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Working with Diverse Learners in Mathematics Classrooms: An Analysis of Elementary Pre-service Teachers' Perceptions Reflected in Lesson Planning

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While the field of mathematics education strives to promote equitable mathematics learning and identifies it as a core instructional practice, less is known about its effective enactment. As teachers' teaching practices are dependent on their views and beliefs, this study investigated 133 elementary pre-service teachers' (PSTs') interpretations of diverse learners' learning experiences and proposed accommodations for them as reflected in their lesson planning process. Findings showed that PSTs came up with some strategies that are often suggested in teacher education literature, such as using multiple modes of representation and various grouping strategies. However, their responses were generic in nature rather than specific to diverse learners. Also, it was noted that many PSTs' interchangeably referred to the English Language Learners (ELLs), struggling learners, and culturally diverse learners, inferring that they thought that culturally diverse students must have been ELLs and that ELLs or culturally diverse students must have been weaker students in math. We found that the PSTs used their own frames while filtering and discarding information about diverse student populations to develop instructional plans, rather than based on the results of assessments of learning. We suggest that it is the critical first step to unwrap PSTs' unproven assumptions to better equip them for working with all of their future students.

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I. INTRODUCTION

Around the world, there has been a continuous and increasing need to ensure equitable educational opportunities for diverse student populations. In the case of the United States, where the diversity is inherent and apparent, many policy documents and recommendations from professional organizations have worked to promote access and attainment for all students in mathematics education (e.g., National Council of Teachers

of Mathematics [NCTM], 2000, 2014). National educational organizations for teacher education programs in the US (e.g., Council of the Accreditation of Educator Preparation Standards, Interstate Teacher Assessment and Support Consortium Standards) also have standards for focusing on teacher preparation to include skills that enable pre-service teachers (PSTs) to work effectively with diverse learners such as those from varied cultural, religious, and socioeconomic backgrounds, English Language Learners, academically lower and higher achieving students, and those with exceptionalities. Populations in the US are becoming more diverse, but more than 80% of the people who teach children belong to the "White" category (U.S. Department of Education, 2016). This figure draws concern since it is not in keeping with the racial makeup of the United States. According to the US Bureau of Labor Statistics, teachers are also primarily female. This discrepancy between the number of diverse students and the number of diverse teachers proposes "new calls for research to explore the attitude, knowledge, and perceptions of PSTs about culturally, linguistically, and underrepresented minority students" (Taylor, Kum-Yeboah, & Ringlaben, 2016, p. 42). In addition, students are also diverse in academic learning potential, disabilities/special education, exceptionalities and motivation/attitudes. These are also areas of diversity that require teachers' skill to work with these groups effectively.

While the field of mathematics education strives to promote equitable mathematics learning and identifies it as a core instructional practice, less is known about its effective enactment. As opposed to some teaching practices that rely on detailed prescribed guidance from a published work, effectively working with diverse learners requires teachers' adaptive expertise and flexibility in structuring rich learning activities. Although the terms of equity and diversity are familiar to many of us, each individual may have a different set of assumptions, resulting in the "easy to say but hard to do" situation.

In this regard, our goal in this article is to study elementary PSTs' interpretations of diverse learners' learning experiences and proposed accommodations for them as reflected in their lesson planning process. More specifically, our study is guided by the following question: What considerations appear in elementary PSTs' mathematics lesson planning with respect to working with diverse learners?

II. RELATED LITERATURE

1. VIEWS ON MATH INSTRUCTION FOR DIVERSE POPULATIONS

As diverse student populations continuously increase, there is a call for mathematics educators to prepare new educators that effectively work with these student populations

as well. Many individual researchers have addressed the need to further push for individual students' statuses and cultural resources to be recognized in learning. For example, Gutierrez (2007, 2012) identified available learning resources (access), visible results (achievement), linguistic, personal, or cultural aptitudes (identity), and issues related to social change (power) as four dimensions of equitable opportunities to learn mathematics by emphasizing that these dimensions should underlay instructional practices (Gutierrez, 2012). As such, Boaler (2008) addressed the importance of identifying students' mathematical contributions from diverse cultural and academic backgrounds. Hand (2012) also highlighted the importance of being inclusive of students from diverse backgrounds. Along with these guidelines and studies, it is considered one of the fundamental capabilities of teachers, which are called "high-leverage practices," to learn about "students' cultural, religious, family, intellectual, and personal experiences and resources for use in instruction" (TeachingWorks, n.d.).

