 4)$.
Let $n=4 t, t \in \mathbb{N}$. Assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{t}$ and assign the label 2 to the vertices $u_{t+1}, u_{t+2}, \ldots, u_{2 t}$. Next we assign the label 3 to the vertices $u_{2 t+1}, u_{2 t+2}, \ldots, u_{3 t}$ then we assign 1 to the vertices $u_{3 t+1}, u_{3 t+2}, \ldots, u_{n-1}$. Finally we assign the label 3 to the vertex $u_{n}$. Next we consider the vertices $v_{i}$ $(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{t}$ and assign the label 2 to the vertices $v_{t+1}, v_{t+2}, \ldots, v_{2 t-1}$ and we assign the label 3 to the vertices $v_{2 t}, v_{2 t+1}, \ldots, v_{3 t-1}$. Next we assign 1 to the vertices $v_{3 t}, v_{3 t+1}, \ldots, v_{n-3}$. Finally, we assign the labels 2,4 to the vertices $v_{n-2}$ and $v_{n-1}$ respectively. Here $t_{f}(1)=t_{f}(2)=t_{f}(3)=t_{f}(4)=5 t-1$.
Case 2. $n \equiv 1(\bmod 4)$.
Let $n=4 t+1, t>1 t \in \mathbb{N}$. As in case 1 , assign the label the vertices to $u_{i}(1 \leq i \leq 3 t), v_{i}(1 \leq i \leq 3 t-1)$. Next we assign the labels $4,3,2$ to the vertices $u_{3 t+1}, u_{3 t+2}, u_{3 t+3}$ respectively. Assign the label 1 to the remaining vertices of the path $P_{n}$. Now we assign the labels $1,3,4$ to the vertices $v_{3 t}, v_{3 t+1}$ and $v_{3 t+2}$ respectively. Finally we assign the label 1 to the vertices $v_{3 t+3}, v_{3 t+4}, \ldots, v_{n-1}$. Clearly $t_{f}(1)=t_{f}(2)=t_{f}(4)=5 t$ and $t_{f}(3)=5 t+1$. For $n=5$ a 4-total prime cordial labelling of $T_{5}$ is shown in Figure 1.


Figure 1

Case 3. $n \equiv 2(\bmod 4)$.
Let $n=4 t+2, t>1 t \in \mathbb{N}$. As in case 2 , we assign the label to the vertices to $u_{i}(1 \leq i \leq 3 t+2), v_{i}(1 \leq i \leq 3 t-1)$. Then we assign the labels 4,2 to the vertices $u_{3 t+3}, u_{3 t+4}$ respectively. Finally we assign the label 1 to the remaining vertices of the path $P_{n}$. Then we assign the labels $4,3,2$ respectively to the vertices $v_{3 t}, v_{3 t+1}$ and $v_{3 t+2}$. Finally we assign the label 1 to the vertices $v_{3 t+3}, v_{3 t+4}, \ldots, v_{n-1}$. It is easy to verify that $t_{f}(1)=t_{f}(3)=5 t+1$ and $t_{f}(2)=t_{f}(4)=5 t+2$. When $n=6$ a 4 -total prime cordial labelling of $T_{6}$ is shown in Figure 2.


Figure 2

Case 4. $n \equiv 3(\bmod 4)$.
Let $n=4 t+3, t>1 t \in \mathbb{N}$. Assign the label to the vertices to $u_{i}(1 \leq i \leq 3 t+2)$, $v_{i}(1 \leq i \leq 3 t+2)$ by case 3 . Next we assign the label $2,3,4,3$ respectively to the vertices $u_{3 t+3}, u_{3 t+4}, u_{3 t+5}$ and $u_{3 t+6}$. Then the remaining vertices of path labelled by 1. Now we assign the label 2 to the vertex $v_{3 t+3}$. Finally we assign the label 1 to the vertices $v_{3 t+4}, v_{3 t+5}, \ldots, v_{n-1}$. Clearly $t_{f}(1)=t_{f}(2)=$ $t_{f}(3)=5 t+3$ and $t_{f}(4)=5 t+2$. When $n=7$ a 4 -total prime cordial labelling of $T_{7}$ is shown in Figure 3.


Figure 3

Theorem 2.6. The double triangular snake $D\left(T_{n}\right)$ is 4 -total prime cordial.

