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Analysis of Pre-Service Teachers' Interpretation and Utilization of Non-Textual Elements in Mathematics Curriculum Materials

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This study explores how pre-service teachers (PSTs) view, interpret, and utilize non-textual elements (NTEs) in mathematics curriculum. Fifty-two PSTs, who enrolled in a mathematics methods course at a Midwestern university in the U.S., engaged in a three-part task that consisted of evaluations and modifications of NTEs in the sample mathematics curriculum materials. We ascertain what PSTs consider to be the strengths and weaknesses of NTEs, how they define the primary goals of NTEs, and how they would work to modify or adapt existing NTEs with effective teaching in mind. By using the Curricular Noticing Framework, we can better understand how PSTs recognize opportunities within mathematics curriculum and gain a deeper understanding regarding how PSTs' prior experiences may affect their curricular-attending habits, which has consequences for their future teaching. Findings indicate that PSTs understand NTEs to be simply a support for traditional mathematics curriculum, rather than tools on their own. Also, they tend to prefer NTEs that are familiar to them. From our findings, we draw implications for teacher educators who support PSTs' interpretation and utilization of NTEs.

Keywords: non-textual elements, mathematical representations, pictorial illustrations, curricular noticing, pre-service teacher education
MESC Classification: D40
MSC2010 Classification: 97D40

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I. INTRODUCTION

Mathematics curriculum materials consist of not only textual elements (e.g., written texts, mathematical signs and symbols), but also various non-textual elements such as figures, diagrams, and various illustrations in print and technological environments (Filloy, Rojano, & Puig, 2008). The general consensus is that, due to the abstract nature of mathematics, various ways of representing or illustrating mathematical concepts and situations are inevitable as placeholders for thought and windows into students' understanding (e.g., Duval, 2006; McKendree, Small, & Stenning, 2002; National Council of Teachers of Mathematics [NCTM], 2014; Woleck, 2001) and that bolstering the capacity to flexibly use various modes of representation improves students' mathematical understanding (Lesh, Post, Behr, 1987). Because teachers are expected to continuously interact with curriculum materials such as textbooks, teachers' manuals, student worksheets and other types of resources as an aide to guide their instruction (Stein, Remillard, & Smith, 2007), we posit that this level of interaction is a capacity to be developed in teacher education.

In this study, we are particularly interested in the interpretation and potential utilization of non-textual elements (NTEs), which refers to "visual representations that is comprised of components that are *not purely* [emphasis added] verbal, numerical, or mathematical symbolic representations" (Kim, 2009, p. 2). For example, "what is 15 + 19?" is not an example of an NTE because this question consists of solely verbal, numerical, and mathematical notations. When the same question is asked alongside an image of two objects with their lengths labeled, it can be considered as an NTE because it is not purely verbal, numeric, or symbolic.

For the purpose of this paper, we define a few terms to be used throughout. We acknowledge that different types of NTEs exist. Adapting Kim's (2012) classifications, which focus on the presence or absence of mathematical connections, we distinguish two different types of NTEs in this paper: (a) mathematical representations and (b) pictorial illustrations. Pictorial illustrations can be further categorized into illustrations with some mathematical information and illustrations that are not specifically mathematical (see Figure 1).

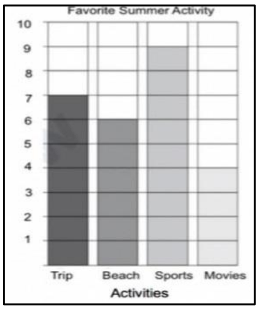
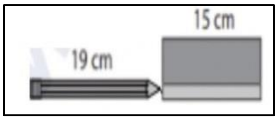
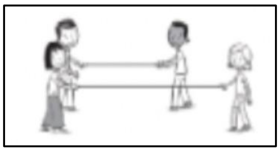
Mathematical representations	NTEs that are typically used to represent mathematical concepts and procedures such as diagrams, graphs, and number lines	
Pictorial illustrations	Illustrations with some mathematical information	
	Illustrations that are not specifically mathematical (i.e., use for non-mathematical purposes such as describing problem context and decorating)	

Figure 1. Types of non-textual elements

Some researchers question whether illustrations contribute to the essential learning process, adding more than decoration and entertainment (e.g., Crisp & Sweiry, 2006; Rasmussen & Bisanz, 2005). Even Berends and van Lieshout's (2009) study reports that illustrations accompanied by arithmetic word problems can have a detrimental effect on students' performance due to irrelevant or redundant sources of information. In mathematics, there have been many studies that focus on specific mathematical representations, such as drawn representations of fractions (e.g., Lee & Lee, 2020; Lee, Brown, & Orrill, 2011). However, studies on the use of NTEs in a broad sense are scarce in mathematics education. That said, even when it comes to mathematical representations, if teachers use representations inappropriately, fundamental mathematical concepts can be distorted and students may become more confused by the use of the representation (Bosse, Lynch-Davis, Adu-Gyamfi, & Chandler, 2016). Therefore, teachers' ability to effectively utilize various NTEs is crucial to student learning, and thus, it is important to provide appropriate opportunities for PSTs to develop this ability in their teacher education program.

Despite the importance of NTEs in teaching and learning mathematics, and the potential for misuse (or lack of use), few guidelines have been developed for teachers and future teachers. In particular, in spite of the wide use of NTEs in recent curriculum materials, the characteristics and roles of NTEs in mathematics curriculum materials are still elusive. Further, there is limited research on PSTs' understanding and performance in this area. Thus, this study aims to examine PSTs' interpretation and utilization of NTEs in mathematics curriculum materials. More specifically, this study explores how PSTs perceive the characteristics and roles of NTEs as reflected in their evaluation of samples from selected mathematics curriculum materials published in the U.S. This effort is particularly relevant because the intended curricula are not necessarily the same as teacher enacted curricula (Remillard, 2005), and the effective selection and use of NTEs ultimately depends on classroom teachers' evaluation of curriculum materials and purposeful enactment. In this regard, we posit that teachers' ability to *attend to* aspects of the curriculum materials, *interpret* what they attended to, and *respond* to the materials in order to make curricular decisions based on the interpretation is important in the effective use of NTEs in curriculum materials. This series of these skills (attending-interpreting-responding) is called *curricular noticing* (Dietiker, Males, Amador, & Earnest, 2018) and it provides a conceptual framework for this study. More specifically, the following research questions guide this project:

(1) What do PSTs perceive the roles and purposes of using NTEs in mathematics curriculum materials to be?

(2) What do PSTs attend to in consideration of the criteria for effective NTEs in mathematics curriculum materials?

(3) How do PSTs interpret sample NTEs' strengths and weaknesses and respond to their interpretations by making modifications to the sample NTEs?

II. LITERATURE REVIEW

This section reviews prior studies associated with NTEs in curricular materials in mathematics education, teacher education, and teacher practice. In a broad sense, NTEs in curricular materials across various subject areas can take many different forms, including photographs, drawings, authentic documents, graphics, diagrams, charts, tables, reproductions of artwork, and typical subject-specific models (e.g., Coleman, McTigue, & Smolkin, 2011; Liu & Qi, 2014; Luo & Lin, 2017). As we distinguish between two types of NTEs (mathematical representations and pictorial illustrations), this review of prior studies is also structured around these two categories.

