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Vibration Analysis of Gearbox for Agricultural UTV using a Reduced-Order Model

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축소 모델 기법을 이용한 농업용 전동식 동력운반차 감속기의 진동 분석

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ABSTRACT

In this study, a model reduction technique was used to develop a precise noise and vibration prediction model for the individual components of a driveline system. The dynamic reduced-order model generated by the Craig-Bampton method was applied to perform dynamic analysis of an electric agricultural power cart. The natural frequency and acceleration response results were analyzed according to the different number of dominant sub-structural modes contained in the reduced-order models. Through the analysis results, it was confirmed that a sufficient number of dominant sub-structures to satisfy the operating conditions should be selected to construct an optimal reduced-order model.

Keywords : Reducing Gearbox(감속기), Component Mode Synthesis(부분구조합성법), Craig-Bampton Method(CB 기법), Dynamic Analysis(동적 해석)

1. Introduction

Today, a variety of agricultural machines such as power carts, brush cutters, and tower wagons, have been changed to be powered by an electric motor to reduce the exhaust gas and promote the health of workers. Although electric agricultural machinery has the advantages of convenience and silence, new noise and vibration issues from a power train system have been emphasized owing to the disappearance of engine noise. Drivers of conventional agricultural machinery are exposed to a large amount of noise and vibration due to the main power source, the engine. To solve this problem, studies on noise and vibration that directly affect drivers, such as cabin noise or suspension vibration,

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have been carried out[1,2,3]. However, to solve the noise and vibration issues of electric power transmission machines, it is necessary to study the noise and vibration of each element of a drive-train system instead of addressing only the noise and vibration in the driver cab. Furthermore, a precise prediction model must be constructed to conduct a study on the noise and vibration of each driveline element.

Finite element models are often used to accurately predict the stiffness, strength, and behavior of individual elements. However, the increase in the size and degree of freedom of the finite element model require much time and a considerable level of equipment performance in the analysis. Therefore, various reduced-order model techniques to reduce finite element models have been developed and used to efficiently use limited temporal and physical resources. Recently, the usability of the model reduction techniques has increased as a variety of model reduction techniques have been utilized to consider the dynamic characteristics of the gearbox housing in commercial programs for gear design and analysis, such as KISSsoft and RomaxDESIGNER[4,5]. In addition, studies on gear transmission error analysis using quasi-static reduced-order models have been published[6].

In this study, the Craig-Bampton method was used to consider the more accurate vibration characteristics of the agricultural machine housing[7]. The Craig-Bampton method is a technique for reducing the degrees of freedom of finite element models by utilizing the constraint mode and normal mode of a fixed interface. Since the development of the method by Craig and Bampton in 1968, it has been most widely used to deal with dynamics vibration problems. Since the 2000s, many studies have been conducted to improve the accuracy and speed of model reduction. In 2004, Bennighof developed the AMLS method through automated multi-level sub-structuring to improve the computational efficiency and analysis speed[8]. Since the 2010s, various studies, such as the those on the method of estimating the error of the Craig-Bampton method[9] and the method of calculating the accuracy of the reduction modeling using the residual mode[10,11], have been conducted by Kim.

In this study, a reduced model of a finite element model of the reducer housing was constructed by using commercial gearbox analysis software, and dynamic analysis was performed with the constructed model. By varying the number of dominant sub-structural modes while constructing the model, it is confirmed that a sufficient number of sub-structures should be contained in the reduced-order model to derive the accurate dynamic analysis results at the target operating conditions.

2. Craig-Bampton method

Consider a global structure $\boldsymbol{\Omega}$ modeled as a finite element as shown in Fig. 1. The linear dynamic equation of this system can be expressed by

$$\mathbf{M}_g\ddot{\mathbf{u}}_g+\mathbf{C}_g\dot{\mathbf{u}}_g+\mathbf{K}_g\mathbf{u}_g=\mathbf{f}_g \tag{1}$$

where

$\mathbf{M}_g$ = square matrices of global mass

$\mathbf{C}_g$ = square matrices of global damping

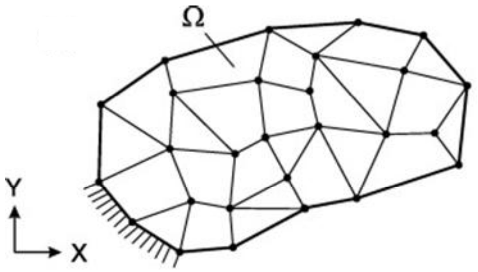
$\mathbf{K}_g$ = square matrices of global stiffness