Proof. Let $P_{n}$ be the path $u_{1} u_{2} \ldots u_{n}$. Let $v_{i}, w_{i}$ be the vertex adjacent to $u_{i}$ and $u_{i+1}$. Obviously $\left|V\left(D\left(T_{n}\right)\right)\right|+\left|E\left(D\left(T_{n}\right)\right)\right|=8 n-7$.
Case 1. $n \equiv 0(\bmod 4)$.
Let $n=4 t, t \in \mathbb{N}$. Assign the label 4 to the vertices $u_{1}, u_{2}, \ldots, u_{t}$ and assign the label 2 to the vertices $u_{t+1}, u_{t+2}, \ldots, u_{2 t}$. Next we assign the label 3 to the vertices $u_{2 t+1}, u_{2 t+2}, \ldots, u_{3 t}$ then we assign 1 to the vertices $u_{3 t+1}, u_{3 t+2}, \ldots, u_{n-1}$. Finally we assign the label 3 to the vertex $u_{n}$. Next we consider the vertices $v_{i}, w_{i}(1 \leq i \leq n-1)$. Assign the label 4 to the vertices $v_{1}, v_{2}, \ldots, v_{t}$ and $w_{1}, w_{2}, \ldots, w_{t}$. Then assign the label 2 to the vertices $v_{t+1}, v_{t+2}, \ldots, v_{2 t-1}$ and $w_{t+1}, w_{t+2}, \ldots, w_{2 t-1}$. Next we assign the label 3 to the vertices $v_{2 t}, v_{2 t+1}, \ldots, v_{3 t-1}$ and $w_{2 t}, w_{2 t+1}, \ldots, w_{3 t-1}$. Now we assign 1 to the vertices $v_{3 t}, v_{3 t+1}, \ldots, v_{n-3}$ and $w_{3 t}, w_{3 t+1}, \ldots, w_{n-3}$. Finally, we assign the labels $2,4,2,4$ to the vertices $v_{n-2}, v_{n-1}, w_{n-2}$ and $w_{n-1}$ respectively. Clearly $t_{f}(1)=t_{f}(2)=t_{f}(3)=8 t-2$ and $t_{f}(4)=8 t-1$.
Case 2. $n \equiv 1(\bmod 4)$.
Let $n=4 t+1, t \in \mathbb{N}$. As in case 1 , assign the label to the vertices $u_{i}(1 \leq i \leq$ $n-3)$, $v_{i}(1 \leq i \leq n-1)$ and $w_{i}(1 \leq i \leq n-3)$. Next we assign the labels $2,3,4,2,3$ respectively to the vertices $u_{n-2}, u_{n-1}, u_{n}, w_{n-2}$ and $w_{n-1}$. Here $t_{f}(1)=t_{f}(3)=t_{f}(4)=8 t$ and $t_{f}(2)=8 t+1$.
Case 3. $n \equiv 2(\bmod 4)$.
Let $n=4 t+2, t \in \mathbb{N}$. As in case 2, assign the label to the vertices $u_{i}(1 \leq i \leq$ $n-4), v_{i}(1 \leq i \leq n-4)$ and $w_{i}(1 \leq i \leq n-2)$. Now we assign the labels $2,3,3,4,4,4,4,2$ to the vertices $u_{n-3}, u_{n-2}, u_{n-1}, u_{n}, v_{n-3}, v_{n-2}, v_{n-1}$ and $w_{n-1}$ respectively. It is easy to verify that $t_{f}(1)=t_{f}(2)=t_{f}(4)=8 t+2$ and $t_{f}(3)=8 t+3$.
Case 4. $n \equiv 3(\bmod 4)$.
Let $n=4 t+3, t \in \mathbb{N}$. As in case 3 , assign the label to the vertices $u_{i}(1 \leq i \leq$ $n-1)$, $v_{i}(1 \leq i \leq n-2)$ and $w_{i}(1 \leq i \leq n-2)$. Finally we assign the labels $4,3,2$ respectively to the vertices $u_{n}, v_{n-1}$ and $w_{n-1}$. Clearly $t_{f}(1)=t_{f}(3)=$ $t_{f}(4)=8 t+4$ and $t_{f}(2)=8 t+5$.

Theorem 2.7. The friendship graph $C_{3}^{(t)}$ is 4-total prime cordial iff $t \equiv 0,1,2$ $(\bmod 4)$.

