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# Rotated-symbol Generalized Spatial Modulation

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***Abstract***

*In spatial modulation (SM), both the signal symbol and spatial symbol, i.e., the index of the antenna from which signal symbol is transmitted, carry information. To increase the number of bits carried by spatial symbols, more transmit antennas are required. In the generalized SM (GSM), the same signal symbol is transmitted from a combination of antennas, resulting in a reduction in the number of antennas required to achieve a given spectral efficiency. In this paper, we propose a rotated-symbol GSM (RGSM), in which the signal symbol is rotated with an angle corresponding to the position of the antenna index within the combination. This increases the number of spatial symbols by a factor equivalent to the length of the antenna combinations of the GSM. Numerically, SM, GSM and RGSM require 128, 17 and 12 transmit antennas to convey seven bits through the spatial symbols. Simulation results show that RGSM performs relatively close to GSM, and in several system settings, their error performances coincide.*

***Keywords:*** *Generalized Spatial Modulation, MIMO Systems, Rotated Symbol, Single Radio Frequency Chain.*

## 1. INTRODUCTION

Spatial modulation (SM) is a multiple-input multiple-output (MIMO) transmission scheme in which a signal symbol and the index of the antenna used for transmission carry information [1]. SM is an attractive technique as it only requires a single radio-frequency (RF) chain thereby keeping the transmitter as simple as that of the single-input communication systems. In SM, the number of bits that are transmitted through the spatial symbols, $m_s$ is equal to $\log_2 n_T$, where $n_T$ is the number of transmit antennas used [2]. For instance, SM transmits seven bits through the spatial symbols, given 128 transmit antennas.

To achieve the same spectral efficiency using a relatively smaller number of transmit antennas, a generalized SM (GSM) scheme was proposed, where rather than using a single antenna for transmission at each channel use, a *combination* of transmit antennas is activated to transmit the same signal symbol [3, 4]. A combination is a set of unique antenna indices where, by definition, order does not matter therefore, $\{1,2,3\} \stackrel{\text{def}}{=} \{3,2,1\} \stackrel{\text{def}}{=} \{2,3,1\}$. Assuming the same number of bits per signal symbol, $q$, GSM and SM require

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7 and 16 transmit antennas, respectively, to transmit four bits using the spatial symbols. A unified precoding scheme for GSM that is based on the maximum minimum Euclidean distance criterion is proposed to improve the system error performance [5]. A variant of GSM called multiple-active SM (MA-SM) is proposed, where a combination of $n_U$ active transmit antennas is activated to transmit $n_U$ different symbols per channel use [6]. The concept of transmitting multiple symbols further increases the spectral efficiency but comes at the cost of additional RF chains and increased system complexity [7].

In this paper, we propose a rotated-symbol GSM (RGSM) to further reduce the number of transmit antennas required to achieve a given spectral efficiency. Instead of transmitting the single signal symbol from the antennas of a given combination, as in the GSM, the selected signal symbol is first rotated by an angle associated with the position of the antenna within the ordered set of antennas available for transmission. As such, instead of using antenna combinations as spatial symbols, ordered tuples of antenna indices become valid spatial symbols. For instance, the spatial symbols $(1,\ 2,\ 3)$, $(2,\ 3,\ 1)$ and $(3,\ 1,\ 2)$, consisting of the same group of antennas, are distinguishable at the receiver. Consequently, the number of spatial symbols is increased for a given number of combinations and, the number of transmit antennas required to achieve a given spectral efficiency is reduced, compared to GSM.

The rest of this paper is organized as follows. Section 2 describes the system model while Section 3 details the proposed rotated-symbol Generalized Spatial Modulation. Simulations results are presented and discussed in Section 4 and Section 5 concludes the paper.

