

## Double Quadrature Spatial Modulation

Tasnim Holoubi<sup>1</sup>, Sheriff Murtala<sup>1</sup>, Nishal Muchena<sup>1</sup>, Manar Mohaisen\*<sup>1</sup>

<sup>1</sup>*Department of Electrical, Electronics and Communication Engineering, Korea University of Technology and Education, Korea.*

*mmohaisen@ieee.org*

### **Abstract**

*Quadrature spatial modulation (QSM) utilizes the in-phase and quadrature spatial dimensions to transmit the real and imaginary parts, respectively, of a single signal symbol. Improved QSM (IQSM) builds upon QSM to increase the spectral efficiency by transmitting the real and imaginary parts of two signal symbols using antenna combinations of size of two. In this paper, we propose a double QSM (DQSM) scheme that transmits the real and imaginary parts of two signal symbols independently through any of the transmit antennas. The two signal symbols are drawn from two different constellations of the same size with the first symbol drawn from any of the conventional modulation sets while the second is drawn from an optimally rotated version of the first constellation. The optimum rotation angle is obtained through extensive Monte Carlo simulations to minimize the bit error rate (BER) of the system. Simulation results show that for a given spectral efficiency, DQSM performs relatively close to IQSM while requiring a smaller number of transmit antennas, and outperforms IQSM by up to 2 dB when the same number of antennas are used.*

**Keywords:** *Quadrature Spatial Modulation, Angle Rotation, Spectral Efficiency, MIMO Systems.*

## 1. INTRODUCTION

Spatial modulation (SM) is a multiple-input-multiple-output (MIMO) technique that conveys information using a signal symbol and the antenna index from which it is transmitted [1]. As such, SM requires a single radio frequency (RF) chain at the transmitter while achieving MIMO capacity. A major drawback of SM is that it requires a large number of transmit antennas in order to increase the spectral efficiency of the system.

A generalized SM (GSM) scheme is proposed where a single signal symbol is transmitted through a combination of antennas, thereby relaxing the power-of-two constraint on the required number of transmit antennas [2]. This improves the spatial spectral efficiency at the cost of a marginal degradation in the bit error rate (BER) performance. Multi-active SM (MA-SM) is a variant of GSM in which each activated antenna transmits a different signal symbol [3]. Consequently, MA-SM improves the spectral efficiency of the system at the cost of increased number of RF-chains and higher system complexity.

Quadrature SM (QSM) extends the spatial dimension into the in-phase and quadrature dimensions to transmit the real and imaginary parts, respectively, of a single signal symbol [4]. Since the real and imaginary parts are transmitted over the orthogonal carriers (*i.e.* sinus and cosinus), QSM avoids inter-channel interference while doubling the spatial spectral efficiency of SM. A complex QSM (CQSM) scheme that transmits two complex signal symbols at each time instance is proposed [5]. The signal symbols are drawn from two different constellations to avoid symbol overlapping in the Euclidean space. The first constellation can be any conventional modulation set, *e.g.* quadrature amplitude modulation (QAM), while the second one is a rotated version of the first. To increase the Euclidean distance in the signal space, an improved CQSM (ICQSM) is proposed [6]. ICQSM uses additional transmit antenna to transmit the second signal symbol when both of the spatial symbols (*i.e.* transmit antennas) are the same. Two generalized versions of CQSM are proposed in [7].

Improved QSM (IQSM) is a recently proposed scheme that transmits the real and imaginary parts of two signal symbols using combinations of two antennas [8]. At each channel use, the real parts are transmitted through one antenna combination and the imaginary parts are transmitted through another one. Since each combination is composed of two distinct antennas, IQSM entirely avoids the real/imaginary overlapping, *i.e.* avoiding the transmission of the real parts or the imaginary parts of the two symbols from the same antenna. As such, the system uses a single constellation set for transmission.

In this paper, we propose a double QSM (DQSM) scheme, where at each channel use, the real and imaginary parts of two signal symbols are transmitted independently through any of the available antennas. The first symbol is drawn from a conventional constellation while the second symbol is drawn from a rotated version of the first. The rotation angle is optimized through extensive Monte Carlo simulations to reduce the BER. Due to its structure, the DQSM significantly reduces the number of required antennas to achieve a given spectral efficiency. For example, the DQSM requires 14 transmit antennas less than the IQSM to achieve a spatial spectral efficiency of 20 bits per channel use (bpcu).