Proof. Let $V\left(C_{3}^{(t)}\right)=\left\{u, u_{i}, v_{i}: 1 \leq i \leq n\right\}$ and $E\left(C_{3}^{(t)}\right)=\left\{u u_{i}, u v_{i}, u_{i} v_{i}: 1 \leq\right.$ $i \leq n\}$.
Case 1. $t \equiv 0(\bmod 4)$.
Let $t=4 m, m>4 m \in \mathbb{N}$.
Subcase 1. m is even.
Assign the label 4 to the central vertex $u$. Next assign the label 4 to the vertex $u_{1}, u_{2}, \ldots, u_{m}$ and $v_{1}, v_{2}, \ldots, v_{m}$. Next assign 2 to the vertices $u_{m+1}, u_{m+2}, \ldots, u_{2 m}$ and $v_{m+1}, v_{m+2}, \ldots, v_{2 m}$. Next assign the label 3 to the vertices $u_{2 m+1}, u_{2 m+2}, \ldots$, $u_{\frac{7 m}{2}}, \ldots, u_{4 m}$ and $v_{2 m+1}, v_{2 m+2}, \ldots, v_{\frac{7 m}{2}}$. Finally assign the label 1 to the remaining vertices.

Subcase 2. $m$ is odd.
As in subcase 1 , assign the label to the vertices $u_{i}, v_{i}\left(1 \leq i \leq \frac{7 m+1}{2}\right)$. Next assign the label 3 to the vertices $v_{\frac{7 m+1}{2}}, \ldots, v_{4 m}$ and assign the label 1 to the remaining vertices. Clearly $t_{f}(1)=t_{f}{ }^{2}(2)=t_{f}(3)=5 m$ and $t_{f}(4)=5 m+1$.
Case 2. $t \equiv 1(\bmod 4)$.
Let $t=4 m+1, m>4 m \in \mathbb{N}$. As in case 1 , assign the label to the vertices $u_{i}, v_{i}(1 \leq i \leq 4 m-1)$. Finally assign the labels 3,2 to the vertices $u_{n}$ and $v_{n}$ respectively. Clearly $t_{f}(1)=t_{f}(2)=5 m+2 t_{f}(3)=t_{f}(4)=5 m+1$.
Case 3. $t \equiv 2(\bmod 4)$.
Let $t=4 m+2, m>4 m \in \mathbb{N}$. Assign the label to the vertices $u_{i}, v_{i}(1 \leq$ $i \leq 4 m-2$ ) by case 1 . Next we assign the labels $4,2,3,3$ respectively to the vertices $u_{4 m-1}, v_{4 m-1}, u_{4 m}$ and $v_{4 m}$. It is clear that $t_{f}(1)=5 m+2$ and $t_{f}(2)=t_{f}(3)=t_{f}(4)=5 m+3$.
Case 4. $t \equiv 3(\bmod 4)$.
Let $t=4 m+2, m>4 m \in \mathbb{N}$. Clearly $\left|V\left(c_{3}^{(t)}\right)\right|+\left|E\left(c_{3}^{(t)}\right)\right|=20 m+16$. Suppose
f is a 4 -total prime cordial of $C_{3}^{(t)}$.
Subcase 1. $f(u)=1$ or 3 .
In this case, either $t_{f}(2)<5 m+4$ or $t_{f}(4)<5 m+4$.
Subcase 2. $f(u)=4$.
To get edge label 3,3 should be labelled to the adjacent vertices. So far the maximum possibility 3 is the label of the adjacent vertices. To get the edge label 2 , either 2 is labelled to the adjacent vertices or 2 and 4 are labelled on the adjacent vertices. Thus for the maximum possibility of 4 is the labels of the adjacent vertices and 2 is the labels of the adjacent vertices. But in this case $t_{f}(4)>5 m+4$, a contradiction.
Subcase 3. $f(u)=2$.
Similar to subcase 2.
Case 2. $t=2,4,5,6,8$.
A 4-total prime cordial labeling is given in Table 1.

| n | 2 | 4 | 5 | 6 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $u$ | 4 | 2 | 2 | 2 | 2 |
| $u_{1}$ | 4 | 4 | 4 | 4 | 4 |
| $u_{2}$ | 2 | 4 | 4 | 4 | 4 |
| $u_{3}$ | 3 | 4 | 4 | 4 | 4 |
| $u_{4}$ | 3 | 3 | 4 | 4 | 4 |
| $u_{5}$ |  | 3 | 3 | 3 | 4 |
| $u_{6}$ |  | 3 | 3 | 3 | 4 |
| $u_{7}$ |  | 4 | 3 | 3 | 3 |
| $u_{8}$ |  | 3 | 3 | 3 | 3 |
| $u_{9}$ |  |  | 2 | 2 | 3 |
| $u_{10}$ |  |  | 1 | 3 | 3 |


| $u_{11}$ |  |  |  | 4 | 3 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $u_{12}$ |  |  |  | 3 | 3 |
| $u_{13}$ |  |  |  |  | 4 |
| $u_{14}$ |  |  |  |  | 3 |
| $u_{15}$ |  |  |  |  | 2 |
| $u_{16}$ |  |  |  |  | 1 |

Table 1:

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