Control of Seesaw balancing using decision boundary based on classification method

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Abstract

One of the key objectives of control systems is to maintain a system in a specific stable state. To achieve this goal, a variety of control techniques can be used and it is often uses a feedback control method. As known this kind of control methods requires mathematical model of the system. This article presents seesaw unstable system with two propellers which are controlled without use of a mathematical model instead. The goal was to control it using training data. For system control we use a logistic regression technique which is one of machine learning method. We tested our controller on the real model created in our laboratory and the experimental results show that instability of the seesaw system can be fixed at a given angle using the decision boundary estimated from the classification method. The results show that this control method for structural equilibrium can be used with relatively more accuracy of the decision boundary.

Keywords: logistic regression, brushless motor, gradient, cost function, machine learning

1. Introduction

Machine Learning, a branch of artificial intelligence, is a scientific discipline that concerned with the design and development of algorithms that allow computers to evolve behaviors based on empirical data. The main purpose of machine learning is to learn automatically and take intelligent decisions based on collected data [1, 2, 3 and 17]. In general, any machine learning problem can be assigned to supervised and unsupervised learning. Supervised learning is the machine learning task of learning a function that maps an input to an output based on example input-output pairs [4, 5]. Supervised learning problems are categorized into "regression" and "classification" problems. In a regression problem, we are trying to predict results within a continuous output, meaning that we are trying to map input variables to some continuous function. In a classification problem, we are instead trying to predict results in a discrete output. In other words, we are trying to map input variables into discrete categories [12]. In the previous researches, we have successfully tested PID and state feedback controls to maintain stability of a two propeller seesaw. Brief intro to these experiments is described below. In order to apply the above control methods, it is necessary to define the dynamic and kinematic models of the system experimental model of which is shown in Fig.1. Here, $l_1$, $l_2$ represent distances of brushless motor from pivot center and $\psi$ represents Euler angle about $x$ body axis, $m_1$, $m_2$ are mass of brushless DC motors with propeller that is fixed at the end of the lever. $F_1$, $F_2$
represent thrust forces produced by brushless DC powered propeller motor, $g$ is the acceleration due to gravity [1-11].

After applied voltage, propellers spin and generate torque to pull up the seesaw. The torque is caused a sum of the forces tangential components to the rotating multiplied with corresponding distances from the pivot point. Neglecting the frictions and the effect of body moments on the translational dynamics, an expression of forces acting on the seesaw according to Newton’s laws is derived as:

$$\ddot{\Psi} = \frac{l_2}{l_1} F_2 - \frac{l_1}{l_1} F_1 - \frac{l_1 m_1 - l_2 m_2}{l_m} g \cos \psi$$  \hspace{1cm} (2)

Here, $\ddot{\Psi}$ is angular acceleration. In order to use the PID or State feedback control to balance the above system, the length of levers ($l_1, l_2$), weights ($m_1, m_2$), lifting forces ($F_1, F_2$) should be precisely defined.

In this research work presented by the article we did not use dynamic and kinematic models of the object. Instead, using the input and output values of this system we build real-time control system based on supervised machine learning algorithm using classification and tested on the microcontroller. Fig. 2 shows real model of the seesaw equipped by propellers.
2. Classifier and decision boundary

The main idea of machine learning is to evaluate the function from collected training data [17]. The purpose of our study is to maintain the stability of a two propeller seesaw on certain angle. We will focus on the binary classification of only two values of 0 and 1. To do this, data is collected by increasing speed of the motor controlled by PWM signal and generate lifting force. The controlled variable for this system is the angle $\psi$ of the seesaw relatively to the horizontal axis and the manipulating variable is rotation speed given to motorized propeller. Rotation speed PWM2 is recorded as training input $x_1$ and PWM1 force as $x_2$. We use $x(i)$ to denote the “input” variables (in our case is motors rotation speed), also called input features, and $y(i)$ to denote the “output” or target variable that we are trying to predict (angle). A pair $(x(i), y(i))$ is training example, and the dataset that will be used to learn a list of $m$ training examples $(x(i), y(i)); i=1,\ldots, m$-is called a training set.

