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UNIT TANGENT SPHERE BUNDLES OF TWO-POINT HOMOGENEOUS SPACES

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Abstract. We characterize two-point homogeneous spaces M by means of the structural operator $h = \frac{1}{2}\mathfrak{L}_\xi\phi$ or the characteristic Jacobi operator $\ell = R(\cdot,\xi)\xi$ on the unit tangent sphere bundles T_1M.

1. Introduction

An intriguing study of a Riemannian manifold (M, g) is to investigate the interaction of the manifold with its unit tangent sphere bundle T_1M endowed with the standard contact metric structure $(\eta, \bar{g}, \phi, \xi)$. In contact metric geometry, apart from the defining structure tensors η, $\bar{g}$, ϕ and ξ, two other operators play a fundamental role, namely the structural operator $h = \frac{1}{2}\mathfrak{L}_\xi\phi$ and the characteristic Jacobi operator $\ell = R(\cdot,\xi)\xi$, where $\mathfrak{L}_\xi$ denotes Lie differentiation in the characteristic direction ξ.

A type of symmetry occurs when some structure tensors are covariantly parallel along the integral curves of ξ. On a contact metric space, it always holds $\bar{\nabla}_\xi\xi = \bar{\nabla}_\xi\eta = \bar{\nabla}_\xi\bar{g} = \bar{\nabla}_\xi\phi = 0$, but the other structure tensors need not be parallel in the ξ-direction. Recently, it was proved that T_1M satisfies the condition $\bar{\nabla}_\xi h = 0$ or, equivalently, $\bar{\nabla}_\xi\ell = 0$, if and only if (M, g) is of constant curvature $c = 0$ or $c = 1$ ([15, 16]). We can easily show these results from the formulas in Section 3.

In this paper, we develop the above results. Specifically we investigate the base manifold (M, g) when the unit tangent sphere bundle T_1M

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satisfies some symmetry conditions of the operators h or ℓ. The main theorems are the following:

Theorem 1.1. *Let M be a real analytic Riemannian manifold and T_1M be the unit tangent sphere bundle with the standard contact metric structure $(\eta, \bar{g}, \phi, \xi)$. Then T_1M satisfies $\bar{\nabla}_\xi h = \mu h\phi + \nu h$ for $(\mu, \nu) \in \mathbb{R}^2$ if and only if either (M, g) is a space of constant curvature or a locally rank 1 symmetric space where the eigenvalues of R_u are* 1 *and* 4 *(or* $\frac{1}{4}$ *).*

Theorem 1.2. *Let M be an n-dimensional Riemannian manifold and T_1M be the unit tangent sphere bundle with the standard contact metric structure $(\eta, \bar{g}, \phi, \xi)$. Then T_1M satisfies $\bar{\nabla}_\xi \ell = k(\phi\ell - \ell\phi)$ $(k \neq 2$, $k \in \mathbb{R})$ if and only if either (M, g) is a space of constant curvature c $(c \neq 2)$ or a locally rank 1 symmetric space where the eigenvalues of R_u are* 1 *and* 4 *(or* $\frac{1}{4}$ *).*

In an earlier work ([9]), we studied the unit tangent sphere bundles whose base manifolds are complex space forms.

2. Preliminaries

All manifolds in the present paper are assumed to be connected and of class C^∞. We start by collecting some fundamental material about contact metric geometry. We refer to [3] for further details. A $(2n-1)$-dimensional differentiable manifold $\bar{M}^{2n-1}$ is said to be a contact manifold if it admits a global 1-form η such that $\eta \wedge (d\eta)^n \neq 0$ everywhere. Given a contact form η, we have a unique vector field ξ, the *characteristic vector field,* satisfying $\eta(\xi) = 1$ and $d\eta(\xi, \bar{X}) = 0$ for any vector field $\bar{X}$ on $\bar{M}$. It is well-known that there exists a Riemannian metric $\bar{g}$ and a $(1,1)$-tensor field ϕ such that

$$(1) \quad \eta(\bar{X}) = \bar{g}(\bar{X}, \xi), \quad d\eta(\bar{X}, \bar{Y}) = \bar{g}(\bar{X}, \phi\bar{Y}), \quad \phi^2\bar{X} = -\bar{X} + \eta(\bar{X})\xi,$$

where $\bar{X}$ and $\bar{Y}$ are vector fields on $\bar{M}$. From (1) it follows that

$$(2) \qquad \phi\xi = 0, \quad \eta \circ \phi = 0, \quad \bar{g}(\phi\bar{X}, \phi\bar{Y}) = \bar{g}(\bar{X}, \bar{Y}) - \eta(\bar{X})\eta(\bar{Y}).$$