## 2. SYSTEM MODEL

We consider a MIMO system with $n_T$ transmit and $n_R$ receive antennas. A single signal symbol is drawn from a conventional $M$-ary quadrature amplitude modulation (M-QAM) constellation set $\Omega$, with $|\Omega| = M$ and $q = log_2(M)$ is the number of bits per signal symbol. In SM, a block of $\log_2(n_T M) = q + m_s$ is transmitted at each channel use. The first $q$ bits modulate a signal symbol $s_k$, for $k = 1, \cdots, M$, and the remaining $m_s$ bits modulate the index $i = 1, \cdots, n_T$ of the antenna from which $s_k$ is transmitted. Accordingly, the system equation is given as follows.

$$\boldsymbol{y} = \boldsymbol{h}_i s_k + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{y} \in \mathbb{C}^{n_R \times 1}$ is the received signal and $\boldsymbol{h}_i$ is the $i^{th}$ column of the channel matrix $\boldsymbol{H} \in \mathbb{C}^{n_R \times n_T}$ whose elements are independent and identically distributed and follow a centered circularly symmetric complex Gaussian distribution with variance of unity. The elements of the noise vector $\boldsymbol{n}$ follow the same distribution as those of $\boldsymbol{H}$ with variance of $\sigma_n^2$, where $\rho = 1/\sigma_n^2$ is the signal-to-noise ratio (SNR).

Assuming $n_U$ active antennas at each channel use, the number of available combinations, *i.e.* spatial symbols, for GSM is $n'_G = \binom{n_T}{n_U}$. Only $n_G = 2^{m_g}$ combinations are used, where $m_g = \left\lfloor \log_2 \binom{n_T}{n_U} \right\rfloor$ and $\lfloor \cdot \rfloor$ is the floor operator. The spectral efficiency of the GSM is therefor $m_g + q$ bits/channel use (bpcu). To achieve a spectral efficiency of 8 bits/channel with $M = 4$, SM requires 64 antennas while GSM requires 12 transmit antennas, with $n_U = 2$. At each transmission instance, each active antenna $l_i$, in combination set $\boldsymbol{l}_a = \{l_1, \cdots, l_{n_U}\}$, for $a = 1 \cdots, n_G$ transmits the modulated signal symbol. The system equation of the GSM is given as follows.

$$\boldsymbol{y} = \boldsymbol{g} + \boldsymbol{n} = \frac{s_k}{\sqrt{n_u}} \sum_{i=1}^{n_U} \boldsymbol{h}_{l_i} + \boldsymbol{n} \tag{2}$$

where $\boldsymbol{g}$ is the noiseless transmitted vector.

## 3. ROTATED-SYMBOL GENERALISED SPATIAL MODULATION

To further reduce the number of transmit antennas required to achieve a given spectral efficiency, an RGSM scheme is proposed in this section where each combination available for GSM is extended to $n_U$ valid antenna tuples by applying rotations to the single signal symbol before transmission. Assuming $n_U = 4$, let $\boldsymbol{l}_a = \{l_1, l_2, l_3, l_4\}$ be the spatial symbol used by GSM. The 4 corresponding spatial symbols available for use by RGSM are obtained through applying repetitive shift of the elements of $\boldsymbol{l}_a$. That is:

$$\boldsymbol{l}_1 = (l_1, l_2, l_3, l_4), \quad \boldsymbol{l}_2 = (l_2, l_3, l_4, l_1), \quad \boldsymbol{l}_3 = (l_3, l_4, l_1, l_2), \quad \boldsymbol{l}_4 = (l_4, l_1, l_2, l_3). \tag{3}$$

Accordingly, the number of spatial symbols that can be used becomes $n'_G = \binom{n_T}{n_U} \times n_U$. Since the number of spatial symbols that can be used for transmission should be a power of two, only $n_G = 2^{m_r}$ symbols are used, where $m_r = \lfloor log_2(n'_G) \rfloor$. The vector of rotation angles for a given value of $n_U$ can be represented as a tuple, $\boldsymbol{\theta} = (\theta_1, \cdots, \theta_{n_U})$. The system equation for RGSM is given by

$$\boldsymbol{y} = \boldsymbol{g} + \boldsymbol{n} = \frac{s_k}{\sqrt{n_U}} \sum_{i=1}^{n_U} \boldsymbol{h}_{l_i} e^{j\theta_i} + \boldsymbol{n}. \tag{4}$$