1. MATHEMATICAL REPRESENTATIONS

In a broad sense, mathematical representations are visible or tangible productions that encode, stand for, or embody mathematical ideas or relationships (Goldin, 2014). The use of various representations is commonly considered necessary in mathematics instruction because abstract mathematical objects (e.g., ideas, concepts, and relations) can only be accessed through representation; thus, activities are affected by how the representation is used (Duval, 2006).

The use of mathematical representations in non-textual forms has its own place in prior studies. The Concrete-Pictorial-Abstract (CPA) approach — or alternatively referred to as Concrete-Representational-Abstract (CRA) approach — is based on Bruner's (1966) conception of the enactive, iconic, and symbolic modes of representation. In the second segment of the sequence (pictorial/representation), various mathematical representations are presented in non-textual forms. Many researchers have explored how the use of mathematical representations in non-textual forms support students' understanding (e.g., Arcavi, 2003; Ruchti & Bennett, 2013).

A well-known classification scheme for types of representations has a place for mathematical representations in non-textual forms. Lesh et al. (1987) highlight the importance of making meaningful connections among contextual, visual, verbal, physical, and symbolic representations. While flexibility and variability in the meaningful use of representations have the utmost importance, some educators (e.g., Arcavi, 2003; Stylianou & Silver, 2004) consider NTEs to be particularly important in mathematics classrooms because students can better attend to relationships among quantities, as well as effectively support their mathematical discourse when they draw diagrams or construct graphs (NCTM, 2014).

Although mathematical representations in non-textual forms are important on their own merit, it should be noted that educators underscore the fluency and flexibility in translating between them and transforming within them, rather than considering one mode of representation to be superior to others (Bruner, 1966; Duval, 2006; Lesh et al., 1987).

2. PICTORIAL ILLUSTRATIONS

Illustrations are used in curriculum materials across all school subjects. Earlier research in the area of pictorial illustrations often identifies the advantages of using illustrations (pictures) in teaching and learning. Additionally, some researchers point out illustrations' conciseness, concreteness, easy access, and their attention-grabbing role as advantages when they are used along with textual information (Levin & Mayer, 1993). However, some researchers also report that illustrations can be distracting if colors are used inappropriately, or unrealistic contexts are provided (Burmark, 2002; Dwyer & Lamberski, 1983; Smith & Watkins, 1972).

There is also another line of research that has focused on identifying types and functions of illustrations in curriculum materials (e.g., Carney & Levin, 2002; Liu & Qi, 2014). In the subject of mathematics, Liu and Qi's (2014) study of NTEs used in Chinese textbooks classified various NTEs into the following types, based on the contents of the illustrations:

(a) Content diagram: An illustration that describes context in mathematics problems and expands mathematical knowledge in realistic situations

(b) Mathematics model diagram: A general model of mathematics that describes basic mathematics concepts and principles

(c) Data figure: An illustration that provides data/information for mathematics problems when the condition is incomplete

(d) Experimental operation diagram: An illustration that shows the experimental process of mathematics activities

The mathematics model diagram in Liu and Qi's (2014) study is similar to the mathematical representation in non-textual form in our study. Additionally, other types of illustrations in their study are similar to pictorial illustrations with or without mathematical connections which we defined in our study.

As pictorial illustrations take different forms, so do they take different functions. Regardless of the slight differences in naming and levels of sophistication, there are several common functions used across different studies in the analysis of curriculum materials. In the review of studies about the effects pictures have on students' textbooks, Carney and Levin (2002) analyzed five functions that pictures have, including a decorative function (when a picture simply decorates the page and has little or no relationship to the text content); a representational function (when the picture mirrors some or all of the text content); an organizational function (when the picture provides a structural framework for the text content); an interpretational function (when the picture helps to clarify difficult text); and a transformational function (when the picture provides systemic mnemonic components to enhance a reader's recall of text information). Liu and Qi (2014) also used similar categories of functions in the analysis of mathematics textbooks (e.g., functions of decoration, characterization, organization, and explanation).

As discussed in the literature (e.g., Levin & Mayer, 1993), illustrations are considered to be important tools for communication when it comes to promoting the effectiveness of curricular materials. However, as Kapyla (2014) points out, "What we see in a picture is affected by our past experiences, our existing knowledge structures and our world view. Many of the personal and social meanings, which we use to 'read' the pictures, are not actually seen in the picture" (p. 233). Because of this, what one student or teacher gains from a pictorial illustration in a textbook may be quite unlike what another student or teacher gains from the same illustration in the same textbook. Because pictorial representations can take various forms and serve different functions, they should be appropriately decoded and interpreted (Metros, 2008). To this end, how pictorial illustrations are implemented in the enacted curriculum relies on the teacher's decoding and interpreting process of illustrations and their functions.

3. QUALITY OF NON-TEXTUAL ELEMENTS

Despite the frequent appearance of NTEs, it is not often that specific frameworks are found that guide or evaluate the selection, interpretation, and usage of NTEs in curriculum materials. One exception is Kim's (2009, 2012) studies that offer an analytical framework around four key concepts: (a) accuracy, (b) connectivity, (c) contextuality, and (d) conciseness. A brief description of each concept is as follows:

(a) Accuracy: Mathematical clarity and rigor of non-textual elements according to the definition of a concept

(b) Connectivity: How closely non-textual elements are related to the mathematical content embodied in the texts

(c) Contextuality: Presentation of mathematical ideas in realistic context(s)

(d) Conciseness: Mathematical succinctness in a non-textual element

In a study that compares the use of NTEs in Korean and U.S. mathematics textbooks, Kim (2009, 2012) reports that pictorial illustrations in particular tend not to be accurate, well connected, or concise enough. Kim's (2009, 2012) exploratory work calls for further systemic analyses regarding how NTEs are perceived by classroom teachers and students and how they are actually used in classrooms.

4. SITUATING THE STUDY

We note that there is a wide range of dimensions of NTEs discussed across multiple studies. However, there is no uniform framework for designing, interpreting, and using NTEs in the classroom context. This means that much is left to the teachers who actually use the curriculum materials that contain various NTEs. Although representational tools are readily available and abundant in the curriculum materials, research urges educators to carefully examine both affordances and constraints of those representational tools for supporting students' learning (e.g., Kamii, Lewis, & Kirkland, 2001) instead of having "magical hope" that the tools will do their job (Ball, 1992).

We also note that, in general, the preceding literature review suggests there is a need to further investigate PSTs' perspectives about how to use NTEs presented in curricular materials. The majority of prior studies in this area have focused on identifying and classifying the types and functions of NTEs used in curricular materials. In addition, while there has been some research conducted regarding how teachers engage and interact with mathematics curriculum, there is not a great deal of research that explores how teachers (and PSTs) use and understand NTEs in curricula. With this gap in literature in mind, this study attempts to address this need by exploring the following curricular noticing skills of PSTs: recognizing opportunities within curriculum materials, understanding their affordances and limitations, and devising strategies for using NTEs in the classroom.

5. CURRICULAR NOTICING: A CONCEPTUAL FRAMEWORK

Considerable research has recently reported that the construct of *professional noticing* is an important teaching practice (Jacobs, Lamb, & Philipp, 2010; Star & Strickland, 2008; van Es, 2011; van Es & Sherin, 2008). Researchers considered noticing to be a series of three interrelated skills teachers should use to make appropriate pedagogical decisions: *attending*, *interpreting*, and *responding*. They called these skills *professional noticing of children's mathematical thinking* (Jacobs et al., 2010).