$\mathbf{u}_g$ = global displacement vector

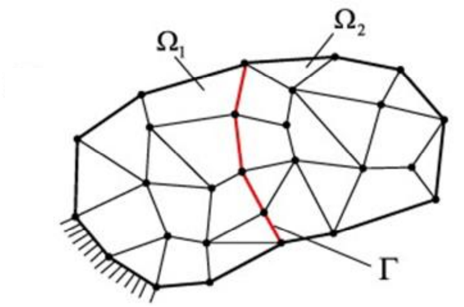
$\mathbf{f}_g$ = global force vector.

As the presence or absence of damping in general structural damping is independent of the coupling of the undamped natural vibration mode, the damping term in Eq. 1 can be ignored.

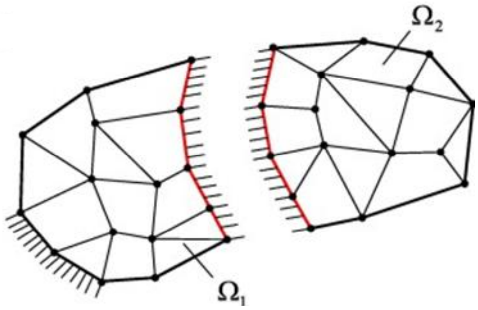
The total mass and stiffness matrices, displacement and force vectors of Eq. 1 can be expressed by sub-structure matrices, coupling matrices and interface matrices by dividing the



(a) The global (non-partitioned) structure $\boldsymbol{\Omega}$



(b) The global structure $\boldsymbol{\Omega}$ partitioned into two sub-structures, $\boldsymbol{\Omega} = \boldsymbol{\Omega}_1 \cup \boldsymbol{\Omega}_2$, $\boldsymbol{\Gamma} = \boldsymbol{\Omega}_1 \cap \boldsymbol{\Omega}_2$



(c) Fixed interface treatment in the CB method

Fig 1. Interface in component mode synthesis[9]

global structure into N_s partial structures, as shown in Eq. 2.

$$\mathbf{M}_g = \begin{bmatrix} \mathbf{M}_s & \mathbf{M}_c \\ \mathbf{M}_c & \mathbf{M}_b \end{bmatrix},\ \mathbf{K}_g = \begin{bmatrix} \mathbf{K}_s & \mathbf{K}_c \\ \mathbf{K}_c & \mathbf{K}_b \end{bmatrix}$$

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix},\ \ \mathbf{f}_g = \begin{bmatrix} \mathbf{f}_s \\ \mathbf{f}_b \end{bmatrix} \tag{2}$$

To convert the displacement vector of the sub-structure into the modal coordinate system, the following process is performed.

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} = \mathbf{T}_{\mathrm{CB}} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{u}_b \end{bmatrix} \tag{3}$$

$$\text{with } \mathbf{T}_{\mathrm{CB}} = \begin{bmatrix} \boldsymbol{\Phi}_s & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

where

$\mathbf{q}_s$ = generalized coordinate vector for the substructural modes

$\mathbf{T}_{\mathrm{CB}}$ = transformation matrix

and $\boldsymbol{\Phi}_{\mathrm{s}}$ is a sub-structural eigenvector matrix computed from the eigenvalue problem of the sub-structure expressed by Eq. 4.

$$\left[\mathbf{K}_s^{(k)} - \lambda_j^{(k)}\mathbf{M}_s^{(k)}\right](\boldsymbol{\phi}^{(k)})_j = 0 \tag{4}$$

$$j = 1,2,\ldots,N_q^{(k)} \ \text{ for } \ k = 1,2,\ldots,N_s,$$

where

$N_q^{(k)}$ = the number of deformable modes in the kth substructure

By dividing the dominant mode and residual mode, Eq. 3 can be expressed as

$$\mathbf{u}_s = \boldsymbol{\Phi}_s\mathbf{q}_s - \mathbf{K}_s^{-1}\mathbf{K}_c\mathbf{u}_b = \left[\boldsymbol{\Phi}_d\ \boldsymbol{\Phi}_r\right]\begin{bmatrix} \mathbf{q}_d \\ \mathbf{q}_r \end{bmatrix} - \mathbf{K}_s^{-1}\mathbf{K}_c\mathbf{u}_b \tag{5}$$

where subscripts d and r denote the dominant and residual terms, respectively.