As an example: Let $\boldsymbol{l}_1 = (1, 2, 3)$, $\boldsymbol{l}_2 = (2, 3, 1)$, $\boldsymbol{l}_3 = (3, 1, 2)$ be three spatial symbols that contain the same group of antennas. For a given signal symbol $s_k$, $\boldsymbol{g}$ in the case of GSM is given as follows;

$$\frac{s_k}{\sqrt{n_U}} \sum_{i \in \boldsymbol{l}_1} \boldsymbol{h}_i = \frac{s_k}{\sqrt{n_U}} \sum_{i \in \boldsymbol{l}_2} \boldsymbol{h}_i = \frac{s_k}{\sqrt{n_U}} \sum_{i \in \boldsymbol{l}_3} \boldsymbol{h}_i. \tag{5}$$

This implies that the above three spatial symbols are not distinguishable at the receiver hence only one of them can be used for transmission. On the other hand, the transmitted vector in RGSM system for $n_U = 3$ is given by

$$\frac{1}{\sqrt{n_U}} \left[ \cdots, \underbrace{s_k e^{j\theta_1}}_{l_1^{th}}, \cdots, \underbrace{s_k e^{j\theta_2}}_{l_2^{th}}, \cdots, \underbrace{s_k e^{j\theta_3}}_{l_3^{th}}, \cdots \right]^T, \tag{6}$$

where $(\cdot)^T$ is the transpose operator. The received noiseless vectors corresponding to the above spatial symbols are given as follows.

$$\begin{aligned} \boldsymbol{g}_1 &= \frac{s_k}{\sqrt{n_U}} \left( \boldsymbol{h}_1 e^{j\theta_1} + \boldsymbol{h}_2 e^{j\theta_2} + \boldsymbol{h}_3 e^{j\theta_3} \right) \\ \boldsymbol{g}_2 &= \frac{s_k}{\sqrt{n_U}} \left( \boldsymbol{h}_1 e^{j\theta_2} + \boldsymbol{h}_2 e^{j\theta_3} + \boldsymbol{h}_3 e^{j\theta_1} \right) \\ \boldsymbol{g}_3 &= \frac{s_k}{\sqrt{n_U}} \left( \boldsymbol{h}_1 e^{j\theta_3} + \boldsymbol{h}_2 e^{j\theta_1} + \boldsymbol{h}_3 e^{j\theta_2} \right). \end{aligned} \tag{7}$$

For unique values of $\theta$, these three vectors are distinguishable at the receiver. For given $n_T$ and $n_U$, RGSM obtains $n_U$ times the number of spatial symbols available for transmission for GSM. To put it differently, RGSM achieves a given spectral efficiency with a smaller number of transmit antennas as compared to GSM. The goal of the rotation vector in RGSM is to render the received spatial symbols as distinguishable as possible, especially for those symbols that consist of the same group of antennas. In this letter, we assume that the transmitter does not have knowledge of the channel state information. Also, based on the assumed channel model, the phase of the channel coefficients is uniform over the period $[0,2\pi]$. Under these conditions, the rotation angles are uniformly distributed over the period $[0,\pi/2]$. Furthermore, to reduce the complexity at the transmitter, $\theta_1$ is normalized to 0 rad and the angle difference between any two adjacent elements of $\boldsymbol{\theta}$ is $\frac{\pi}{2n_U}$ rad i.e. the $i^{th}$ rotation angle is defined as

$$\theta_i = \frac{\pi}{2n_U}(i-1). \tag{8}$$

As an example, using (8), the rotation vectors for $n_U = 2$ and 3 are $\boldsymbol{\theta} = \left(0, \frac{\pi}{4}\right)$ and $\boldsymbol{\theta} = \left(0, \frac{\pi}{6}, \frac{\pi}{3}\right)$ respectively. The receiver employs the maximum-likelihood (ML) principle to recover the signal symbol $s$ and the spatial symbol $\boldsymbol{l}$ as follows.