Another line of research has focused on the interactions between mathematics teachers and curriculum materials. Remillard (2005) theorized that teachers both draw from and shape the curriculum materials as they actively *participate with* the curriculum materials. In other words, curriculum materials are not a static entity to be used by teachers. Rather, teachers continuously interact with the curriculum materials by adapting and manipulating them with their specific purpose and context in mind (Remillard, 2013).

Building on the work of professional noticing of children's mathematical thinking (Jacobs et al., 2010) and Remillard's (2005) notion of participation with curriculum materials, a group of researchers (Dietiker et al., 2018) proposed the term, *Curricular Noticing,* which uses a similar mechanism to professional noticing but for the analysis of teachers' interactions with curriculum materials, rather than that of students' mathematical thinking. Dietiker et al. (2018) define curricular noticing as "a process of interaction between the teacher and the curriculum materials that is initiated by attending and is extended to include interpreting and responding, which are each dependent on the preceding phase(s)" (p. 524). Curricular noticing includes the dynamic and ongoing work of attending, interpreting, and responding in the context of curriculum (see Figure 2).

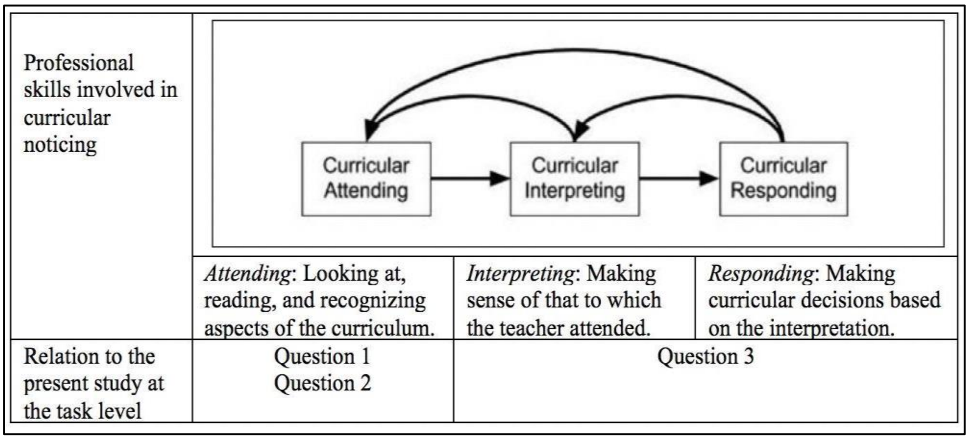


Figure 2. The curricular noticing framework and relation to the present study (adapted from Dietiker et al., 2018)

Curricular attending involves the skills of "viewing information within curriculum materials to inform the teaching and learning of mathematics" (Dietiker et al, 2018, p. 525). Drake and Sherin's (2009) notion of curriculum vision, which refers to knowledge of the design, content, and philosophy of a curriculum, may influence teachers' habits of curricular attending.

Curricular interpreting describes teachers' skills of interpreting what they attend to (Dietiker et al., 2018). Researchers claim that teachers' interpretations of curriculum materials is closely associated with their prior experiences, goals, background knowledge, and their understanding of the design rationale of curriculum materials (Choppin, 2011; Rosenblatt, 1988).

Curricular responding refers to the phase when teachers use "their interpretations to make decisions about what curriculum materials to use and how to use these materials as they both plan and enact instruction" (Dietiker et al., 2018, p. 527). In this phase, teachers may decide to adapt or replace resources in the curriculum materials both before a lesson and in the moment, and their patterns of responding may be changed as they learn through enactment (Brown, 2009; Choppin, 2011; Dietiker et al., 2018). Similar to the notion of curricular responding, there is a considerable line of research on adapting and modifying curriculum. This includes "modified content, instruction, and/or learning outcome to meet diverse student needs" (Hall, Vue, Koga, & Silva, 2004, p. 3). Although the concept of curricular modification is slightly different and bigger than what we want to examine in this study, we recognize that curriculum modification and the curricular responding component in the curricular noticing framework fall under a shared overarching idea.

The researchers of the studies discussed above used the notion of curricular noticing in two contexts: at the particular task level and at the multiple published curricula level. As reflected in the aforementioned research questions, this study uses the notion of curricular noticing at the particular task level involving NTEs in mathematics curriculum materials. The first and second question relate to the first component of curricular noticing — *attending*. While the first question surveys PSTs' conceptions of NTEs' roles and purposes in the general sense, the second question probes what aspects they attend to when they look at sample NTEs. The third question relates to the other two components of curricular noticing — *interpreting* and *responding*.

III. RESEARCH METHODOLOGY

This study adopted an exploratory character offering plausible explanations for further investigation in mathematics teacher education, rather than providing conclusive evidence regarding the quality of prospective teachers' understanding or performance (Yin, 2009). Considering the exploratory nature of this study, we aimed to investigate PSTs' perceptions of the characteristics and roles of NTEs while they engaged in a mathematics methods course in a pre-service teacher-training program. To do so, we designed evaluation and modification tasks that could help explore PSTs' perceptions. Cognizant of the research questions and the nature of the study, this section describes our participants and context, tasks used for data collection, and the data analysis process employed.

1. PARTICIPANTS AND CONTEXT

This study involved 52 PSTs enrolled in a required elementary mathematics methods course at a Midwestern university in the United States. All participants were pursuing their initial elementary teaching certifications. As part of their elementary education major requirements, they had taken two prerequisite mathematics content courses focusing on number theory and geometry. PSTs were required to have some field experiences throughout the program at local schools for participatory observation and limited levels of instruction under the supervision of cooperating teachers. The university's educational resources library had reserved published curriculum materials in print format, including teacher guides, student textbooks, and other resources from various publishers for PSTs' perusal.

Because the course was the only mathematics methods course offered to PSTs in the program, it was challenging to focus on specific topics. At the time of data collection, PSTs had reviewed several documents, including Common Core State Standards for Mathematics (CCSSM) and the accompanying Standards for Mathematical Practices (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and guidelines for effective teaching practices (NCTM, 2014), which underscore the use of and connections among various mathematical representations. Throughout the semester-long course, PSTs were exposed to mathematical representations while discussing how specific mathematics concepts and procedures can be represented with NTEs (e.g., visually represent the process of ¼ x ½). Other than that, the limited time allotted for the course did not allow for in-depth discussions of various NTEs, in particular of pictorial illustrations, their strengths and limitations, or how they are used in enacting curriculum prior to administering the tasks presented in the following section. Thus, this exploratory study focused on PSTs' perceptions of NTEs in the curriculum materials and the curricular noticing skills they bring to the methods course.

2. TASKS AND DATA COLLECTION

A task-based written questionnaire was developed by the researchers and provided to PSTs as an individual assignment to be completed outside of class. The assignment consisted of three parts. To answer the first and second research questions, Part 1 intended to check PSTs current perceptions of the role and function of NTEs in curriculum materials. As particular beliefs and expectations may influence what PSTs attend to as it relates to curricular noticing (Dietiker et al., 2018; Drake & Sherin, 2009), we first examined PSTs perceptions. In Part 1, PSTs were asked to briefly describe their opinions on the following two open-ended questions: (a) What are the roles, functions, or purposes of non-textual elements in mathematics curriculum materials (e.g., textbooks, teacher guides, worksheets)? (b) List at least three criteria for exemplary non-textual elements in mathematics curriculum materials (e.g., textbooks, teacher guides, worksheets).