The rest of this paper is organized as follows. In section 2 we explain the system model and review related work. The DQSM is introduced in section 3. The simulation results are explained in section 4 and finally we conclude our work in section 5.

## 2. SYSTEM MODEL AND RELATED WORK

### 2.1 SYSTEM MODEL

We consider a MIMO system with  $n_T$  transmit antennas and  $n_R$  receive antennas. The  $n_R \times n_T$  channel matrix  $\mathbf{H}$ , and  $n_R \times 1$  noise vector  $\mathbf{n}$ , have elements that follow a circularly symmetric complex Gaussian distribution with zero mean and variance of unity and  $\sigma^2$ , respectively. The signal symbol is drawn from the modulation set  $\Omega$ , where  $|\Omega| = 2^q$  and  $q$  is the number of bits per signal symbol. The  $n_T \times 1$  transmitted vector is denoted by  $\mathbf{s}$  and has a unit energy such that the signal-to-noise (SNR)  $\rho = 1/\sigma^2$ . The number of antenna combinations available for transmission is  $2^N$  spatial symbols, where  $N$  is the number of bits per spatial symbol. The total spectral efficiency of the system is composed of spatial and signal parts.

### 2.2 IMPROVED QUADRATURE SPATIAL MODULATION

The IQSM uses  $\binom{n_T}{2}$  antenna combinations to transmit the real and imaginary parts of two signal symbols over the in-phase and quadrature spatial dimensions, respectively. The  $\binom{\cdot}{\cdot}$  operator refers to the binomial coefficient. At each channel use, the message to be transmitted is split into four parts. Two parts modulate the

signal symbols  $s_a$  and  $s_b$ , while the remaining two parts will be used to choose  $\mathbf{l}_{\Re} = \{l_{1\Re}, l_{2\Re}\}$  and  $\mathbf{l}_{\Im} = \{l_{1\Im}, l_{2\Im}\}$ , where  $\mathbf{l}_{\Re}$  and  $\mathbf{l}_{\Im}$  are the in-phase and quadrature antenna combinations, respectively. As such, the antennas from  $\mathbf{l}_{\Re}$  (i.e.  $l_{1\Re}$  and  $l_{2\Re}$ ) are used to transmit the real parts  $s_{a\Re}$  and  $s_{b\Re}$  and those from  $\mathbf{l}_{\Im}$  (i.e.  $l_{1\Im}$  and  $l_{2\Im}$ ) are used to transmit the imaginary parts  $s_{a\Im}$  and  $s_{b\Im}$ . Assuming that  $l_{1\Re} < l_{2\Re}$  and  $l_{1\Im} < l_{2\Im}$ , the received vector  $\mathbf{y}$  is given as follows [8].

$$\mathbf{y} = \mathbf{h}_{l_{1\Re}} s_{a\Re} + \mathbf{h}_{l_{2\Re}} s_{b\Re} + \mathbf{h}_{l_{1\Im}} s_{a\Im} + \mathbf{h}_{l_{2\Im}} s_{b\Im} + \mathbf{n} \quad (1)$$

Where  $\mathbf{h}_l$  denotes the  $l$ -th column of the channel matrix. The information bits are mapped only to  $2^N$  combinations and  $N = \lfloor \log_2 \binom{n_T}{2} \rfloor$ . The total spectral efficiency of the IQSM is  $M = 2(N + q)$  bpcu.

### 2.3 MULTIPLE-ACTIVE SPATIAL MODULATION

MA-SM uses  $\binom{n_T}{n_U}$  antenna combinations to transmit  $n_U$  signal symbols at each channel use. In this case, the message to be transmitted is split into  $(n_U + 1)$  parts. One part is used to select  $\mathbf{l} = \{l_1, \dots, l_{n_U}\}$  antenna combination and each of the  $n_U$  parts is used to choose a signal symbol  $s_i$ ;  $i = 1, \dots, n_U$ . The received vector is then given as follows [3].

$$\mathbf{y} = \sum_{i=1}^{n_U} \mathbf{h}_{l_i} s_i + \mathbf{n}. \quad (2)$$

As such, the system achieves high multiplexing gain while benefitting from the simple structure of SM. MA-SM achieves spectral efficiency of  $M = N + n_U \times q$  bpcu, where  $N = \lfloor \log_2 \binom{n_T}{n_U} \rfloor$ .