As the Craig-Bampton method generates a reduced model that approximates the original finite element model with only the dominant modes of the sub-structure, the residual modes are removed, and the transformation matrix is rewritten as

$$\mathbf{u}_g = \begin{bmatrix} \mathbf{u}_s \\ \mathbf{u}_b \end{bmatrix} \approx \overline{\mathbf{u}}_g = \overline{\mathbf{T}}_{\mathrm{CB}} \begin{bmatrix} \mathbf{q}_d \\ \mathbf{u}_b \end{bmatrix} \tag{6}$$

$$\text{with } \overline{\mathbf{T}}_{\mathrm{CB}} = \begin{bmatrix} \boldsymbol{\Phi}_d & -\mathbf{K}_s^{-1}\mathbf{K}_c \\ \mathbf{0} & \mathbf{I}_b \end{bmatrix}$$

where

$\overline{\mathbf{T}}_{\mathrm{CB}}$ = reduced transformation matrix.

Finally, using Eq. 6 in Eq. 1, the following reduced mass matrix and stiffness matrix are obtained.

$$\overline{\mathrm{M}}_p \ddot{\overline{\mathrm{u}}}_p + \overline{\mathrm{K}}_p \overline{\mathrm{u}}_p = \overline{\mathrm{f}}_g,$$

$$\overline{\mathrm{M}}_p = \overline{\mathrm{T}}_0^T \mathrm{M}_g \overline{\mathrm{T}}_0 = \begin{bmatrix} \mathrm{I}_d & \overline{\mathrm{M}}_c \\ \overline{\mathrm{M}}_c & \widehat{\mathrm{M}}_b \end{bmatrix},$$

$$\overline{\mathrm{K}}_p = \overline{\mathrm{T}}_0^T \mathrm{K}_g \overline{\mathrm{T}}_0 = \begin{bmatrix} \Lambda_d & 0 \\ 0_c & \widehat{\mathrm{K}}_b \end{bmatrix},$$

$$\overline{\mathrm{u}}_p = \begin{bmatrix} \mathrm{q}_d \\ \mathrm{u}_b \end{bmatrix}, \quad \overline{\mathrm{f}}_p = \overline{\mathrm{T}}_0^T \begin{bmatrix} \mathrm{f}_s \\ \mathrm{f}_b \end{bmatrix} \tag{7}$$

where

$$\mathrm{I}_d = \Phi_d^T \mathrm{M}_s \Phi_d,$$

$$\overline{\mathrm{M}}_c = \Phi_d^T [\mathrm{M}_c - \mathrm{M}_s \mathrm{K}_s^{-1} \mathrm{K}_c],$$

$$\widehat{\mathrm{M}}_b = \mathrm{M}_b + \mathrm{K}_c^T \mathrm{K}_s^{-1} \mathrm{M}_s \mathrm{K}_s^{-1} \mathrm{K}_c - \mathrm{M}_c^T \mathrm{K}_s^{-1} \mathrm{K}_c - \mathrm{K}_c^T \mathrm{K}_s^{-1} \mathrm{M}_c,$$

$$\Lambda_d = \Phi_d^T \mathrm{K}_s \Phi_d$$

$$\widehat{\mathrm{K}}_b = \mathrm{K}_b - \mathrm{K}_c^T \mathrm{K}_s^{-1} \mathrm{K}_c.$$

3. Dynamic Analysis of the Gearbox

3.1 Analysis model

For the dynamic analysis of the reducer housing, a 7-kW electric power cart reducer of Nara Samyang Gear Inc. was modeled by commercial gearbox analysis software. The actual reducer is shown in Fig. 2, and the modeled gearbox is shown in Fig 3. The reducer model includes two stage helical gear sets, a differential gear set, a differential gearbox case, and a gearbox housing.