To answer the third research question, the researchers designed Parts 2 and 3. Part 2 asked PSTs to evaluate the effectiveness of 11 sample problems that contain NTEs and to identify the strengths and weaknesses of each example. This evaluation task intended to explore PSTs' curricular interpreting (Dietiker et al., 2018) patterns. The following prompts were used:

- Evaluate the problems in order to indicate the effectiveness of each non-textual element. Use a scale of 1–10 with 10 representing the most effective case of using non-textual elements and 1 representing the least effective case of using non-textual elements. Please note that each problem is being ranked on its own and not in comparison to the other visuals provided.

- Indicate strengths of each non-textual element.
- Indicate weaknesses of each non-textual element.

All examples used in Part 2 were from *Bridges in Mathematics* (2017), which is a K-5 curriculum guided by the CCSSM. The Appendix shows 11 sample problems used in Part 2. We limited our selections to NTEs used in the presentation of mathematical problems. Two examples (Examples 4 and 5) contain two types of NTEs to probe how this influences PSTs' evaluation.

In the process of selecting a variety of NTEs, we considered several aspects. First, we considered types of NTEs explained earlier in Figure 1: (a) mathematical representations, and (b) pictorial illustrations with or without mathematical information. Then, we looked into the essentiality and contextuality of each example considering the functions of NTEs in the literature. Essentiality refers to whether or not the NTEs themselves provide critical information for problem solving. We focused on this aspect for item selection, adapting the idea of *connectivity* in Kim's (2009, 2012) work and Carney and Levin's (2002) distinction between decorative function and representational function. High essentiality means that visuals can present stand-alone information and problems cannot be solved without them. For example, pictorial illustrations in Examples 1 and 3 are noted as high in essentiality because the illustrations contain the amounts of two addends, which are not stated in the text. Low essentiality means the absence of the visual would not affect problem context because the text provided all necessary information.

Contextuality is adapted from Kim's (2009, 2012) framework, which refers to the level of contextualization of mathematical ideas in realistic contexts. High contextuality means that mathematizing realistic objects/contexts is apparent. Low contextuality means that either realistic objects/contexts are missing or realistic objects/contexts are shown without a mathematizing component. For example, the one apple in Example 2 (see Appendix) is noted as a low contextuality item because it just shows a realistic object without key mathematical ideas. Whereas the pictorial illustration in Example 3 (see Appendix) is noted as a high contextuality item because the realistic context (riding a train) illustrates the amount of two addends. All mathematical representations are noted as low in contextuality because they do not have realistic contexts to explain the concept or process. We indicate the characteristics of each example in Table 1.

Table 1. Characteristics of samples used

Example	NTE Type	Essentiality	Contextuality
1	Pictorial Illustration	High	High
2	Pictorial Illustration	Low	Low
3	Pictorial Illustration	High	High
4	Mathematical Representation Pictorial Illustration	Low (number line) Low (illustration)	Low (number line) High (illustration)
5	Mathematical Representation	High	Low
6	Pictorial Illustration	Low	High
7	Mathematical Representation Pictorial Illustration	Low (circle) Low (illustration)	Low (circle) High (illustration)
8	Pictorial Illustration	High	High
9	Pictorial Illustration	High	High
10	Mathematical Representation	High	Low
11	Pictorial Illustration	Low	High

Part 3 asked for PSTs' modification strategies. To accomplish this, we used the following prompt: "Choose at least two examples from the 11 examples in Part 2 that you rated as the least effective use of non-textual elements. (a) How would you modify the existing examples to make them more effective? (b) Provide justification for your modification in detail." This task was designed to probe PSTs' engagement in the third phase of the curricular noticing framework, that is, curricular responding (Dietiker et al., 2018).

3. DATA ANALYSIS

We analyzed the data both qualitatively and quantitatively. The quantitative analysis provided descriptive information to give an overall picture of PSTs' perceptions. Those included the mean scores of the numeric ratings of the 11 sample NTEs. For the qualitative data (i.e., PSTs' written responses via the task-based questionnaire), we followed the inductive content analysis approach by employing data-driven open coding (DeCuir-

Gunby, Marshall, & McCulloch, 2011; Grbich, 2013). The initial stage open coding involved low-level coding by focusing on drawing out the themes explicitly presented by the PSTs (Carspecken, 1996). We used these initial raw codes to include the participants' opinions. Following open coding, responses were analyzed again using high-level codes that required investigators' informed abstraction and interpretation (Carspecken, 1996). We carried out constant comparative analysis during the coding process in order to maximize consistency of grouped units and ensure clear distinctions between categories. We (coders) secured a consensus among categories and their definitions. Before coding the entire data, the investigators obtained the inter-coder reliability in the form of percent agreement. Once the themes for coding were identified, two researchers independently coded a random sample of about 10% of the data. The concordance between the two coders was 90%. For the rest of the data, the two researchers jointly coded to resolve coding discrepancies. After we completed the coding, we identified the frequencies of coded themes. We used descriptive analysis to report data from the ranking tasks of 11 samples. Finally, the investigators completed a horizon analysis (Carspecken, 1996), which involves making objective, subjective, and normative claims, while working to distinguish between each of these types of claims.

IV. FINDINGS

This section summarizes the characteristics of PSTs' perceptions of NTEs in mathematics curriculum materials as expressed in their declarative accounts and their evaluation and modification of the sample NTEs. In the following subsections, frequency is reported as the percentage of specific themes present in individual PSTs' responses. Some PSTs addressed multiple aspects and thus we coded their work into multiple categories, which explains why some columns total more than 100%.

1. ROLES AND PURPOSES OF NON-TEXTUAL ELEMENTS IN MATHEMATICS CURRICULUM MATERIALS

To answer the first research question, we analyzed the data collected from the first prompt in Part 1 of the survey. From PSTs' responses regarding the roles and purposes of NTEs in mathematics curriculum materials, we observed multiple perspectives. Table 2 shows the major inductive categories that emerged in the PSTs' written statements by using the inductive content analysis approach.

Table 2. PSTs' perceived roles and purposes of non-textual elements

Inductive categories identified	Examples (representative phrases in PSTs' responses)	Frequency (n=52)
Explaining the problem's context: NTEs take a supporting role to the given textual information and focus on explaining the problem's context	· Clarify the given text by providing an alternative source of information · Further explain and elaborate on the given text · Help students comprehend or visualize the problem's context, which is described in the text · Display important information in the text · Add additional information that was not clearly provided in the text	37 PSTs (71%)
Relating mathematics content: NTEs show the mathematics content in the problem more explicitly	· Enhance understanding of mathematics content · Make mathematics concepts more accessible · Organize relevant mathematics concepts · Assign an image to a mathematics concept	31 PSTs (60%)
Accommodating individual differences: NTEs help students with unique traits or special needs	· Support – visual learners – struggling readers – younger students – students with a language barrier – mathematically struggling students	14 PSTs (27%)
Turning abstract to concrete: NTEs make abstract concepts concrete	· Make abstract concepts concrete and easier to understand · Provide concrete examples	11 PSTs (21%)
Informing process/methods: NTEs hint toward or show steps or methods to use when finding the solution	· Inform students of the steps to take · Help students know/understand the process · Offer simplified methods/processes · Provide or hint at problem solving methods	7 PSTs (13%)
Providing a language for communication: NTEs provide students a language for better communication	· Promote discussion of new concepts · Allow students to express their thinking · Provide a common language to aid in communication	7 PSTs (13%)
Relating problems to real life: NTEs connect mathematics to real life	· Relate mathematics problems to the real world or students' lives	6 PSTs (12%)
Motivating/entertaining students: NTEs motivate students and add an aspect of entertainment	· Motivate students · Provoke student interest · Make the process enjoyable	5 PSTs (10%)
Using for future reference: NTEs can be useful for future use	· Provide a tool for students' future use · Provide a tool for students' future reference	4 PSTs (8%)
Assisting teachers/instruction: NTEs assist teachers with instruction	· Assist teachers when designing their instruction or teaching students	2 PST (4%)

The majority of PSTs considered explaining the problem's context by taking a supporting role to the written texts to be the main role of NTEs. In their view, while the written texts can be used independently or stand alone, the NTEs complement the texts. Thus, texts are considered essential while NTEs take a supporting role. In terms of what form NTEs take and how they support the written texts, two dominant inductive categories were identified. About 71% of PSTs stated that NTEs would be helpful in *explaining the problem's context*, and about 60% of PSTs considered NTEs to be a tool for *relating mathematics content* by organizing relevant mathematics concepts in a more accessible way and offering an explicit image associated with the concept.