A Riemannian manifold $\bar{M}$ equipped with structure tensors $(\eta, \bar{g}, \phi, \xi)$ satisfying (1) is said to be a contact metric manifold and is denoted by $\bar{M} = (\bar{M}; \eta, \bar{g}, \phi, \xi)$. Given a contact metric manifold M, we define the *structural operator* h by $h = \frac{1}{2}\mathfrak{L}_\xi\phi$, where $\mathfrak{L}$ denotes Lie differentiation. Then we may observe that h is symmetric and satisfies

$$(3) \qquad h\xi = 0 \quad \text{and} \quad h\phi = -\phi h,$$

$$(4) \qquad \bar{\nabla}_{\bar{X}}\xi = -\phi\bar{X} - \phi h\bar{X},$$

where $\bar{\nabla}$ is the Levi-Civita connection. We denote by $\bar{R}$ the Riemannian curvature tensor defined by

$$\bar{R}(\bar{X},\bar{Y})\bar{Z} = \bar{\nabla}_{\bar{X}}\bar{\nabla}_{\bar{Y}}\bar{Z} - \bar{\nabla}_{\bar{Y}}\bar{\nabla}_{\bar{X}}\bar{Z} - \bar{\nabla}_{[\bar{X},\bar{Y}]}\bar{Z}$$

for all vector fields $\bar{X}, \bar{Y}$ and $\bar{Z}$. From (3) and (4) we see that ξ generates a geodesic flow. Furthermore, we know that $\bar{\nabla}_\xi \phi = 0$ in general (cf. p.67 in [3]). From the second equation of (3) it follows also that

$$(\bar{\nabla}_\xi h)\phi = -\phi(\bar{\nabla}_\xi h). \tag{5}$$

Along a geodesic flow ξ, the Jacobi operator $\ell = \bar{R}(\cdot,\xi)\xi$ is a symmetric $(1,1)$-tensor field. We call it the *characteristic Jacobi operator*. From the definition of $\bar{R}$ by using (4) we have

$$\ell = \phi\bar{\nabla}_\xi h - (h^2 + \phi^2). \tag{6}$$

From (6), using the second equation of (3) and (5), we have

$$\bar{\nabla}_\xi h = \frac{1}{2}(\ell\phi - \phi\ell). \tag{7}$$

3. The contact metric structure of the unit tangent sphere bundle

The basic facts and fundamental formulae about tangent bundles are well-known (cf. [12, 14, 18]). We only briefly review some notations and definitions. Let $M = (M, g)$ be an n-dimensional Riemannian manifold and let TM denote its tangent bundle with the projection $\pi : TM \to M,\ \pi(p,u) = p$. For a vector $X \in T_pM$, we denote by X^h and X^v, the *horizontal lift* and the *vertical lift*, respectively. Then we can define a Riemannian metric $\tilde{g}$, the *Sasaki metric* on TM, in a natural way by

$$\tilde{g}(X^h, Y^h) = \tilde{g}(X^v, Y^v) = g(X,Y)\circ\pi, \quad \tilde{g}(X^h, Y^v) = 0$$

for all vector fields X and Y on M. Also, a natural almost complex structure tensor J of TM is defined by $JX^h = X^v$ and $JX^v = -X^h$. Then we easily see that $(TM;\tilde{g}, J)$ is an almost Hermitian manifold. We note that J is integrable if and only if (M, g) is locally flat ([12]). Now we consider the unit tangent sphere bundle (T_1M, g'), which is an isometrically embedded hypersurface in $(TM, \tilde{g})$ with unit normal vector field $N = u^v$. For $X \in T_pM$, we define the *tangential lift* of X to $(p, u) \in T_1M$ by

$$X^t_{(p,u)} = X^v_{(p,u)} - g(X,u)N_{(p,u)}.$$

Clearly, the tangent space $T_{(p,u)}T_1M$ is spanned by vectors of the form X^h and X^t where $X \in T_pM$. We put