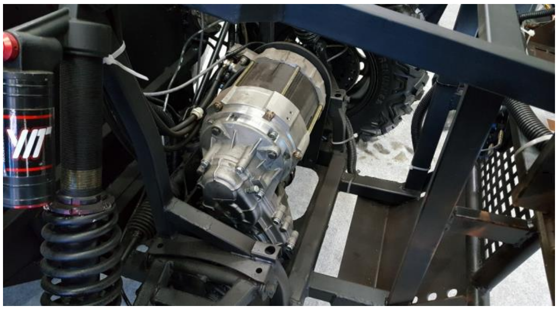
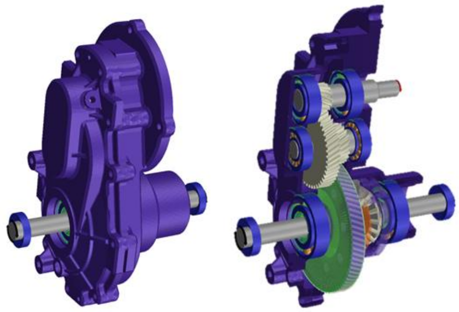


Fig. 2 7-kW Helical reducer, Nara Samyang Gear Inc.



(a) Isometric view (b) Cross sectional view

Fig. 3 A gearbox modeled by commercial gearbox analysis software:

Table 1 Housing material property

Property	Aluminum alloy
Material type:	Isotropic
Youngs modulus: (MPa)	0.33
Density (kg/m^3)	2740
Poisson’s ratio:	0.33
Coeff. expansion: (μm/mC)	21.1

The gearbox housing is fixed to six bearings that are attached to both ends of the two shafts and the differential gearbox case. For realistic modeling, the differential gearbox case, the rim of the helical gear, which is attached to the differential gearbox case, and the gearbox housing were modeled by a finite element model software and imported to gearbox analysis software. The material properties of the housing are listed in Table 1.

The dynamic analysis of the gearbox housing consists of dynamic characteristic analysis and dynamic response analysis. The dynamic characteristic of the housing is calculated through the eigenvalue problem of the reduced-order model.

The dynamic response is calculated from the excitation by the fluctuating loads of the bearings fixing the housing, which are caused by the dynamic response of the housing, a response node was created at the weak point on the housing.

To examine the analytical results according to the size of the reduced-order models of the housing, analytical models were constructed by varying the number of dominant sub-structural modes contained in the reduced-order model. For the smallest reduced-order model, three dominant modes are contained in the model. For a relatively large model, 20 and 40 dominant modes are contained in the model.

3.2 Reduced-order model

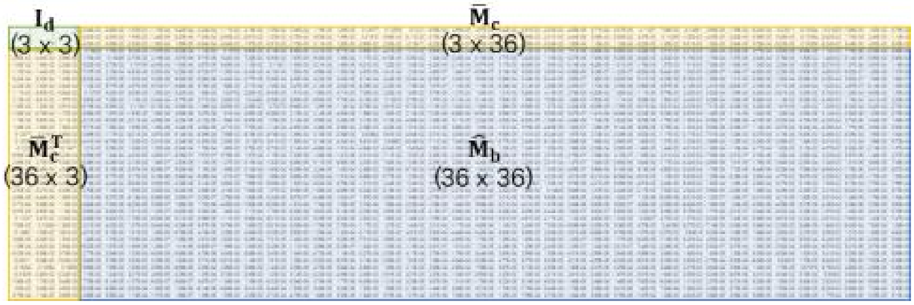
The reduced-order model of the housing produced by the Craig-Bampton method was extracted in the gearbox model. The reduced mass and stiffness matrices were constructed by Eq. 7 and the extracted reduced-order models are shown in Fig. 5-7.

As shown in the figures, the sizes of the dominant sub-structure sub-matrix in the constructed reduced-order model are 3 × 3, 20 × 20, and 40 × 40, respectively, depending on the number of dominant sub-structural modes contained in each reduced-order model. As the housing is fixed to six bearings and each bearing node has six degrees of freedom in the x, y, z axis and rotation direction, the boundary sub-matrix size is 36 × 36. Finally, the size of the coupled sub-matrix is a combination of the number of dominant modes and the number of boundary conditions.

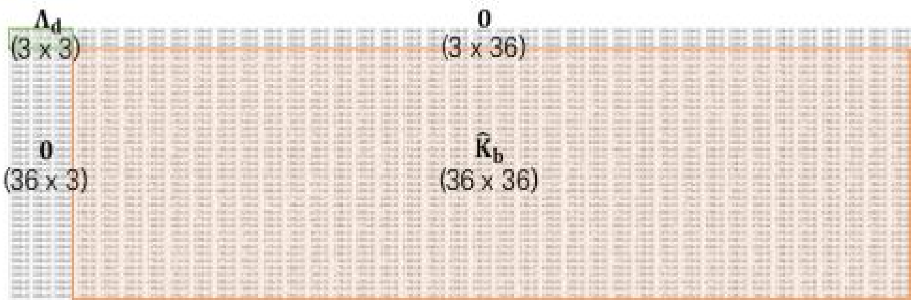
3.3 Analysis results

3.3.1 Dynamic characteristic results

Through the constructed reduced-order model, the natural frequency of the housing according to the order of mode was analyzed with the eigenvalue problem described above.