Other roles and functions were also identified, but by a much smaller number of PSTs. For example, 27% of PSTs noted that NTEs could be used for *accommodating individual differences* for those students with unique traits or special needs instead of as a general tool for all students. A relatively small number of PSTs considered NTEs' roles in *motivating/entertaining students* (about 10%) and in *informing process/methods* toward the solution (about 13%). Notably, only two PSTs mentioned NTEs would be helpful for teachers when designing instruction and teaching students.

2. CRITERIA FOR EFFECTIVE NON-TEXTUAL ELEMENTS

To answer the second research question, we analyzed the data collected from the second prompt in Part 1 of the survey. When asked to indicate at least three criteria for good NTEs to use when presenting mathematics problems, PSTs named various aspects of NTEs, as shown in Table 3.

Table 3. PSTs' criteria for effective non-textual elements

Inductive categories identified	Examples (representative phrases in PSTs' responses)	Frequency (n=52)
Easiness	· Should be easy to understand/follow · Should be easy to see, read, identify	26 PSTs (50%)
Alignment with the text	· Support the text's role · Align with the text · Clearly depict/describe the written problem's contexts · Must match with the written texts · Directly relate to the problem · Add additional information that is relevant to the written texts	19 PSTs (37%)
Enhance student	· Provoke an appropriate thought process	18 PSTs

understanding/learning	· Provide scaffolding for current or future lessons · Further/extend the understanding of the questions · Should not lead students to any misconceptions about the concept being taught	(35%)
Accuracy/Precision	· Precise drawing/proportional drawing/pictures are to the scale · Accurate labeling	16 PSTs (31%)
Connection	· Relatable to students' lives · Connection to students' prior knowledge/experience · Real-world application/connection	15 PSTs (29%)
Appropriate/Relevant level of content	· Should not hinder the students' learning by being too in-depth · Match the grade level, age, content, objectives, and standards	11 PSTs (21%)
Appropriate level of complexity	· Not too complicated/overwhelming for students · Efficiently, clearly, and simply organize the information	10 PSTs (19%)
Offer a process to follow	· Offer steps to take · Offer problem solving tools/methods	9 PSTs (17%)
Ensure student engagement	· Allow student engagement and exploration · Keep students engaged	7 PSTs (13%)
Other	· Visual appeal (e.g., use of colors, appropriate sizing) · Versatility (e.g., flexible, reusable, transformative) · Diversity (e.g., include diverse people and culture) · Concrete representation (e.g., use concrete visuals) · Use of mathematical representations (e.g., use number lines, graphs, charts, graphic organizers) · Meaningful (e.g., should not just be a picture for the sake of being a picture) · Consistency (e.g., should be used in a consistent manner) · Multiple forms (e.g., multiple representations should be used)	5 PSTs (10%) 5 PSTs (10%) 3 PSTs (6 %) 3 PSTs (6%) 3 PSTs (6%) 3 PSTs (6%) 2 PSTs (4%) 2 PSTs (4%)

Many criteria for effective NTEs proposed by PSTs were aligned with their perceived roles and purposes of NTEs that are summarized in the previous section. In addition, PSTs addressed additional criteria, such as the merits of being easily understandable, presented with precision, and having an appropriate content level or complexity.

3. EVALUATIONS AND MODIFICATIONS OF SAMPLE PROBLEMS

To answer the third research question, we analyzed the data collected from Parts 2 and 3 of the survey. Figure 3 shows PSTs' average rating for each example in the evaluation task (see sample in Appendix). PSTs indicated their evaluations on a scale of 0 to 10, with 10 being the highest score, and described each item's strengths and weaknesses. PSTs also selected at least two NTEs they felt were used ineffectively and suggested modifications for improvement.

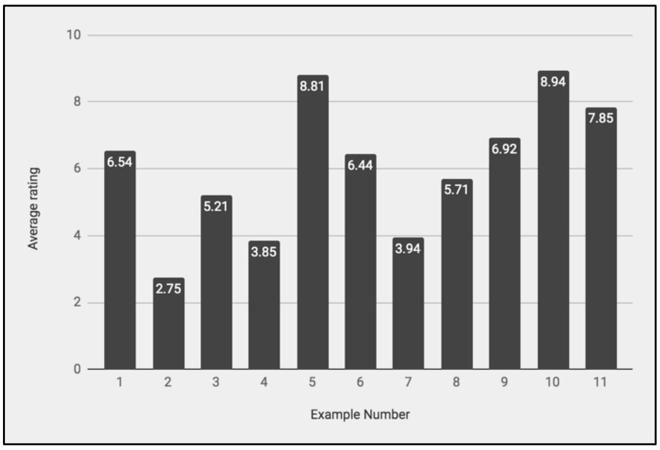


Figure 3. PSTs' average rating for each example

Note. This figure shows overall rating of each example. That means that the two NTEs in Examples 4 and 7 were not evaluated separately. The PSTs' written justifications reported in the following sections offer further explanation.

To provide an overview, the examples with high essentiality (see Table 1) generally received good evaluations (all average ratings are higher than 5). The two most highly rated examples (Examples 5 and 10) contain only mathematical representations. The PSTs' evaluations of examples with high contextuality (see Table 1) show the mixed results. In particular, the examples with two NTEs (Examples 4 and 7), which contain not only pictorial illustrations with high contextuality, but also mathematical representations with low contextuality, were not perceived well by the PSTs. Because the overall ratings for these two examples do not tell which NTEs were more influential in PSTs' overall ratings, the written justifications presented in the following section can offer more nuanced opinions. In the following sub-sections, we report illustrative examples of strengths and weaknesses of NTEs, as described by the PSTs, as well as modifications for three lowly rated examples (Examples 2, 4, and 7) and three highly rated examples (Examples 5, 10, and 11).