Although many teachers are aware of students' individual needs, many teachers may not have enough knowledge to put their awareness and commitment into their day-to-day practice (Brevik, Gunnulfson & Renzulli, 2018). Dee (2011) cites several research studies that also support this conclusion that teachers lack the training, skills, and experience necessary to be inclusive in instruction and states that preparation in inclusion for general education teacher programs remains deficient.

Some studies address a prevalent assumption underlying this situation; considering mathematics as a neutral and universal subject. Jackson, Taylor and Buchheister (2018) found that many math teachers and PSTs do not attend to equity issues as relevant because they view mathematics as a culture-free, neutral subject. To address this issue, the NCTM (2014) says it is critical for teacher educators "to promote meaningful learning experiences by emphasizing explicit strategies for connecting mathematical concepts to students' lived experiences and the mathematical content addressed in the classroom setting, thus contributing to meaningful, high-quality instruction" (Jackson, et al., pp. 264-265). Leonard, Brooks, Barnes-Johnson, and Berry (2010) also asserted that mathematics is not neutral in their study of race and mathematics teaching; rather, "Mathematics, like all other forms of knowledge is situated within a cultural context" (p.262). Although educators voice that mathematics can be used to develop both individual and social agency through meaningful engagement with real-world problems and can motivate diverse students to use math in "emancipatory" ways (Leonard et al., 2010, p.268), it seems that a gap still exists between the ideal status of inclusive mathematics education and the enactment of it at the classroom level. Teachers' own views on mathematics and mathematics instruction may explain the reason for the gap.

2. STUDIES ON ENACTMENT OF DIFFERENTIATION AND CULTURALLY RESPONSIVE TEACHING

Although scarce, there are several studies that investigate the enactment of differentiation and culturally responsive teaching by PSTs in various areas and with different types of learners. De Araujo, Smith, I and Sakow (2018) examined the instructional strategies used by PSTs when implementing cognitively demanding mathematics tasks with English Language Learners (ELLs). The PSTs predominantly used four categories of strategies: 1) using multiple modes of communication that allowed students to communicate meaning and thinking in ways that encouraged non-verbal or native language communication, 2) planned use of visual supports, 3) asking pressing questions to elicit further student thinking, and 4) checks for understanding not only about the math but about the language and instruction. The researchers concluded that the supports for the ELLs were most often for language rather than math, which resulted in the teachers doing the mathematical thinking rather than the children thinking mathematically themselves. One finding was that the PSTs drew on these strategies despite having no explicit instruction about them related to mathematics education of ELLs.

In another study of learning to teach English Language Learners, Kelly (2017) asked PSTs in a field experience to draw pre and post course pictures depicting a teacher teaching ELLs. Most drawings showed a teacher using direct instruction at the front of the room and passive students listening. There was little difference in the before and after content pictures. This result calls for teacher educators' explicit attention to PSTs' beliefs, which may be one of the most influential factors in how PSTs adapt lessons. Kelly (2017) further noted, "Presumably when teachers begin teaching, they provide instruction congruent with their beliefs about teaching, learning, and their student population… research has identified that many teachers hold inaccurate views about language learning and biased views toward language learners" (p. 110).

Akiba's (2011) review of a more general multicultural awareness beyond English Language learners shows several other studies have investigated the awareness of PSTs at the time they begin teacher education. Akiba bases the entry point awareness on students' own demographic characteristics, families, and prior experiences related to diversity (e.g. travel, friends). The author states that it is important to consider how these factors may affect how and to what extent university courses can influence PSTs' development of diversity beliefs. Mills and Ballantyne (2010) found several studies that suggest that stand alone courses in diversity often do not impact development but just reinforce PSTs' existing beliefs.