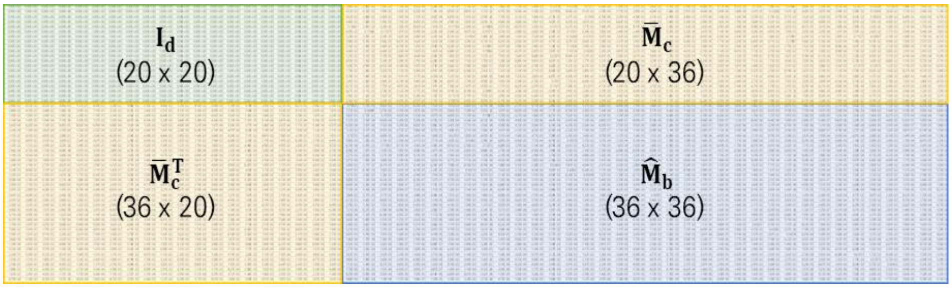


(a) Mass matrix

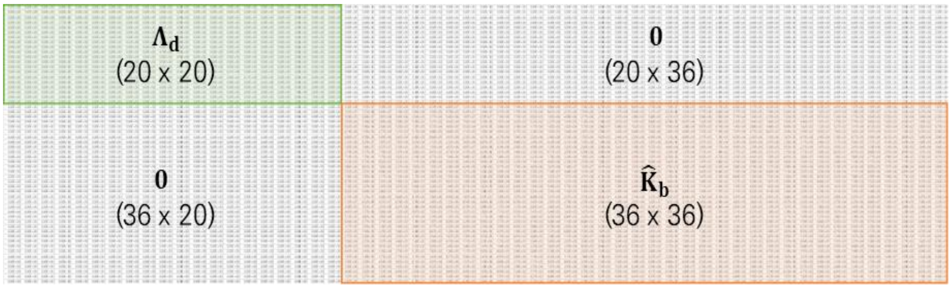


(b) Stiffness matrix

Fig. 4 Reduced model of the gearbox housing containing three dominant modes:

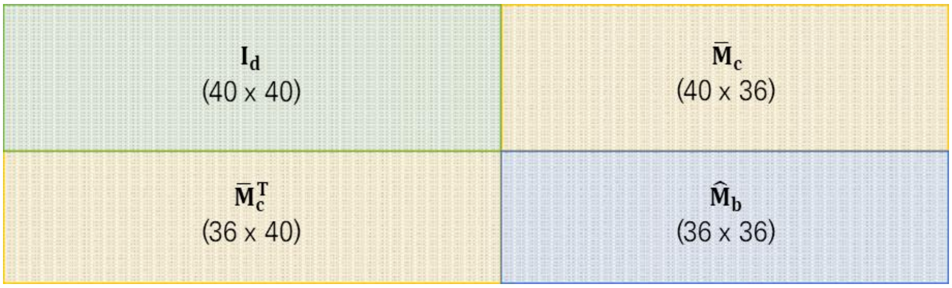


(a) Mass matrix



(b) Stiffness matrix

Fig. 5 Reduced model of the gearbox housing containing 20 dominant modes:



(a) Mass matrix

(b) Stiffness matrix

Fig. 6 Reduced model of the gearbox housing containing 40 dominant modes:

The calculation results show that the natural frequency results differ according to the number of dominant sub-structural modes contained in the reduced-order model. Because the component mode synthesis is a method that generates a reduced-order model that approximates an original finite element model by using the dominant sub-structural modes, it can be predicted that the result would represent a similar result to the original finite element model as the contained number of dominant modes becomes larger.