$$\xi' = -JN, \quad \phi' = J - \eta' \otimes N.$$

Then we find $g'(\bar{X}, \phi'\bar{Y}) = 2d\eta'(\bar{X}, \bar{Y})$. By taking $\xi = 2\xi'$, $\eta = \frac{1}{2}\eta'$, $\phi = \phi'$, and $\bar{g} = \frac{1}{4}g'$, we get the standard contact metric structure $(\eta, \bar{g}, \phi, \xi)$. Indeed, we easily check that these tensors satisfy (1). Here we notice that ξ determines the geodesic flow. The tensors ξ and ϕ are explicitly given by

$$\xi = 2u^h, \quad \phi X^t = -X^h + \frac{1}{2}g(X,u)\xi, \quad \phi X^h = X^t \tag{8}$$

where X and Y are vector fields on M. From now on, we consider $T_1M = (T_1M; \eta, \bar{g})$ with the standard contact metric structure. We list the fundamental formulae which we need for the proof of our theorems. They are derived in [6, 5, 11]. The Levi-Civita connection $\bar{\nabla}$ of $(T_1M, \bar{g})$ is given by

$$\begin{aligned}
\bar{\nabla}_{X^t}Y^t &= -g(Y,u)X^t,\\
\bar{\nabla}_{X^t}Y^h &= \frac{1}{2}(R(u,X)Y)^h,\\
\bar{\nabla}_{X^h}Y^t &= (\nabla_XY)^t + \frac{1}{2}(R(u,Y)X)^h,\\
\bar{\nabla}_{X^h}Y^h &= (\nabla_XY)^h - \frac{1}{2}(R(X,Y)u)^t.
\end{aligned} \tag{9}$$

For the Riemann curvature tensor $\bar{R}$, we give only the two expressions, which we need for the characteristic Jacobi operator ℓ:

$$\begin{aligned}
\bar{R}(X^t,Y^h)Z^h = &-\frac{1}{2}\{R(Y,Z)(X-g(X,u)u)\}^t\\
&+\frac{1}{4}\{R(Y,R(u,X)Z)u\}^t\\
&-\frac{1}{2}\{(\nabla_YR)(u,X)Z\}^h,\\
\bar{R}(X^h,Y^h)Z^h = &(R(X,Y)Z)^h + \frac{1}{2}\{R(u,R(X,Y)u)Z\}^h\\
&-\frac{1}{4}\{R(u,R(Y,Z)u)X - R(u,R(X,Z)u)Y\}^h\\
&+\frac{1}{2}\{(\nabla_ZR)(X,Y)u\}^t
\end{aligned} \tag{10}$$

for all vector fields X, Y and Z on M. In the above, we denote by ∇ the Levi-Civita connection and by R the Riemannian curvature tensor associated with g. From (8) and (9), it follows that

$$\bar{\nabla}_{X^t}\xi = -2\phi X^t - (R_u X)^h, \quad \bar{\nabla}_{X^h}\xi = -(R_u X)^t, \tag{11}$$

where $R_u = R(\cdot, u)u$ is the Jacobi operator associated with the unit vector u. From (4) and (11), it follows that

$$\begin{aligned} hX^t &= X^t - (R_u X)^t, \\ hX^h &= -X^h + \frac{1}{2}\, g(X,u)\xi + (R_u X)^h. \end{aligned} \tag{12}$$

Using the formulae (10), we get

$$\begin{aligned} \ell X^t &= (R_u^2 X)^t + 2(R_u' X)^h, \\ \ell X^h &= 4(R_u X)^h - 3(R_u^2 X)^h + 2(R_u' X)^t, \end{aligned} \tag{13}$$

where $R_u' = (\nabla_u R)(\cdot, u)u$ and $R_u^2 = R(R(\cdot, u)u, u)u$. By using (7), (8) and (13) we obtain

$$\begin{aligned} h' X^t &= -2(R_u X)^h + 2(R_u^2 X)^h - 2(R_u' X)^t, \\ h' X^h &= -2(R_u X)^t + 2(R_u^2 X)^t + 2(K_u' X)^h, \end{aligned} \tag{14}$$

where we put $h' = \bar{\nabla}_\xi h$.

The above formulae (11)–(14) are also found in [4, 5]. Finally, from (9) and (13) we compute

(15)

$$\begin{aligned} \ell' X^t &= 4(R_u' R_u X + R_u R_u' X)^t + 4(R_u'' X + R_u^2 X - R_u^3 X)^h, \\ \ell' X^h &= 8(R_u' X - R_u' R_u X - R_u R_u' X)^h + 4(R_u'' X + R_u^2 X - R_u^3 X)^t, \end{aligned}$$

where $\ell' = (\bar{\nabla}_\xi \bar{R})(\cdot, \xi)\xi$. We also refer the formula (15) to [8, 10].