With exceptional student populations, Dee (2011) analyzed types of accommodations in units of instruction employed with students who had Individualized Education Programs (usually based on exceptionalities). The PSTs used grouping/partner work, teacher/student one-on-one work, seating arrangements, repetition of directions, giving extra time, and giving less work as strategies for these students. However, the data did not show any strategies that reveal PSTs' insight into the specific learning needs of students. They note that PSTs have "an undeveloped or inaccurate understanding of special education and its terminology" (Dee, 2011, p. 63). Interesting is that more than 70% of the learning units written by the PSTs specifically stated that services were provided outside the general education classroom, but no plans referred to collaboration with any special teacher. A study of differentiation practices by Brevik et al. (2017) used focus groups to ask over 300 Norwegian student teachers about their confidence in enacting differentiation strategies with high-achieving students. Though the student teachers "suggested that although they seemed confident about 'who' the high-potential students were, 'what' their traits and needs might be, and 'why' they should plan and enact differentiated instruction for these students, they were uncertain about the 'how'" (Brevik et al, 2017, p. 41). We question if this may be true for PSTs working with any group. Figuring out "how" to differentiate strategies for any diverse group seems to be a deficient skill in teacher candidates.

Increasingly diverse populations require teachers to have the skills to create, adapt and modify lessons and strategies so that all students learn to their highest potentials. This means not only understanding differences and effectively assessing needs of students but having the knowledge and skills to act on and with this information. They must be prepared to use knowledge and strategies about diverse populations, pedagogy, and content to teach equitably. Research suggests that this needs to be more explicitly taught in teacher education programs, but that teaching may or may not have a large effect due to PSTs' prior beliefs. The literature reviewed has explored PSTs' thoughts about teaching students in diverse populations and offer some insights into how we may better prepare students to become teachers of diverse learners.

III. RESEARCH METHODOLOGY

1. PARTICIPANTS AND CONTEXT

This study involved 133 PSTs enrolled in a required elementary mathematics methods course at a Midwestern university in the United States over two years. Participants were predominantly White females. Male participants or non-White populations were less than

10% of the total participants. All participants were pursuing their initial elementary teaching certifications. As part of their elementary education major requirements, they had taken several education courses prior to this methods course, including educational foundations, classroom management and diversity, instructional design and assessment, and educational technology. PSTs were required to have some field experiences throughout the program at local schools for participatory observation and limited levels of instruction under the supervision of cooperating teachers. One of the authors was the instructor of this methods course. The other author was the instructor of the instructional design and assessment course.

2. TASKS AND DATA COLLECTION

As a part of course activities, PSTs engaged in lesson planning activities using the Thinking Through a Lesson Protocol (TTLP) developed by Smith, Bill, and Hughes (2008). The TTLP was originally designed to help teachers implement tasks that help students engage in high-level thinking. According to Smith et al. (2008), it "provides a framework for developing lessons that use students' mathematical thinking as the critical ingredient in developing their understanding of key disciplinary ideas" (p. 4) and intends to help teachers anticipate what students will do and generate good questions prior to the lesson being taught. The main components of the TTLP are briefly outlined in Table 1.

Table 1. Thinking Through a Lesson Protocol (Smith, Bill, & Hughes, 2008)

Part/Descriptor	Selected examples of related prompts
Part 1. Selecting and setting up a mathematical Task: This part lays the groundwork for subsequent planning by asking the teacher to identify the mathematical goals for the lesson and set expectations regarding how students will work.	• What are your mathematical goals for the lesson? • In what ways does the task build on students' previous knowledge, life experiences, and culture? • ***What particular challenges might the task present to struggling students or students who are English Language Learners (ELL)? How will you address these challenges?*** • How will you introduce students to the activity so as to provide access to all students while maintaining the cognitive demands of the task?
Part 2. Supporting students' exploration of the task: This part focuses on monitoring students as they explore the task (individually or in small groups).	• As students work independently or in small groups, what questions will you ask? • How will you ensure that students remain engaged in the task? • How will you orchestrate the class discussion so that you accomplish your mathematical goals?
Part 3. Sharing and discussing the task: This part focuses on orchestrating a whole-group discussion of the task that uses the different solution strategies produced by students to highlight the mathematical ideas that are the focus of the lesson.	• How will you orchestrate the class discussion so that you accomplish your mathematical goals? • How will you ensure that each student has the opportunity to share his or her thinking and reasoning with their peers?