## 3. DOUBLE QUADRATURE SPATIAL MODULATION

In DQSM, the real and imaginary parts of two signal symbols are transmitted through any of the transmit antennas;  $N = \log_2(n_T)$ . The first and the second signal symbols,  $s_a$  and  $s'_b$  are drawn from the modulation set  $\Omega_a$  and  $\Omega_b$  respectively, such that

$$\Omega_b = \{s'_b = s_a e^{j\theta} \mid s_a \in \Omega_a\} \quad (3)$$

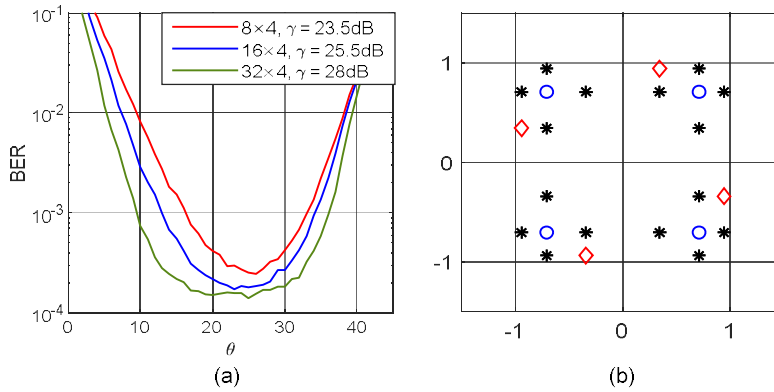
where  $\theta$  is the rotation angle. At each channel use, the  $l_{\Re 1}$ -,  $l_{\Im 1}$ -,  $l_{\Re 2}$ - and  $l_{\Im 2}$ -th transmit antennas are activated to transmit  $s_{a\Re}$ ,  $s_{a\Im}$ ,  $s'_{b\Re}$  and  $s'_{b\Im}$ , respectively. The received vector is then given as

$$\mathbf{y} = \mathbf{h}_{l_{\Re 1}} s_{a\Re} + \mathbf{h}_{l_{\Im 1}} s_{a\Im} + \mathbf{h}_{l_{\Re 2}} s'_{b\Re} + \mathbf{h}_{l_{\Im 2}} s'_{b\Im} + \mathbf{n} \quad (4)$$

Accordingly, the DQSM achieves a spectral efficiency of  $M = 4N + 2q$  bpcu. At the receiver, the signal and spatial symbols are recovered using the maximum likelihood detector as follows.

$$\begin{aligned} (s_{a\Re}^*, s_{a\Im}^*, s'_{b\Re}, s'_{b\Im}, l_{\Re 1}^*, l_{\Im 1}^*, l_{\Re 2}^*, l_{\Im 2}^*) &= \arg \min_{\substack{s_{a\Re}, s_{a\Im} \in \Omega_a, s_{b\Re}, s_{b\Im} \in \Omega_b \\ l_{\Re 1}, l_{\Im 1}, l_{\Re 2}, l_{\Im 2}}} \|\mathbf{y} - \mathbf{g}\|^2 \\ &= \arg \min_{\substack{s_{a\Re}, s_{a\Im} \in \Omega_a, s_{b\Re}, s_{b\Im} \in \Omega_b \\ l_{\Re 1}, l_{\Im 1}, l_{\Re 2}, l_{\Im 2}}} \|\mathbf{g}\|^2 - 2\Re(\mathbf{y}^H \mathbf{g}) \end{aligned} \quad (5)$$

where  $\mathbf{g} = \mathbf{h}_{l_{\Re 1}} s_{a\Re} + \mathbf{h}_{l_{\Im 1}} s_{a\Im} + \mathbf{h}_{l_{\Re 2}} s'_{b\Re} + \mathbf{h}_{l_{\Im 2}} s'_{b\Im}$  and  $\Re(\cdot)$  is the real operator.