Example of collected data is shown in Table 1.

| $x_1$ | 900 | 900 | 900 | 900 | 900 | 900 | 900 | 910 | 910 | 910 | 910 | 920 | 920 | 930 | 930 | 940 | 950 |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $x_2$ | 900 | 902 | 904 | 906 | 910 | 912 | 902 | 904 | 906 | 908 | 910 | 912 | 914 | 916 | 918 | 920 | 930 |
| $y$   | 0   | 0   | 0   | 0   | 0   | 0   | 0   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   |

We tried to stabilize the seesaw at three different angles (-10°, 0° and +10°). First experiment was to stabilize seesaw at angle -10°. If $\psi < -10°$ we recorded as $y=1$ and represented with red circle and if $\psi > -10°$, $y=0$ with blue circle. The relation of training data is illustrated in Figure 3a.

![Figure 3a](image-a)

![Figure 3b](image-b)
Using the classification method, it’s possible to define the line between two fields filled by colored circles. This line is the decision boundary which is used to stabilize the seesaw in a given angle. From the equation of this line we can estimate second motor rotation speed when first motor rotation speed is fixed.

3. Estimation of decision boundary

Figure 4 shows the machine learning algorithm. Collected training data set will be included in the training algorithm and the training algorithm is used to determine the hypothesis function $h$. From hypothesis function we can determine decision boundary for a given angle and rotational speed of one of the propellers is calculated (PWM).

To describe the supervised learning problem, our goal is, given a training set, to learn hypothesis function $h$: $X \rightarrow Y$ so that $h(x)$ is a “good” predictor for the corresponding value of $y$ [2, 12-17].

$$0 \leq h_\theta(x) \leq 1$$

Here we use sigmoid function to ensure the above conditions.

$$h_\theta(x) = g(\theta^T x)$$

$$z = \theta^T x = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$
\[ \bar{h}_\theta(x) = g(z) = \frac{1}{1 + e^{-x}} = \frac{1}{1 + e^{-\theta^T x}} \]  

(5)

The function \( g(z) \) converts any real number into \((0, 1)\) intervals which converts any values function into a more appropriate classification function. We can write \( h_\theta(x) \) as probability function (6).

\[ h_\theta(x) = P(y = 1|x; \theta) = 1 - P(y = 0|x; \theta) \]  

(6)

Here, \( h_\theta(x) \) – hypothesis function, \( \theta_0, \theta_1, \theta_2 \) – parameters.

To derive the discrete 0 or 1 class, we can convert the hypothesis function into the following:

\[ h_\theta(x) \geq 0.5 \text{ and } y = 1 \]
\[ h_\theta(x) < 0.5 \text{ and } y = 0 \]

The experiment was conducted to determine \( \theta_0, \theta_1, \theta_2 \) parameters which are estimated so that the line most closely aligned with the training data. To do this, the difference between hypothesis function and output value should be minimal.

\[ h_\theta(x^{(i)}) - y^i \]

The cost function of the linear regression cannot be used to classification tasks which creates a number of local minimums. In other words, this is not a convex function. The cost function \( J(\theta) \) of the system is determined by expression (7).

\[ J(\theta) = \frac{1}{m} \sum_{i=1}^{m} C(h_\theta(x^{(i)}), y^{(i)}) \]  

(7)

If \( y = 1 \), \( C(h_\theta(x^{(i)}), y^{(i)}) = -\log(h_\theta(x)) \)  

(8)

If \( y = 0 \), \( C(h_\theta(x^{(i)}), y^{(i)}) = -\log(1 - h_\theta(x)) \)  

(9)

We can write two conditional cases of the cost function in one case.

\[ J(\theta) = -\frac{1}{m} \sum_{i=1}^{m} [y^{(i)} \log(h_\theta(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_\theta(x^{(i)}))] \]  

(10)

Using gradient descent algorithm, we define the minimum value of the cost function parameters.

\[ \min_{\theta_0, \theta_1, \theta_2} J(\theta_0, \theta_1, \theta_2) \]  