Using already existing lesson ideas, PSTs were asked to respond to each prompt in the TTLP. The following were original lesson ideas taken from an elementary textbook series (Elsbury & Roberts, 2017, *Bridges in Mathematics*): Fractions on a Number Line, Fractions on a Ruler, Egg Carton Fractions, Equivalent Egg Carton Fractions, and Comparing, Adding and Subtracting Fractions.

For the purpose of this study, we collected PSTs' responses from Part 1: What particular challenges might the task present to struggling students or students who are English Language Learners (ELL)? How will you address these challenges? While some discussion was made prior to this activity (e.g., questioning techniques, talk moves, etc.), we did not specifically discuss this specific prompt in order for PSTs to fully express their own perceptions without the instructor's influence.

3. DATA ANALYSIS

We analyzed the data both qualitatively and quantitatively. All PSTs' responses for the TTLP prompt were analyzed using the inductive content analysis approach by developing data-driven codes (DeCuir-Gunby, Marshall, & McCulloch, 2011; Grbich, 2007). Multiple stages of analysis were implemented to accomplish the open-coding process. The initial stage involved low-level coding by focusing on pulling out the themes explicitly presented by the PSTs (Carspecken, 1996). These initial raw codes were used to include the participants' opinions as they are voiced. Following open coding, responses were analyzed again using high-level, theoretical codes that require investigators' informed abstraction and interpretation (Carspecken, 1996). Two researchers independently read all PSTs' responses and compared the codes and emerging themes to secure a consensus among categories and their definitions. Once the themes for coding were identified, two researchers jointly coded the data. When disagreement arose, two researchers jointly discussed until the agreement was reached. After coding was completed, the frequencies of coded themes were identified.

IV. FINDINGS

This section summarizes PSTs' perceived anticipations of diverse students' challenges in mathematics classrooms and their suggestions for instructional strategies to address the anticipated challenges. In the following sub-sections, frequencies are reported to show the overall tendencies.

1. PSTs' ANTICIPATIONS OF DIVERSE LEARNERS' CHALLENGES

Because the given question in the TTLP specifically asked to consider three types of diverse learners, we first report how PSTs considered students' challenges using these categories. We then report on other areas of consideration made by PSTs as shown in Table 2.

Table 2. Anticipated challenges of diverse learners considered by PSTs

Main Category (frequency)[a]	Subcategory (frequency)[b]	Example
Struggling students 36 statements (32%)	Complexity of the mathematical task (56%)	"For students who are already struggling with basic fraction concepts, having them to compare fractions with different denominators can be overwhelming…"
	Weak understanding of math terms/ vocabulary/ definition (11%)	"Some challenges…are interpreting and understanding the question being asked, understanding words and defining them…"
	Lack of (or no) prior mathematics knowledge (5%)	"They may not have required prior knowledge."
	Lack of independent problem-solving skills (5%)	"They might not be ready to independently work on marking fractions on a number line."
	Low reading level (3%)	"Student might have trouble with recognizing the numbers or reading the problems…"
	Other types of struggles (e.g., physical disabilities, learning disabilities) (19%)	"…learners with disabilities might find this activity more difficult."
English language learners 63 statements (55%)	Challenges in understanding vocabulary and instruction (87%)	"If they are ELL then they might have an even more difficult time because they might not understand the instructions, they might have trouble understanding the words used as in equivalent, compare, especially when they have to work just with numbers."
	Challenges in understanding mathematical representations (e.g., charts, graphs, manipulatives) (2%)	"The charts, graphs or manipulatives that we use to complete the problems might cause challenges for those who are ELL…"
	Challenges in understanding contexts (e.g., American currency, non-metric system, etc.) (11%)	"The biggest challenge the task presents to ELL or culturally diverse learners is that this lesson involves understanding the value of American currency…"
Culturally diverse learners 15 statements (13%)	Challenges in understanding contexts (87%)	"Culturally diverse students, depending on their culture and familiarity with American culture, might have no context for what licorice is and why we would want to share it with friends."
	Unfamiliar instructional style (7%)	"…certain math concepts may be taught in a way that is not "normal" for them compared to what they are told at home."
	Cultural conventions, religious rules (7%)	"Certain lessons may require them to do things that their religion or culture does not allow."