**Figure 1. a) BER performance versus  $\theta$  for  $n_R = 4$  and  $q = 2$ , b) DQSM constellation without real/imaginary overlapping.**

The DQSM achieves a high spatial spectral efficiency while significantly reducing  $n_T$  in the system. Both IQSM and DQSM require only two RF-chains. However, unlike IQSM which avoids transmitting the real/imaginary parts of the signal symbols from the same antenna, the DQSM has no such constraint. When the real/imaginary parts are overlapping, the Euclidean space of the modulation set will become dense leading to a performance degradation. In this case, using the same modulation set for transmission can lead to ambiguity at the receiver side, making it impossible to recover the signal symbols (for more details, refer to [5] and [6]). In the sequel, we will elaborate on the symbols' overlapping probability of the DQSM and show that for high  $n_T$ , the overlapping probability approaches zero. Figure 1(a) depicts the DQSM's BER performance versus the rotation angle and Figure 1(b) depicts the conventional QPSK (circle), the rotated version (diamond) with  $\theta = 25^\circ$ , and symbols resulted from  $(s_{aR} + s_{bI})$  or  $(s_{aI} + s_{bR})$  of  $\Omega_a$  and  $\Omega_b$  elements (asterisk).

Let  $\mathcal{V} = \{v_1, \dots, v_n\}$  be the set of all possible relational conditions between the activated antennas  $l_{1R}, l_{1I}, l_{2R}$  and  $l_{2I}$ , such that  $\Pr[v_i] = \frac{f_i}{n_T^4}$ , where  $f_i$  is the frequency of  $v_i$ . Between the four index parameters, there are a total of 64 relational conditions with only 15 non-contradicting ones;  $n = 15$ . Table 1. shows all the possible relational conditions for transmission in DQSM. Since the real and imaginary parts are independent, the performance of the system will be affected only when two real or imaginary parts are transmitted from the same antenna. Therefore, only the occurrence of  $\mathcal{V}_l = \{v_1, v_2, v_3, v_4, v_5, v_6, v_9, v_{14}\}$  is new to the DQSM and can affect its performance as compared to IQSM. Note that all the other events  $\mathcal{V}_j = \{v_7, v_8, v_{10}, v_{11}, v_{12}, v_{13}, v_{15}\}$  belongs also to the IQSM, where the overlapping of real and imaginary parts happens.

**Table 1. All possible transmission events of the DQSM with their corresponding frequencies**

$i$	Relational Condition	$f_i$	$i$	Relational Condition	$f_i$
1	$l_{1R} = l_{2R} = l_{1I} = l_{2I}$	$n_T$	9	$l_{1R} = l_{2R} \neq l_{1I} \neq l_{2I}$	$n_T(n_T - 1)(n_T - 2)$
2	$l_{1R} = l_{2R} = l_{1I} \neq l_{2I}$	$n_T(n_T - 1)$	10	$l_{1R} = l_{1I} \neq l_{2R} \neq l_{2I}$	
3	$l_{1R} = l_{2R} = l_{2I} \neq l_{1I}$		11	$l_{1R} = l_{2I} \neq l_{1I} \neq l_{2R}$	
4	$l_{1R} = l_{1I} = l_{2I} \neq l_{2R}$		12	$l_{2R} = l_{1I} \neq l_{1R} \neq l_{2I}$	
5	$l_{2R} = l_{1I} = l_{2I} \neq l_{1R}$		13	$l_{2R} = l_{2I} \neq l_{1R} \neq l_{1I}$	
6	$l_{1R} = l_{2R} \neq l_{1I} = l_{2I}$		14	$l_{1I} = l_{2I} \neq l_{1R} \neq l_{2R}$	

7	$l_{1\Re} = l_{1\Im} \neq l_{2\Re} = l_{2\Im}$	15	$l_{1\Re} \neq l_{2\Re} \neq l_{1\Im} \neq l_{2\Im}$	$n_T(n_T - 1)(n_T - 2)(n_T - 3)$
8	$l_{1\Re} = l_{2\Im} \neq l_{1\Im} = l_{2\Re}$			

Based on Table 1, let  $f_l$  and  $f_j$  be the frequencies associated with  $\mathcal{V}_l$  and  $\mathcal{V}_j$  respectively,

$$f_l = n_T + 5n_T(n_T - 1) + n_T(n_T - 1)(n_T - 2) \quad (6)$$

$$f_j = 2n_T(n_T - 1) + 4n_T(n_T - 1)(n_T - 2) + n_T(n_T - 1)(n_T - 2)(n_T - 3) \quad (7)$$

As shown in Table 2, when  $n_T$  increases,  $f_j$  grows more rapidly than  $f_l$  until the probability of the former converges to one and that of the latter to zero. As such, the system performance of the DQSM significantly improves while increasing  $n_T$  until it coincides with that of the IQSM for the same spatial spectral efficiency.