$$(s^*, \boldsymbol{l}^*) = \arg \min_{s\in\Omega, \boldsymbol{l}\in\boldsymbol{L}} ||\boldsymbol{y} - \frac{s}{\sqrt{n_u}} \sum_{i=1}^{n_U} \boldsymbol{h}_{\boldsymbol{l}_i} e^{j\theta_i} \ ||^2 \tag{9}$$

where $\boldsymbol{L}$ is the set of all used spatial symbols. For the same spectral efficiency, both GSM and RGSM use the same number of spatial symbols. That is, the reduction in the number of transmit antennas by RGSM comes at no cost in the computational complexity of the receiver.

## 4. PERFORMANCE ANALYSIS

Let $\boldsymbol{g} = \frac{s_k}{\sqrt{n_U}} \sum_{i=1}^{n_U} \boldsymbol{h}_{l_i} e^{j\theta_i}$ and $\widehat{\boldsymbol{g}} = \frac{\hat{s}_k}{\sqrt{n_U}} \sum_{i=1}^{n_U} \widehat{\boldsymbol{h}}_{l_i} e^{j\theta_i}$ denote two noiseless received vectors corresponding to the transmitted vectors $\boldsymbol{s}$ and $\widehat{\boldsymbol{s}}$, respectively. The pairwise error probability (PEP) is given by the following equation.

$$\Pr[\boldsymbol{g} \to \widehat{\boldsymbol{g}} \mid \mathbf{H}] = Q\left(\sqrt{\frac{\|\boldsymbol{g} - \widehat{\boldsymbol{g}}\|^2}{2\sigma_n^2}}\right) \tag{10}$$

In the equation, $Q(.)$ is the Gaussian tail function, and $d_{\mathrm{rx}}^2 = \|\boldsymbol{g} - \widehat{\boldsymbol{g}}\|^2$ is the squared Euclidean distance at the receiver. Similarly, $d_{\mathrm{tx}}^2 = \|\boldsymbol{s} - \hat{\boldsymbol{s}}\|^2$ is the squared Euclidean distance at the transmitter. Note that $\mathbb{E}_{\mathbf{H}}\{d_{\mathrm{rx}}^2\} = (\boldsymbol{s} - \hat{\boldsymbol{s}})^H(\boldsymbol{s} - \hat{\boldsymbol{s}}) = d_{\mathrm{tx}}^2$. By averaging (10) over $\mathbf{H}$, we get the following unconditional PEP (UPEP). The integral evaluation of the is taken from [8, 9].

$$\Pr[\boldsymbol{g} \to \widehat{\boldsymbol{g}}] = \mu^{n_R} \sum_{l=0}^{n_R-1} \binom{n_R-1+l}{l} [1-\mu]^l \tag{11}$$

In the equation, $\mu = \frac{1}{2}\left(1 - \sqrt{\frac{\rho \cdot d_{\mathrm{tx}}^2}{4+\rho \cdot d_{\mathrm{tx}}^2}}\right)$. The union bound on the PEP is then given by averaging all pairwise probabilities as follows.

$$\Pr[e] \le \frac{1}{2^L} \sum_{i=1}^{2^L} \sum_{k=1}^{2^L} \Pr[\boldsymbol{g}_i \to \widehat{\boldsymbol{g}}_k]. \tag{12}$$

Accordingly, the average bit-error-rate (BER) is given as follows.