In this study, the model with the greatest number of dominant sub-structural modes is a model that contains 40 dominant modes. After assuming the result of the dynamic characteristic analysis of the model containing 40 dominant modes is the result of the original finite element model, the relative natural frequency error is compared with the result of the analysis of the model containing fewer dominant modes. As shown in Fig. 7-8, in lower-order mode, models with 3 and 20 dominant modes show low error rates of less than 1%. However, it can be seen that the relative natural frequency error rate of the model with three dominant modes in the higher order increases sharply compared with the model containing the 20 dominant modes. Through the analysis results, it is confirmed that a sufficient number of sub-structures

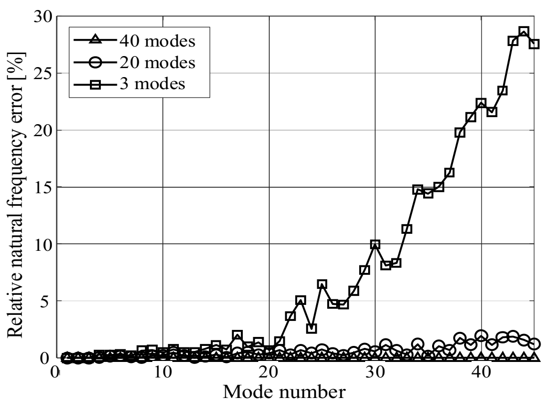


Fig. 7 Relative natural frequency error according to mode number

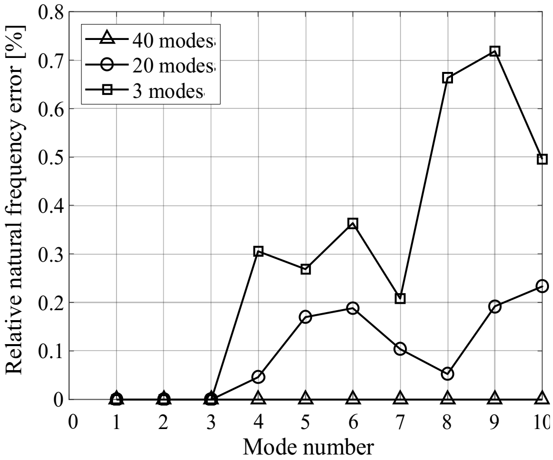
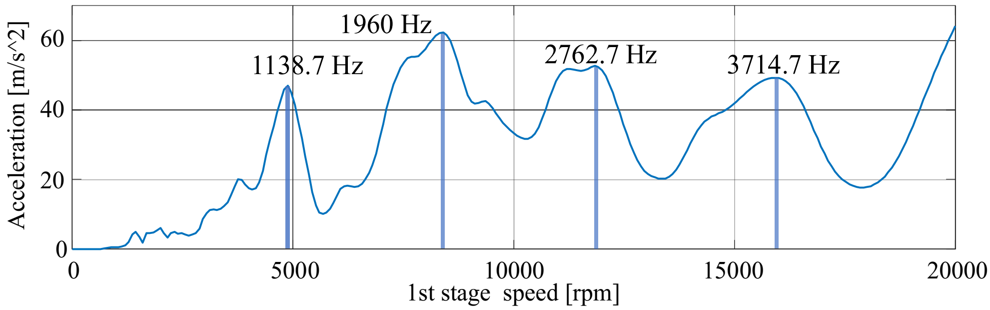


Fig. 8 Relative natural frequency error according to mode number; an enlarged version of the lower-modes

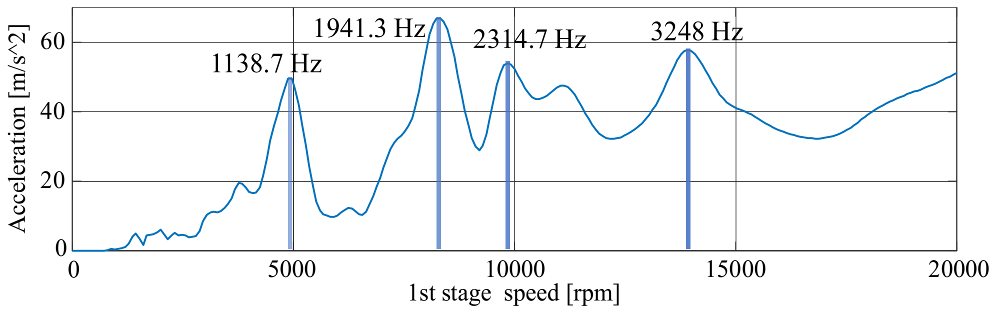
corresponding to the target order of mode should be contained in the reduced-order model in order to analyze the housing natural frequency of the higher-order mode through the dynamic characteristic analysis.