**Table 2. The convergence of  $\mathcal{V}_l$  and  $\mathcal{V}_j$  probabilities while increasing  $n_T$**

$n_T$	$\Pr[\mathcal{V}_l]$	$\Pr[\mathcal{V}_j]$
8	0.095	0.766
16	0.053	0.879
32	0.029	0.939

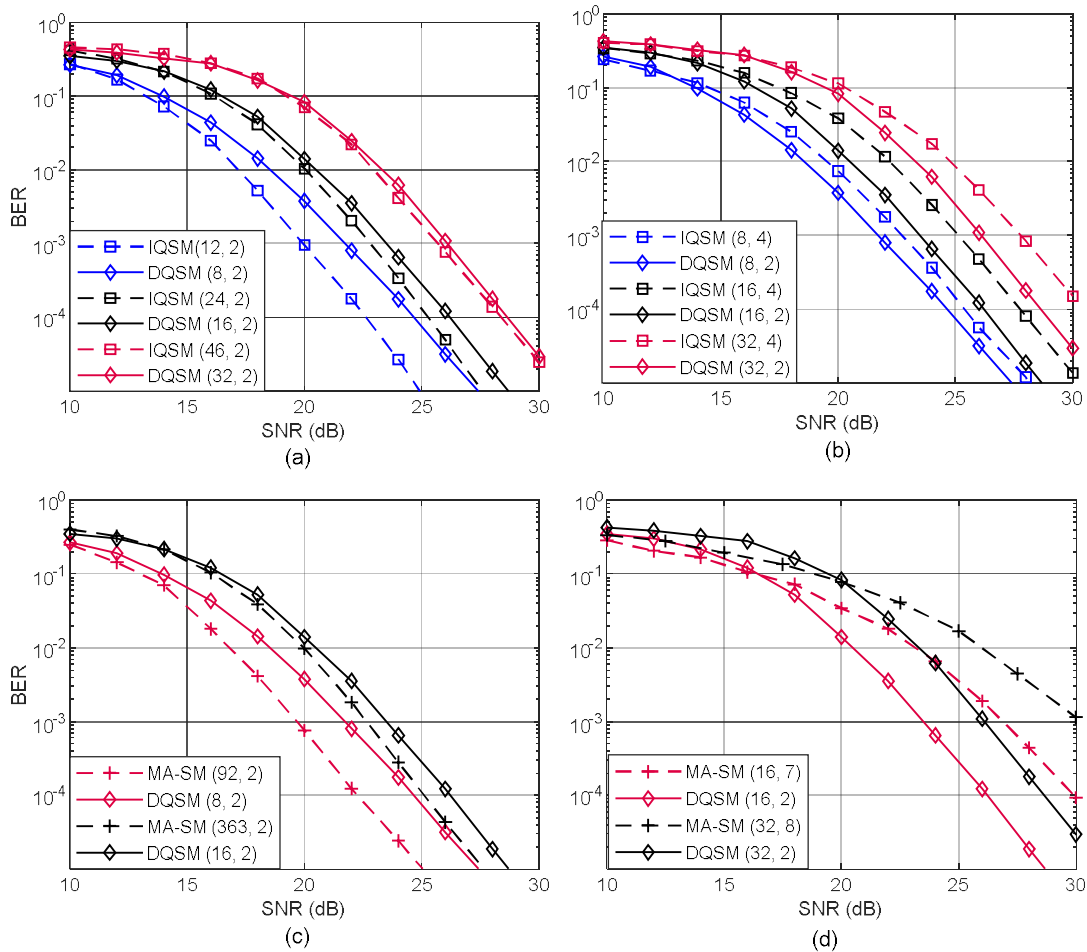
## 4. SIMULATION RESULTS

In this section, the channel state information (CSI) is assumed to be perfectly known only at the receiver. The simulation results are obtained using  $n_R = 4$  and QAM with varying  $(n_T, q)$  values as indicated in the legend of each sub-figure.