$$\mathrm{Pr}_e \leq \frac{1}{L2^L} \sum_{i=1}^{2^L} \sum_{k=1}^{2^L} \mathrm{D}_{s,\hat{s}} \mathrm{Pr}[\boldsymbol{g}_i \to \widehat{\boldsymbol{g}}_k] \quad (13)$$

In the equation, $\mathrm{D}_{s,\hat{s}}$ is the hamming distance between $\boldsymbol{s}$ and $\hat{\boldsymbol{s}}$ that corresponds to the number of errors associated with the event $[\boldsymbol{g}_i \to \widehat{\boldsymbol{g}}_k]$ and $L = m_r + q$ is the spectral efficiency of the system.

## 5. SIMULATION RESULTS

In this section, the bit-error rate (BER) performance of RGSM is evaluated and compared to that of SM and GSM for several system configurations. The following simulation results are obtained for a fixed value of $n_R = 4$, and on the assumption that only the receiver has knowledge of the channel state information.
Table 1 summarizes the number of required transmit antennas to achieve a given spectral efficiency by SM, GSM and RGSM systems. For a fair comparison, the performance is evaluated for an equal spectral efficiency of the three systems. For instance, assuming $q = 4$, $m = 9$, and $n_U = 3$, SM, GSM and RGSM require 512, 16 and 12 transmit antennas to achieve the same spectral efficiency of 13 bpcu. In this case, RGSM requires 4 less antennas, compared to GSM. This difference in $n_T$ increases to 6 for $m = 10$.

**Table 1. Number of transmit antennas required to achieve the same spectral efficiency of $q + m$ for SM, GSM and the proposed RGSM, assuming $m = m_s = m_g = m_r$**

| $(q, m, n_U)$ | SM | GSM | RGSM |
|---|---|---|---|
| (2/4,4,2) | 16 | 7 | 5 |
| (2/4,4,2) | 128 | 17 | 12 |
| (2/4,4,2) | 512 | 16 | 12 |
| (2/4,4,2) | 1024 | 20 | 14 |

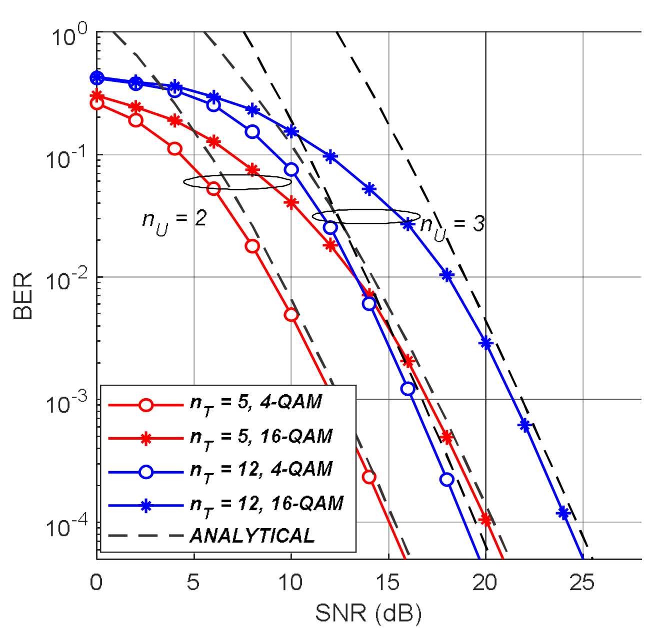


**Figure 1. Analytical and simulation results of RGSM for system configurations using 4-QAM and 16-QAM, with $n_T = \{5, 12\}$ and $n_U = \{2, 3\}$.**