3.3.2 Dynamic response results

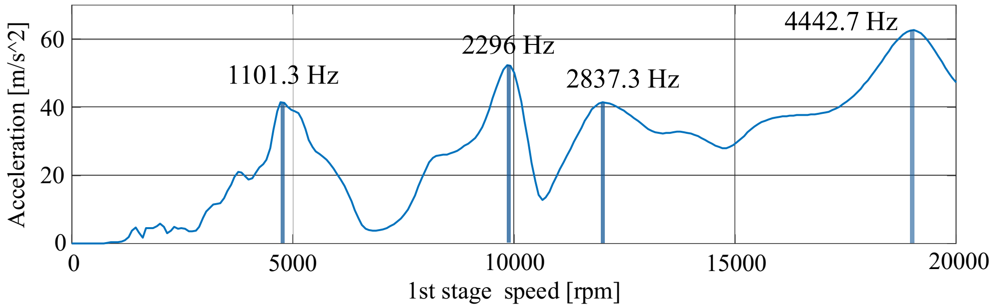
After generating the reaction node at the weak point of the housing, the acceleration due to the transmission error was analyzed as the dynamic response result of the housing. The results of the acceleration analysis are shown in Fig. 9. The input shaft speeds, frequencies and the orders of mode at the acceleration peak points are shown in Tables. 2-5. Similar to the dynamic characteristic analysis, the dynamic response analysis result of the model containing 40 dominant modes is assumed to be the result of the original finite element model. Then, the dynamic response analysis results of the model containing 3 and 20 dominant modes were compared with the result of the model containing 40 dominant modes.



(a) Acceleration result of the gearbox housing containing 40 dominant modes



(b) Acceleration result of the gearbox housing containing 20 dominant modes



(c) Acceleration result of the gearbox housing containing three dominant modes

Fig. 9 Acceleration results according to input shaft speed

As shown in the figures, the model with 20 dominant modes shows similar results to those of the model with 40 dominant modes up to the second acceleration peak point. However, there is a difference in the acceleration peak point from the third peak point.

However, in the case of the model with 3 dominant modes, the result from the second acceleration peak point differs from the model with 40 dominant modes. Similar to the dynamic characteristic analysis, it is confirmed that a sufficient number of sub-structures corresponding to the target driving speed should be contained in the reduced-order model through the dynamic response analysis.

Table 2 Driving condition at the first acceleration peak point

The number of dominant modes contained in the reduced model	Speed (rpm)	Frequency (Hz)	Mode
40 modes	4880	1138.67	11
20 modes	4880	1138.67	11
3 modes	4720	1101.3	10

Table 3 Driving condition at the second acceleration peak point

The number of dominant modes contained in the reduced model	Speed (rpm)	Frequency (Hz)	Mode
40 modes	8400	1960	22
20 modes	8320	1941.3	22
3 modes	9840	2296	24

Table 4 Driving condition at the third acceleration peak point

The number of dominant modes contained in the reduced model	Speed (rpm)	Frequency (Hz)	Mode
40 modes	11840	2762.7	32
20 modes	9920	2314.7	25
3 modes	12160	2837.3	29

Table 5 Driving condition at the fourth acceleration peak point

The number of dominant modes contained in the reduced model	Speed (rpm)	Frequency (Hz)	Mode
40 modes	15920	3714.7	44
20 modes	13920	3248	37
3 modes	19040	4442.67	42

4. Conclusion

In this study, a model reduction method was used to develop a prediction model that predicts noise and vibration for agricultural machinery. By using the Craig-Bampton method, a reduction model of a reducer gearbox housing in electric power cart was constructed. During the model reduction, the number of sub-structure dominant modes was set to be a parameter to produce accurate results under object operating conditions. As a result, three different models were constructed that contain different numbers of dominant modes: 3, 20, and 40 dominant sub-structure modes. With the constructed models, the natural frequencies of the housing models according to the order of mode and the acceleration responses of the weak point on the housing according to the input shaft speed are analytically estimated. Through the analysis results, the following conclusions were obtained:

1. With the increase in the number of dominant sub-structural modes of the reduced-order model, more accurate natural frequency results can be obtained, even in the higher-order modes.
2. With the increase in the number of dominant sub-structural modes of the reduced-order model,

more accurate acceleration response results can be obtained, even under high-speed operating conditions.

3. To construct a reduced-order model to analyze a finite element model, a moderate number of dominant sub-structural modes must be contained in the reduced model.

Acknowledgement

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