1) Justifications For Lowly Rated Examples And Suggestions For Modifications

Examples 2, 4, and 7 received less than 4 points on average (see Figure 4). Example 2 contains one decorative pictorial illustration with low essentiality and low contextuality. Examples 4 and 7 are similar in that both provide two NTEs: one pictorial illustration of the problem's context with low essentiality and high contextuality), in addition to a mathematical representation with low essentiality and low contextuality. Several criteria for effective NTEs as reported in Table 3 were heavily used in addressing the strengths and weaknesses PSTs perceived. Those included *aligning with the text*, *offering process to follow*, *accuracy/precision*, and *meaningfulness.*

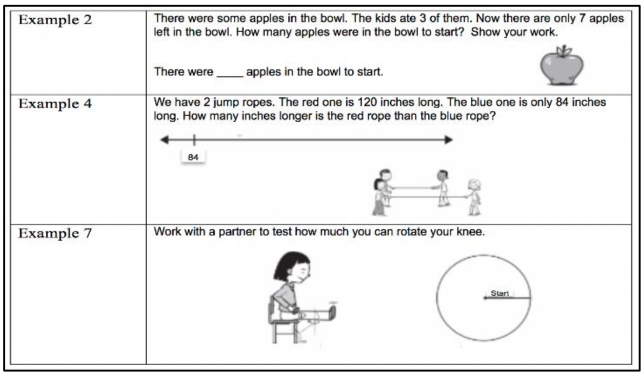


Figure 4. Lowly rated examples

In the examination of Example 2, nearly 60% of PSTs stated that the NTE had no strengths. Other weaknesses of the NTE that PSTs identified were a lack of alignment with the text (54%) and a lack of meaningfulness (44%). Some PSTs pointed out that one apple did not accurately depict the problem's context and that it could be distracting or confusing due to misalignment with the text, and that this NTE (apple) was nothing but a decoration without any meaning. In response to the weaknesses addressed, PSTs attempted to modify this example by providing the related quantities of apples more explicitly in the visuals, as shown in the following samples of PSTs' work (see Figure 5):

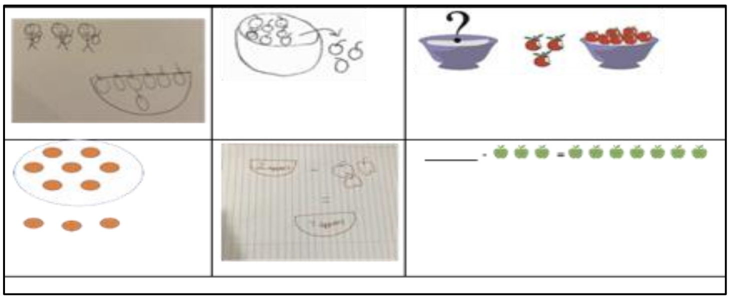


Figure 5. PSTs' suggested modifications for Example 2

For Example 4, there was a split discussion on the strengths and weaknesses mainly due to the fact that this example contained two types of NTEs. Mostly referring to the pictorial illustration, 67% of PSTs stated that *aligning with the text*, one of the criteria in Table 3, was the strength of Example 4. About 65% of PSTs indicated that *aligning with the text* was a weakness of Example 4 because the mathematical representation (an incomplete number line) was not aligned with the text. This is expressed in the following excerpts of PSTs' responses:

- "The number line does not contain all numeric information provided (e.g., 120)."
- "There should be two number lines to represent two ropes."
- "Arrows on the number line are not relevant to the problem."
- "Number line is not relevant to the problem's context."

In Example 4, 31% of PSTs negatively evaluated the NTEs by referring to the criterion of accuracy/precision. It is notable that PSTs interpreted and applied the criterion of accuracy/precision from many different perspectives, as shown in the following statements:

- "It [the pictorial illustration] is not to scale."
- "Number line is not accurate because it is incomplete."
- "Ropes in the illustration are not to the same scale as the number line given."
- "Illustration does not show actual difference between the two lengths (shorter rope looks like it is more than half the length of the longer rope)."

When it comes to the other criterion, *offering process to follow*, 21% of PSTs considered it to be a strength. They made reference to the mathematical representation (number line), expressing it met this criterion well. However, 19% of PSTs considered Example 4 to be weak in this area because the pictorial illustration did not offer any process to follow.

All PSTs who suggested modifications for Example 4 attempted to modify the number line by providing more details as shown in the following samples (see Figure 6):

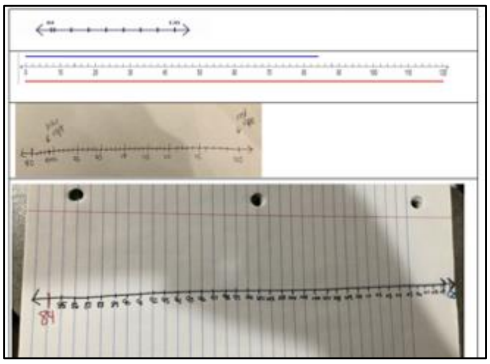


Figure 6. Suggested modifications for Example 4

As for Example 7, 69% of PSTs considered this example to be effective, mainly because of the pictorial illustration that was closely aligned with the text. The 50% of PSTs who considered this example to be ineffective mostly referred to the mathematical representation (circle), explaining that a simple circle does not offer any needed process. Similarly, about 21% of PSTs pointed out that the mathematical representation (circle) was not meaningful, or it was even more confusing than helpful when it came to think about how to proceed. Additionally, PSTs mentioned that the two NTEs provided did not communicate well with each other. Below are some excerpts:

- "Many may be confused about what to do. It does not say which way to rotate or how fast. It could be dangerous."
- "The full circle with the line marked "start" means nothing. No one can rotate their knees in a full circle."
- "I don't understand why there is a circle with a labelled radius."
- "Pictures are not self-explanatory. The two illustrations don't match up with each other."

As for modifications of Example 7, PSTs suggested using different contexts (other than knee rotation), a more realistic illustration, and more signs/descriptors to make the directions clear (see Figure 7):

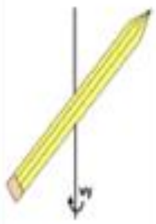
		"Darken the outline of the leg movement, and include arrows for the direction of movement."	"If I was to change this question, I would have had the numbers written within the visual representation instead of having students measure their own knee rotation. I believe that it would get confusing for students to measure that on their own and their answers could be way off from what a reasonable answer would and could be."

Figure 7. Suggested modifications for Example 4

2) Justifications for highly rated examples and suggestions for modifications

Examples 5, 10, and 11 were the three most highly rated items. They received more than 7 points on average, out of 10 possible points. As shown in Figure 8, Examples 5 and 10 contain mathematical representations (e.g., area model of a fraction, a graph) with high essentiality, in that NTEs provide all necessary information, and low contextuality due to the absence of the depiction of realistic contexts. Example 11 contains one NTE with low essentiality and high contextuality. Several criteria for effective NTEs, as shown in Table 3 were used when PSTs discussed the strengths and weaknesses of these examples, including *aligning with the text*, *easiness*, and *accuracy/precision*.