[a] n = 114 statements. Some PSTs addressed multiple aspects and strategies and their work was coded into multiple categories. Also, some PSTs did not address anticipated challenges and directly address what they could offer for those students. This frequency is based on the total number of statements from PSTs. [b]This frequency is based on the total number of PSTs' statements that mentioned the main category. (e.g., 56% of 36 statements that mentioned struggling students considered the complexity of the mathematical task was the reason for students' difficulties.)

When considering culturally diverse learners, the most frequently anticipated challenge was being unable to understand the problem contexts. However, just like the tendency showed in anticipating ELL students' challenges, there was a wide range of expectations regarding this. The following PSTs' statements implicitly exhibited their portrayal of diverse learners: "Students may not understand what the ice cream cone is because they have never had it in a cone or because they only ever had shaved ice."; "If the student has never eaten or seen eggs before, this may serve as a problem because the problem context will no longer be relatable to the student." Also, in some cases, PSTs equated culturally diverse learners to ELL students as shown in the following statement: "Culturally diverse learners may have a problem saying or learning some of the vocabulary words and they might even have a hard time learning the topic or concept if there is a language barrier."

2. PSTs' SUGGESTED STATEGIES TO ADDRESS THE CHALLENGES OF DIVERSE LEARNERS

When asked about suggested strategies for diverse learners, PSTs suggested modifications of their lesson ideas considering the challenges of their diverse students as described in the previous section. However, some PSTs suggested strategies without explicitly commenting on the aforementioned categories. Table 3 summarizes what types of modifications of lessons PSTs thought of.

In addition to the main categories summarized above, PSTs referred to different groups of students including advanced students and general groups of students with no specifications. Five PSTs addressed strategies that would be helpful in assisting advanced students in mathematics by offering more challenging and abstract level tasks. Twenty-two PSTs proposed some general strategies that could be applicable for any student (e.g., use of various modes of representation, cooperative learning, interacting with parents, etc.).

It is notable that there were seven statements, which are not included in Table 2, which considered mathematics as a universal language. Thus, there would not be as many challenges in this subject as compared to other areas. Also, there were some responses where PSTs did not expect any challenges due to the nature of specific lesson content or format. Thus, they thought that there would be no additional accommodations needed. These responses were noted as "none" in Table 3.