4. Proof of Theorems

In this section, we prove our main theorems.

Proof of Theorem 1.1. Suppose that $T_1 M = (T_1 M; \eta, \bar{g})$ satisfies

$$\bar{\nabla}_\xi h = \mu h\phi + \nu h \tag{16}$$

for $(\mu, \nu) \in \mathbb{R}$. From (8) and (12), we compute

$$\begin{aligned} (\mu h\phi + \nu h)(X^t) &= \mu(X - R_u X)^h + \nu(X - R_u X)^t, \\ (\mu h\phi + \nu h)(X^h) &= \mu(X - R_u X)^t - \nu(X - R_u X)^h \end{aligned} \tag{17}$$

for all $X \perp u$. From (14) and (17), we have

$$2R'_u X + \nu X - \nu R_u X = 0 \tag{18}$$

and

$$2R_u^2 X + (\mu - 2)R_u X - \mu X = 0 \tag{19}$$

for all $X \perp u$. From (18), we get

$$\nu(R'_u R_u X - R_u R'_u X) = 0. \tag{20}$$

At first, we take a look at the case (i) $\nu = 0$. Then (18) yields again $R'_u X = 0$. Due to Cartan's result ([7]), this condition is satisfied if and only if (M, g) is locally symmetric. Further, from (19), we see that the eigenvalues λ of R_u are equal 1 or $-\frac{\mu}{2}$, that is, (M, g) is a globally Osserman space (i.e., the eigenvalues of R_u neither depend on the point p nor on the choice of unit vector u at p). However, a locally symmetric globally Osserman space is locally flat or locally isometric to a rank 1 symmetric space ([13]). To show the converse, we treat first a Riemannian space (M, g) of constant curvature c. Then we have $R_u X = cX$ and $R'_u X = 0$ for all $X \perp u$. Thus, from (14) we get

$$h'X^t = 2c(c-1)X^h,$$
$$h'X^h = 2c(c-1)X^t.$$

Moreover, we have from (8) and (12)

$$(\mu h\phi + \nu h)X^t = (1-c)(\mu X^h + \nu X^t),$$
$$(\mu h\phi + \nu h)X^h = (1-c)(\mu X^t - \nu X^h).$$

Hence, we see that the condition (16) is satisfied with $\mu = -2c$ and $\nu = 0$. Next, we consider a two-point homogeneous spaces of non-constant sectional curvature. Those are the compact symmetric spaces of rank 1: $\mathbb{C}P^{n/2}$, $\mathbb{H}P^{n/4}$, the Cayley plane $\mathrm{Ca}P^2$ and their non-compact duals. In such spaces the Jacobi operator R_u has only two eigenvalues α and $\frac{\alpha}{4}$. Thus, we see that $\mu = -8$ (or $\mu = -\frac{1}{2}$) and $\nu = 0$. For example, assuming $R_u X = X$ and $R_u Y = 4Y$, where X and Y are orthogonal, then we get

$$\begin{aligned} h'X^t = 0,\ h'Y^t = 24Y^h, \\ h'X^h = 0,\ h'Y^h = 24Y^t. \end{aligned} \tag{21}$$

Moreover, we have from (12)

$$\begin{aligned} (\mu h\phi + \nu h)X^t = 0,\ (\mu h\phi + \nu h)Y^t = 24Y^h, \\ (\mu h\phi + \nu h)X^h = 0,\ (\mu h\phi + \nu h)Y^h = 24Y^t. \end{aligned} \tag{22}$$

Next, we treat the case (ii) $\nu \neq 0$. Then from (20), we get $R'_u \circ R_u = R_u \circ R'_u$, that is, two operators R'_u and R_u commute. But, since M is real analytic and the eigenvalues of R_u are constants from (19), we find that M is locally symmetric, that is, $\nabla R = 0$ (cf. [1]). By using (18) again we get $R_u X = X$, which implies that M is a space of constant curvature 1. But, from (12), if M is a space of constant curvature 1, then we have $h = 0$. And thus, T_1M satisfies the condition (16). Therefore, M is either a space of constant curvature or a locally symmetric space of rank 1 with $\lambda = 1, 4(\text{or } \frac{1}{4})$. This completes the proof of Theorem 1.1. □

From Theorem 1.1, we obtain the following result.