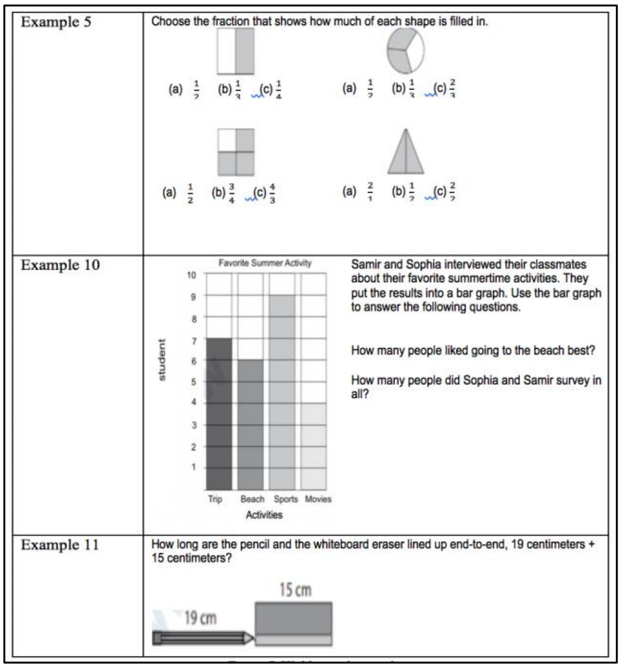


Figure 8. Highly rated examples

In reviewing Example 5, 33% of PSTs stated that there was no weakness in the presented NTEs. About 25% of PSTs considered this example to be effective due to its easiness to understand, especially as these are familiar NTEs for fractions. Additionally, 33% of PSTs stated that the NTEs met the criterion of accuracy/precision. For some aspects addressed as weaknesses, 31% of PSTs pointed out that these NTEs are too simple and obvious to enhance students' learning, and 8% of PSTs considered these NTEs to not have merits capable of engaging students in the learning process. As for modifications, no PSTs selected this as one of two items they would improve.

Example 10 was also perceived positively by the majority of PSTs due to its easiness (44%), accuracy (40%), visual appeal (38%), and alignment with the written text (25%). While the majority of PSTs (about 44%) considered there to be no weaknesses in this item, a small number of PSTs (2%) did say there was not much incentive for student engagement in the straightforward, mathematical representation used. About 19% of PSTs addressed the weaknesses of the problem itself, not necessarily the weaknesses of the NTEs provided. The following statements show the main argument PSTs made for Example 10:

- "This problem asked us to solve a problem without all the necessary information, so you have to make assumptions (e.g., Did students all respond just once? Were students permitted to mark more than one activity? It is not clear whether multiple selections were allowed or not.).”

• “Do we assume that all students voted for just one activity?”
• “Are there any students who did not respond at all?”

For modifications, no PSTs selected this as one of two examples they would improve.

For Example 11, 25% of PSTs stated that there was no weakness and a majority of PSTs (about 67%) thought *alignment with the text* was a strength of this example. About 31% of PSTs referred to *accuracy/precision* to address the presented NTE’s strength, by pointing out that the NTE is drawn to scale, includes labels, and shows objects side by side. In regard to weaknesses, PSTs mentioned a lack of precise illustration of the real objects (33%) and a lack of an ability to enhance student understanding (25%). Additionally, one PST argued that this problem would not need any visuals, as a specific number sentence was given. Two PSTs selected this example for modification and both attempted to improve accuracy/precision by including pictures of real objects or adding a number line/ruler (see Figure 9).

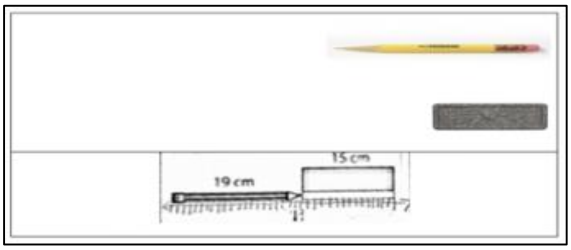


Figure 9. Suggested modifications for Example 11

PSTs’ responses to the evaluation and modification tasks uncovered their collective perceptions of the purposes of NTEs in mathematics curriculum materials and suggest that there is a wide range of individual criteria and preference when it comes to the use of NTEs in mathematics curriculum. Our findings also suggest that when thinking about, interpreting, and responding to NTEs in the curriculum materials, PSTs’ interpreting and responding skills are pretty much bounded by that to which they initially attend. This is a critical part of this particular study, as not all PSTs attended to the same pieces of each sample NTE.

Before presenting our discussion and implications, we recognize some limitations of our research. First, we limited our sample NTEs to problems presented in students’ textbooks from one publisher. Second, the generalizability of our results might also be limited, given our choice to rely on written responses as the only measure, without opportunities to follow up with clarifying questions. In spite of these limitations, we believe that PSTs had multiple opportunities to demonstrate their thinking. We used a series of tasks where the PSTs exhibited their ideas in different situations (e.g., evaluation and modification). We believe the use of open-ended, written measures, which asked PSTs to describe their thoughts, adequately reflect the PSTs' overall perceptions of, and ability to use, NTEs.

V. DISCUSSION AND IMPLICATIONS

A series of tasks in this exploratory study helped us to look into the PSTs' collective perceptions of quality NTEs as they attended, interpreted, and responded to the NTEs presented in existing curriculum material. This section revisits the findings from this study and suggests areas for further exploration in mathematics teacher education.

1. PSTS' PERCEIVED ROLES OF NON-TEXTUAL ELEMENTS: LIMITATIONS AND POSSIBILITIES

The inductive categories presented in Table 2 show that PSTs attend to a wide range of aspects of NTEs. We note that the two dominant inductive categories – *explaining the problem's context* and *relating mathematics content* by organizing relevant mathematics concepts in a more accessible way and offering an explicit image associated with the concept – were in line with some of the forms and functions prior studies identified. Those included the role of describing context in mathematics problems and expanding mathematical knowledge in realistic situations (Liu & Qi, 2014), as well as NTEs' representational function that mirrors some or all of the text content (Carney & Levin, 2002).

We also note that a small number of PSTs addressed other roles and functions, which were not explicitly shown in the previous studies (e.g., Carney & Levin, 2002; Liu & Qi, 2014). For example, some PSTs (about 27%) stated that NTEs were for *accommodating individual differences*, especially for those students with unique traits or special needs. This may indicate that some PSTs see NTEs as more of an accommodation for struggling learners, rather than as a curricular element that needs attending to in order to instruct all students. Additionally, only two PSTs addressed the role of NTEs in *assisting teachers' instruction*, implying that consideration of NTEs is an important part of instructional planning and enactment.

Through data collection and analysis, we found that most PSTs exhibited somewhat limited understanding of how and why NTEs can or should be used during instruction. A critical mass of PSTs sees the value of NTEs as simply a 'supporting tool' for textual elements rather than a piece of curricular material that has merit on its own, or that can be used for a specific educational purpose.

Some PSTs' limited understanding of NTEs also presented itself when a small number of PSTs reported seeing NTEs as more of an accommodation for students with unique traits or special needs, rather than as a curricular element that needs attending to in order to instruct all students. The idea of NTEs acting only as accommodations rather than as a teaching tool speaks to the possibility that the intended curriculum might differ from the enacted curriculum. This can occur if NTEs are not used deliberately in teaching which can result in missed opportunities in the mathematics classroom (Kapyla, 2014). Additionally, when curricular noticing requires attending to, interpreting, and then responding to NTEs, these steps might be compromised if NTEs are viewed as a tool for only some students, or as a tool that takes on only a supporting role. Kapyla (2014) explains that the reading of pictures is not explicitly taught, but it should be. This aligns with the work of Ball and Forzani (2009) as well as the Curricular Noticing Framework, which support the idea that teachers need direction when it comes to attending to NTEs and interpreting them in a way that will make them a more valuable part of instruction. Teachers (and PSTs) need direct, targeted practice concerning 'reading,' understanding, and interpreting NTEs. This study supports the notion that PSTs need greater support in their mathematics methods classes in addition to clinical experiences in which they work with non-textual elements of mathematics curriculum. PSTs specifically need to learn the value that NTEs bring when teaching mathematics. Additionally, instructors should direct PSTs' attention to an NTE's ability to make a problem's meaning clearer for students. Instructors should also ensure PSTs understand the other main functions of NTEs (Kapyla, 2014).