Table 3. Suggested strategies for diverse learners considered by PSTs

Main Category (Frequency)[a]	Subcategory (Frequency)[b]	Example
Struggling students 62 statements (27%)	Partnering with an advanced student (19%)	"Teachers may place struggling students by "star" students who understand math and would be willing to help."
	Using various representations (e.g., manipulatives, models, visuals) (18%)	"…provide real life, tangible manipulatives/ representation"
	One-on-one assistance by teacher (13%)	"I [teacher] would work with them one-on-one when the individual task begins."
	Offering accommodations for physical disabilities (10%)	"For students that may have a physical disability, I will pre-cut their fraction strips for them."
	Small group work with a teacher or adult helper (8%)	"I may create math workshops where I could work with a group of struggling students.."
	Providing easier work or less work (6%)	"…they [struggling students] could do a smaller portion of the assignment."
	Using real-world examples (5%)	"Equivalency could be shown using candy bars (provide a real-life example)."
	Giving a completed model/example (3%)	"Have them be able to reference off a modeled completed sheet of different ways the fractions may be represented…"
	Offering extended time (2%)	"I may give extra time in class for them to ask me questions or complete the work."
	None (2%)	
English language learners 147 statements (64%)	Offering various representations (27%)	"ELL students might need more visuals…"
	Having the student work with another adult (18%)	"They may be accompanied by a bilingual specialist."
	Paring up or grouping with another student or group of students (same native speakers or other fluent students) (14%)	"It could also help to partner this student strategically with a native speaker who is patient and willing to guide his or her partner through the activity."
	Translating materials into students' native languages (13%)	"Worksheets can be translated into the language known by that student."
	Offering technology tools for videos (8%)	"…show videos… so that they can see how each item is used."
	Offering additional resources (e.g., definitions, flashcards, vocab list, etc.) (8%)	"Give them flashcards for each mathematical word that is used in the task."
	Other ideas (e.g., the teacher proximity, allowing various modes of communication) (2%)	"If a student is struggling with writing in English…they may be able to complete the task by telling what they think orally to an aid or the teacher…"
Culturally diverse learners 21 statements (9%)	Using examples from students' own culture to explain (33%)	"Change the "licorice rope" in the activity to something that the student would be more familiar with (something that is prevalent in that student's culture)."
	Teaching the U.S. contexts (e.g., measurement system, currency) (33%)	"If there are students who are from another place that measures with the metric system, the teacher should take the time to teach the differences to the students."
	Offering manipulatives (5%)	"Culturally diverse learners…need to use other manipulatives to help them with this task."
	Becoming familiar with cultural needs; Learning about students' cultures (14%)	"I would learn about my student's culture. Understanding the culture can inform me on their perceptions and values."
	Adapting instructional methods from other countries (5%)	"Teachers learn how math is done in other countries and adapt those methods…"
	Consulting with parents (5%)	"It is most beneficial to converse with the parents and come up with a mutual understanding and decide on boundaries to follow."
	None (5%)	

[a] n = 230 statements. Some PSTs addressed multiple aspects and strategies and their work was coded into multiple categories. Also, some PSTs did not address anticipated challenges and directly address what they could offer for those students. This frequency is based on the total number of statements from PSTs. [b]This frequency is based on the total number of statements that mentioned the main category. (e.g., 19% of 62 statements that mentioned struggling students suggested partnering with an advanced student to help struggling students.

V. DISCUSSION AND IMPLICATIONS

As De Araujo et al. (2018) found in their study, although PSTs in this study did not have specific instructional activities designed to teach mathematics to diverse students, they were able to come up with strategies that are often suggested in teacher education literature. In particular, the use of multiple modes of representation and various grouping strategies were popular responses from the PSTs across different types of learners. However, we found that PSTs' responses were generic in nature rather than specific to diverse learners. As Brevik et al. (2017) discussed, student teachers can identify learners in need of lesson modification, however, figuring out how to actually modify effectively is a deficient skill for PSTs. Here, we discuss some areas of concerns and areas that warrant further probing. In addition, we will briefly discuss the implications for teacher educators.

For struggling students, most PSTs interpreted them as low-achieving students' in mathematics. A small number of PSTs noted other causes of struggles such as various types of disabilities. For this group of students, one of the popular strategies was partnering them with advanced students or having them working with small groups under adult guidance, which were also common modifications in Dee's 2011 study. Our PSTs seemed to suggest that being together with high-achieving students or others might offer a better understanding. However, there is no guarantee that the weaker student will benefit and this setting may reinforce the weaker student to take on a more passive role as a learner (Chapin, O'Connor, Anderson, 2013). Effective teaching of mathematics provides all students the opportunity to engage in productive struggle (NCTM, 2014), it would also be important to gauge whether the grouping strategies would benefit all (both the lower level and the higher level) students involved or not.