Corollary 4.1. *(1) T_1M satisfies $\bar{\nabla}_\xi h = 2h\phi$ (i.e., $\mu = 2$ and $\nu = 0$) if and only if M is a space of constant curvature 1 or −1 (cf. [2]).*
(2) T_1M satisfies $\bar{\nabla}_\xi h = 2\phi h$ (i.e., $\mu = -2$ and $\nu = 0$) if and only if M is a space of constant curvature 1.

Proof of Theorem 1.2. Suppose that T_1M satisfies $\nabla_\xi \ell = k(\phi\ell - \ell\phi)$, $k \neq 2$ ($k \in \mathbb{R}$). From (8) and (13), we compute

$$(\phi\ell - \ell\phi)(X^t) = 4(R_u X - R_u^2 X)^h + 4(R'_u X)^t,$$
$$(\phi\ell - \ell\phi)(X^h) = 4(R_u X - R_u^2 X)^t - 4(R'_u X)^h \tag{23}$$

for all $X \perp u$. Then from (15) and (23), we have

$$R'_u R_u X + R_u R'_u X - kR'_u X = 0, \tag{24}$$

$$R''_u X - kR_u X + (k+1)R_u^2 X - R_u^3 X = 0 \tag{25}$$

and

$$(k+2)R'_u X - 2(R'_u R_u X + R_u R'_u X) = 0 \tag{26}$$

for all $X \perp u$. Combining (24) and (26), then we get $(k-2)R'_u X = 0$. From the assumption($k \neq 2$), we have M is locally symmetric, and from (25) we have $R_u^3 X - (k+1)R_u^2 X + kR_u X = 0$. Assuming $R_u X = \lambda X$, then we get $\lambda(\lambda - 1)(\lambda - k) = 0$. Using similar arguments in the proof of Theorem 1, we find that M is a space of constant curvature or a rank 1 locally symmetric space with $\lambda = 1, 4$ (or $\frac{1}{4}$). Conversely, we assume that M is a space of constant curvature c. Then from (15) and (23), we get

$$\ell' X^t = 4c^2(1-c)X^h,$$
$$\ell' X^h = 4c^2(1-c)X^t$$

and

$$(\phi\ell - \ell\phi)X^t = 4c(1-c)X^h,$$
$$(\phi\ell - \ell\phi)X^h = 4c(1-c)X^t.$$

From the above equations, we see that T_1M satisfies the condition $\nabla_\xi \ell = k(\phi\ell - \ell\phi)$ $(k = c)$. Actually, assuming $R_u X = X$ and $R_u Y = 4Y$, then we have from (15)

$$\ell' X^t = 0, \quad \ell' Y^t = 4(16Y - 64Y)^h$$
$$\ell' X^h = 0, \quad \ell' Y^h = 4(16Y - 64Y)^t.$$

Moreover, we have from (13)

$$(\phi\ell - \ell\phi)X^t = 0, \quad (\phi\ell - \ell\phi)Y^t = 4(4Y - 16Y)^h$$
$$(\phi\ell - \ell\phi)X^h = 0, \quad (\phi\ell - \ell\phi)Y^h = 4(4Y - 16Y)^t.$$

After all, we have that M is either a space of constant curvature or a locally symmetric space of rank 1 with $\lambda = 1, 4(\text{or } \frac{1}{4})$. This completes the proof of Theorem 1.2. □

From Theorem 1.2, we immediately have the following:

Corollary 4.2. *(1) T_1M satisfies $\bar{\nabla}_\xi \ell = \phi\ell - \ell\phi$ $(k = 1)$ if and only if M is a space of constant curvature* 0 *or* 1.
(2) T_1M satisfies $\bar{\nabla}_\xi \ell = \ell\phi - \phi\ell$ $(k = -1)$ if and only if M is a space of constant curvature 0, 1 *or* -1.

Also, from (23), we easily obtain that

Corollary 4.3. *T_1M satisfies $\phi\ell - \ell\phi = 0$ if and only if M is a space of constant curvature* 0 *or* 1.

Remark 4.4. *From the result of Perrone(*[15]*), Corollary 4.2 and Corollary 4.3, we see that the following conditions on T_1M are equivalent:*

(1) $\bar{\nabla}_\xi \ell = 0$,
(2) $\bar{\nabla}_\xi \ell = \phi\ell - \ell\phi$,
(3) $\phi\ell = \ell\phi$.

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