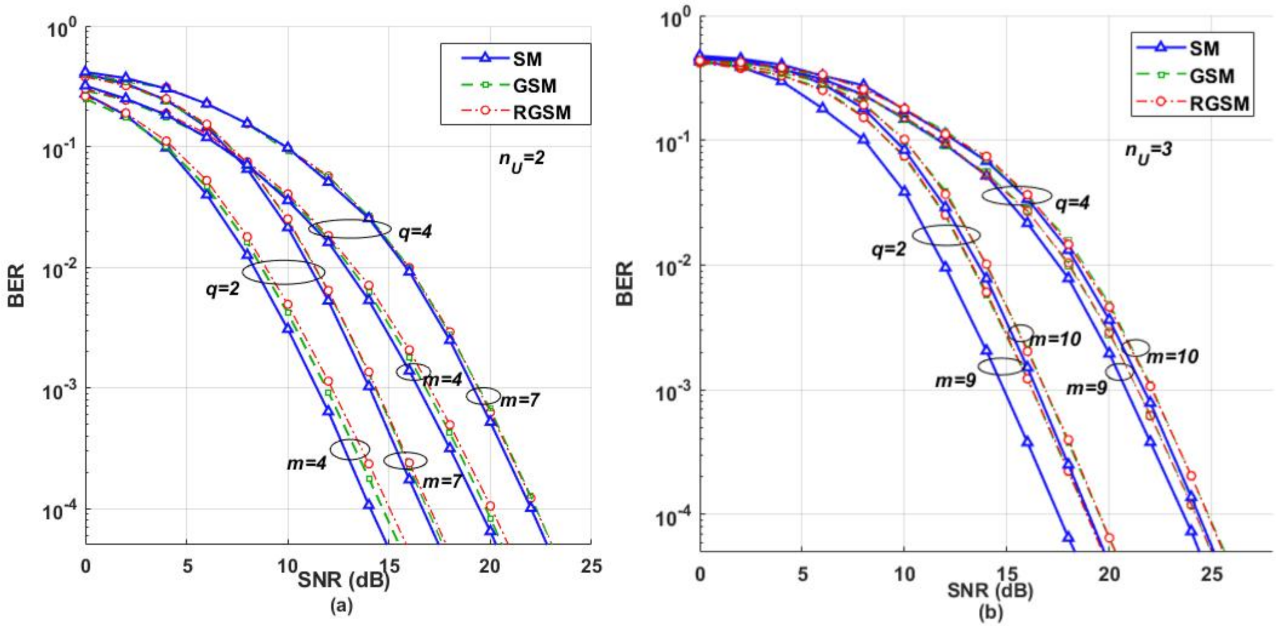


**Figure 2. BER performance of SM, GSM and RGSM for several values of spectral efficiency using 4- and 16-QAM, and (a) $n_U = 2$, (b) $n_U = 3$ for GSM and RGSM systems.**

As mentioned earlier, this gain is achieved at no computational cost. Figure 1 depicts the BER performance for simulated and analytical results of RGSM configurations with $n_U = 2$ and 3, $n_T = 5$ and 12, using 4- and 16-QAM schemes. The simulation results perform within the confines of the analytical results in all scenarios. Figure 2 (a) depicts the BER performance of the three systems, assuming $n_U = 2$ and using 4- and 16QAM schemes. Both GSM and RGSM perform close to SM, especially for high values of $m$. The gap in the BER performance between GSM and RGSM is negligible and vanishes as $m$ increases. Figure 2 (b) depicts the BER performance for $n_U = 3$ using 4- and 16-QAM schemes. For all the simulated scenarios, the performances of GSM and RGSM coincide. The gap in the BER performance between SM on the one side, and GSM and RGSM on the other is bridged for high values of $m$.

## 6. CONCLUSION

GSM achieves the same spectral efficiency of SM while using fewer transmit antennas. In this letter, we propose a rotated-symbol GSM (RGSM) system which further reduces the number of transmit antennas to achieve the same spectral efficiency of SM and GSM. In RGSM, the set of spatial symbols is increased through rotating the signal symbol with predefined angle vector. This gain in the number of transmit antennas is achieved at no computational cost. Furthermore, the performance gap between SM, GSM and RGSM is negligible.

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