Overall, ELLs were the students that PSTs anticipated the most challenges from and for which they proposed a high number of strategies. It is notable that many PSTs' statements interchangeably used the ELLs, struggling students, and culturally diverse students. These statements assumed that culturally diverse students must have been ELLs and that ELLs or culturally diverse students must have been weaker students in math. For example, the following statement is an example of equating these populations: "Culturally diverse learners may have a problem saying or learning some of the vocabulary words and they might even have a hard time learning the topic or concept if there is a language barrier." We were concerned about this type of assumption. Each individual ELL or culturally diverse student may have different mathematical backgrounds. However, we found no PSTs' suggestions for the diagnostic assessment as an effort to provide appropriate intervention and were concerned about these types of beliefs PSTs held. This finding seems to reinforce Kelly's (2017) assertion that "many

teachers hold inaccurate views about language learning and biased views toward language learners" (p.110).

For the culturally diverse students, it was interesting to see that two main strategies proposed by PSTs were either using examples from students' own culture or teaching the U.S. context more explicitly. We found similar assumptions as shown in the cases of ELLs while PSTs talked about culturally diverse learners. With a small number of exceptions, the majority of PSTs considered these students as being unfamiliar with the mainstream culture or daily life situations, and some of the examples made us wonder if they were talking about very extreme cases only (e.g., how many students would not know what the egg carton is, what pizza is, etc.). We were concerned about this level of understanding. Akiba (2011) suggests that a PSTs' own demographics and prior experiences with diverse groups and situations impact the beginning point into and development of diversity beliefs. These few extreme case comments have us pondering if some PSTs have no diversity experiences at all or what entry point perceptions would prompt them to think of these extreme possible causes.

At the day-to-day instructional practice level, such as lesson planning, working with diverse student populations means that teachers and teacher candidates should be prepared to create lessons with appropriate attention to learner needs and differences by making adaptations, modifications, and accommodations necessary to address the needs of all children. These adaptations should be based on the results of assessments of learning as well as a deep understanding of the individual child and his/her community. However, our study suggests PSTs do not consider this process as a critical step. None of the PSTs suggested providing necessary modifications/accommodations based on the assessment. Rather than informed by data, many of the instructional strategies PSTs proposed are based on their assumptions. We found that it is critical to unwrap their unproven assumptions first.

We admit there are some limitations to this study. Since the goal of this study was to identify PSTs' perceptions, we did not attempt to provide interventions through formal instruction or training on effective mathematics pedagogical strategies for diverse learners. The absence of intervention, lack of specifications in the questions asked, and absence of the enactment with real students may be considered limitations of this study. Also, it is plausible that each individual PSTs' own personal backgrounds and limited levels of experiences in field placement settings might have played out as completely different frames to see the proposed question.

In spite of these limitations, this study contributes to the ongoing effort to probe PSTs' optimal learning opportunities in teacher education programs. In particular, this study encouraged us, who interacted with PSTs at the different stages of the program, to talk to each other. We (a math methods course instructor and an instructional design and

assessment course instructor) found that we both utilized lesson planning activities in the courses but addressed the issue of working with diverse students in a very general way. The math methods course instructor assumed that this topic must have been fully addressed in the instructional design course. The instructional design course instructor assumed that the more math-specific strategies with diverse learners would be discussed in the math methods course. We found that, just like the PSTs' unproven assumptions on diverse learners, we also have many unproven assumptions about PSTs' perceptions and prior experiences. Noting these issues, the authors' program is currently restructuring the program to provide better learning experiences using practice-based techniques and cohesive learning trajectories woven through all the professional courses to our PSTs. One consideration in this process is how we can help our PSTs get more experiences with diverse learners across the program with guided practice and how this effort can help our PSTs equip better to facilitate the development of mathematical learning goals for all students.

In this study, we presented the case of American elementary PSTs. As PSTs' perceptions and beliefs about equitable mathematics instruction may be dependent on their cultural and educational contexts (e.g., Lee, Kim, Lim & Kim, 2018), we suggest the similar inquiry presented in this paper to be probed in other international contexts in the future.

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