Figure 2 depicts the BER performance of the DQSM compared to the IQSM and MA-SM using  $n_U = 2$ . For several values of  $M$ , the results are obtained for two scenarios. First, assuming the same modulation order, DQSM requires less number of transmit antennas to achieve the same spectral efficiency of IQSM and MA-SM. The second scenario is when the same number of transmit antennas are used, DQSM uses lower modulation order and outperforms the other schemes.

In Figure 2(a), DQSM requires 4, 8, and 14 transmit antennas less than IQSM to achieve  $M = 16, 20$ , and 24 bpcu, respectively. While using the same  $n_T$ , DQSM outperforms IQSM by about 1 dB for  $n_T = 8$  and 2 dB for both  $n_T = 16$  and 32 as shown in Figure 2(b). Compared to MA-SM, Figure 2(c) shows that DQSM reduces the number of required antennas by 84 and 347 to achieve  $M = 16$  and 20 bpcu, respectively. Finally, in Figure 2(d), DQSM outperforms MA-SM by about 4 dB for both  $n_T = 16$  and 32.

It is clear from Figure 2(a) that when  $n_T$  is low the system's performance of DQSM degrades because of the probability of the symbols' overlapping. However, this gap shrinks significantly as  $n_T$  increases until the curves of DQSM and IQSM coincides for  $n_T = 32$ . This is evident since high values of  $n_T$  reduces the overlapping probability as analytically explained in Section 3.



**Figure 2. BER performance of DQSM compared to a) IQSM with equal  $q$ , b) IQSM with equal  $n_T$ , c) MA-SM with equal  $q$ , and finally d) MA-SM with equal  $n_T$ .**

## 5. CONCLUSIONS

In this paper, we proposed a DQSM scheme, where the real and imaginary parts of two signal symbols are transmitted through any of the available transmit antennas. The first symbol is drawn from a conventional constellation and the second symbol is drawn from a rotated version of the former. The optimum rotation angle is obtained through Monte Carlo simulations to minimize the BER of the system. Since the DQSM has no constraint over the real/imaginary parts overlapping, we derived the overlapping probability and showed that for large number of transmit antennas this probability tends to zero. Simulation results were consistent with our analysis and showed that the BER performance of the proposed scheme improves as the number of transmit antennas increases. Furthermore, for the same number of transmit antennas, the DQSM has performance gain of 4 dB over MA-SM.

## Acknowledgement

This work was supported by research stipend by Korea University of Technology and Education for the period 2019-2020.

## References

- [1] R. Mesleh, H. Haas, S. Sinaovic, C. W. Ahn, and S. Yun "Spatial modulation" *IEEE Trans. Veh. Technol.* vol. 57 no. 4 pp. 2228-2241 July 2008.
- [2] A. Younis, N. Serafimovski, R. Mesleh, and H. Haas "Generalised spatial modulation" *Proc.2010 Signals Syst. Comput.* pp. 1498-1502.
- [3] J. Wang, S. Jia, and J. Song "Generalised spatial modulation system with multiple active transmit antennas and low complexity detection scheme" *IEEE Trans. Wirel. Commun.* vol. 11 no. 4 pp. 1605-1615 2012.
- [4] R. Mesleh, S. Ikki, and H. Aggoune, "Quadrature spatial modulation," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 6, pp. 2738-2742, 2014.
- [5] M. Mohaisen and S. Lee, "Complex quadrature spatial modulation," *ETRI Journal*, vol. 39, no. 4, pp. 514–524, 2017.
- [6] M. Mohaisen, "Increasing the minimum Euclidean distance of the complex quadrature spatial modulation," *IET Communications*, vol. 12, no. 7, pp. 854–860, 2018.
- [7] M. Mohaisen, "Generalized Complex Quadrature Spatial Modulation," *Wireless Communications and Mobile Computing*, vol. 2019, pp. 1–12, Apr. 2019.
- [8] B. Vo and H. H. Nguyen, "Improved Quadrature Spatial Modulation," in *2017 IEEE 86th Vehicular Technology Conference (VTC-Fall)*, Toronto, ON, pp. 1–5, 2017.