(11)

First, \( \theta_0, \theta_1, \theta_2 \) parameters selected randomly. Usually \( \theta_0 \), \( \theta_1 \) and \( \theta_2 \) are chosen equal to zero and we change the value of the parameters be decreasing the value of the function \( J(\theta_0, \theta_1, \theta_2) \). It is described as a program algorithm,

\[ \text{Repeat until converge ()} \]
\[ \{\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1, \theta_2)\} \]

Here, \( j = 0, 1, 2 \) and \( \alpha \) is the learning rate. To find minimum value of parameters we take partial derivative from cost function \( \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1, \theta_2) \).
4. Implementation and Experimental results

The seesaw shown in Figure 1 swings between ±25º. We try to stabilize the seesaw in given angles -10º, 0º, and +10º and for this reason have been collected 500 training data in each learning process. The calculations were performed on the Atmega32 controller that is operated at 8 MHz with learning rate α=0.02. To reach cost function $J(\theta_0, \theta_1, \theta_2)=0.03$ it takes 3 hours. The following parameters were calculated as a result of the training.

**Table 2. Estimated parameters**

<table>
<thead>
<tr>
<th>Angle</th>
<th>$\theta_0$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10º</td>
<td>-1.6399</td>
<td>0.0109</td>
<td>-0.0085</td>
</tr>
<tr>
<td>0º</td>
<td>-2.6249</td>
<td>0.0107</td>
<td>-0.0073</td>
</tr>
<tr>
<td>+10º</td>
<td>-0.9186</td>
<td>0.0107</td>
<td>-0.0092</td>
</tr>
</tbody>
</table>

Hypothesis functions are:

- For angle -10º: $h_\theta(x) = -1.6399 + 0.0109x_1 - 0.0085x_2$
- For angle 0º: $h_\theta(x) = -2.6249 + 0.0107x_1 - 0.0072x_2$
- For angle +10º: $h_\theta(x) = -0.9186 + 0.0107x_1 - 0.0092x_2$

The hypothesis function is defined as $h_\theta(x) = 0.5$ then by selecting one of the propeller rotation speed $x_1$, speed value of the second propeller rotation is calculated from the decision boundary. Decision boundary at given angle -10º, speed of the second propeller can be estimated by the following formula:

$$x_2 = \frac{0.5 + 1.6399 - 0.0109x_1}{-0.0085}$$

Results of computed decision boundaries are illustrated by green dotted line in Figure 3. By experiment, we choose $x_1=930$ for the first propeller then $x_2$ calculated and the results are shown in Table 3.

**Table 3. Estimated second motor rotation speed from decision boundary**

<table>
<thead>
<tr>
<th>Angle</th>
<th>$x_1$</th>
<th>$x_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10º</td>
<td>930</td>
<td>940.8</td>
</tr>
<tr>
<td>0º</td>
<td>930</td>
<td>935.1</td>
</tr>
<tr>
<td>+10º</td>
<td>930</td>
<td>927.4</td>
</tr>
</tbody>
</table>

In Figure 5 shown experimental results of seesaw angle stabilization with 3000 samples, (one sample =5ms).
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5. Conclusion

Unstable seesaw system was trained by classification method. The advantage of this control is that we are trying to stabilize the seesaw basing on the training data only without modeling of the physical data of the system. However, there are disadvantages of classification methods. With the increasing number of training data, the calculation speed is decreased drastically. This control system acts like open loop system without any feedback therefore output value cannot be stabilized precisely. The system with learning algorithm can be stabilized in a given angle by re-learning with different voltage supply or with different weights.

Figure 5. Stabilization of seesaw in given angles. a - stabilization in given angle -10º, b - stabilization in given angle 0º, c- stabilization in given angle +10º.
References

[10] Tengis Tserendondog, Batmunkh Amar. “Quadcopter stabilization using state feedback controller by pole placement method”. International Journal of Internet, Broadcasting and Communication Vol.9 No.1, 1-6, E-ISSN number, 2288-4939. DOI: https://www.earticle.net/Article/A297898