Above are the main starting points toward improving the ways in which PSTs understand, interpret and implement NTEs in their instruction. There are many steps involved in using NTEs effectively while teaching, and this skill set must be developed with PSTs while they are learning content. For example, Dietiker et al. (2018) suggest that teachers and PSTs be allowed more curricular opportunities to attend to and interpret curriculum materials. It is possible with more time spent attending to and interpreting curriculum materials, teachers will become more likely to later design and enact lessons that involve the same opportunities for their students. Additionally, with more purposeful practice, teachers can become more skilled at noticing different aspects of curriculum and visuals.

Our findings also bring about a few questions, many of which Dietiker et al. (2018) struggled with; namely, how do teachers' skill levels in working with NTEs interact with their actual use of mathematical representations and illustrations in the classroom? How do the phases of curricular noticing manifest in the transformation from policy documents to what is planned and then enacted in the classroom? And finally, how can teacher educators support the act of curricular noticing and shift their practices to support teaching that is ambitious and attends to visuals as part of the curriculum, and not just as 'add-ons' to the text?

2. PSTS' CRITERIA FOR EFFECTIVE NON-TEXTUAL ELEMENTS

Overall, PSTs had some simplistic ideas regarding what makes an effective NTE. Half of the respondents noted that the NTE must be easy to follow. Additionally, 37% of the PSTs who participated noted that the NTEs should be aligned with the text and should relate to the problem. While the overall simplicity of these ideas stands out, it is worthwhile to examine some discrepancies among PSTs' list of criteria for effective NTEs. Some PSTs noted that pictorial illustrations with high contextuality are critical while others thought that the presence of mathematical representations (e.g., number lines, graphs) are more important even though they lack contextuality. This matters when thinking about how curriculum will be enacted by the PSTs in the future, as some respondents attended more to pictorial illustrations, while others attended to mathematical representations. Because of this difference in attending, when enacting curriculum, PSTs may interpret NTEs in different ways and with different levels of attention, which could then manifest in how they interpret the curricular materials they are presenting to students. Some PSTs highlighted the flexible use of NTEs while others preferred to see NTEs used in a consistent manner. While the criteria most PSTs proposed addressed the explicit features of NTEs, it is notable that three PSTs touched on implicit features, focusing on how NTEs represent diverse people and cultures.

We would like to pay attention to two other criteria for 'effective' NTEs proposed by PSTs, because they speak to the importance of teachers' knowledge of their students. Twenty-nine percent of PSTs noted *connection* as a criterion for an effective NTE, which relates to students' lives, prior knowledge, and experiences. About 21% of PSTs considered *appropriate/relevant level of content* as a criterion, highlighting that NTEs should match the grade level, age, and content. This seems to be an issue, not just from an equity standpoint, but because meaning making is based on prior knowledge, experiences and beliefs. (Are teachers always clear on their students' prior knowledge before teaching? Do they understand their cultural and personal backgrounds?) What the teacher believes to be relatable or age-appropriate may be based on his or her own prior experiences in school or in life, rather than on the experiences and knowledge of his or her students. Cooper, Sidney, and Alibali (2018) noted, ". . . it seems possible that students' interest in mathematics and their attitudes about its value may have differential effects on their processing of diagrams and illustrations" (p. 25). This seems to support the data we collected from PSTs: namely, that their attitudes and judgments about the value of NTEs might be directly linked to their own interests and attitudes. These interests and attitudes not only influence how they teach mathematics, but also how they understand and implement NTEs.

3. PSTS' EVALUATION OF NON-TEXTUAL ELEMENTS AND PROPOSED MODIFICATION STRATEGIES

It is interesting to note that initially, PSTs listed a wide range of roles and purposes of using NTEs. When asked the criteria for effective NTEs, PSTs mentioned simplistic aspects. When asked to evaluate or modify sample NTEs, PSTs tended to rely on even fewer criteria. In other words, PSTs' perceived roles of NTEs or criteria for effective NTEs did not always carry over into PSTs' critiques of NTEs or their suggestions for modifications. In this section, we specifically pay attention to three aspects noted when PSTs responded to the curriculum materials through evaluation and modification activities: familiarity, essentiality, and contextuality.

1) Familiarity

In the findings above, the data illustrates the idea that PSTs prefer the familiar, meaning they rated NTEs more highly if they were already familiar with the NTE. Otherwise, they rated NTEs less highly. For example, two very familiar mathematical representations (Examples 5 and 10) were most highly rated. In contrast, the empty number line in Example 4 was not perceived well by some PSTs. The PSTs considered it to be incomplete without all the labels and marks that they are accustomed to seeing on a number line. The empty number line might not be a familiar NTE for the PSTs based on their experiences as students, or at this point in their teacher training. Subsequently, PSTs showed efforts to make this example a more familiar form of NTE when they were asked to modify it for improvement. As shown in Figure 6, all PSTs who suggested modifications for Example 4 tried to modify the number line by providing more detailed labels and marks. Because PSTs were interpreting the number line as incomplete or irrelevant to the problem, it is possible they will respond to this interpretation by not utilizing that particular NTE while enacting curriculum, without seeing its potential for students' learning.

We note that a very small portion of PSTs (2%) addressed the disadvantages of using NTEs that are too familiar to students as student engagement and learning would not be encouraged. Presmeg (1997) argued that the "one-case concreteness" of NTEs could be the sources of students' difficulties in mathematical reasoning. Thus, we may need to encourage PSTs to get out of their comfort zone with familiar and straightforward NTEs and instead explore the potential of intentionally vague, incomplete, or unfamiliar NTEs.

2) Essentiality

In PSTs' modifications, we observed substantial efforts to put as much relevant information as possible into NTEs. For instance, in Example 2, some modifications showed all remaining apples, as well as apples that were taken away. In Example 4, modification of number lines contained the starting and ending numbers, as well as detailed interval marks. These types of modifications could possibly lead students to lower-level problem solving strategies (e.g., counting one by one) rather than using higher-level strategies (e.g., various composing/decomposing strategies). While NTEs should contain essential information, PSTs' modifications suggest that the high level of essentiality does not always guarantee successful students' learning experiences.

3) *Contextuality*

In this study, contextuality refers to mathematizing realistic objects/contexts. PSTs' modifications seem to focus more on how to make NTEs more realistic, rather than on the mathematizing process, as shown in the more realistic drawings of knees (Example 7) and more realistic pencils and erasers (Example 11). While the modified drawing in Example 7 showed various angles to support the mathematizing process, modified drawings of Example 11 do not seem to work to improve contextuality. In this regard, some questions are left for teacher educators and PSTs: How do the more realistic NTEs contribute to the mathematizing process? How much is real enough when it comes to NTEs in mathematics curriculum materials?

4. THE TASK FOR TEACHER EDUCATORS

These findings indicate teacher educators need to focus part of their instruction on providing more curricular opportunities for PSTs to attend to and interpret NTEs in curriculum materials. As many teacher educators have made clear over the years, professional teaching practices must be intentionally learned and developed (Ball & Forzani, 2009; Grossman, 2018; Zeichner, 2018). Part of these teaching practices in the field of mathematics can include focusing on curricular noticing.

In courses that are devoted to teaching mathematics, teacher educators can help PSTs develop their own professional vision as it relates to the use of curriculum materials. A big piece of this professional skill development should include the three phases of curricular noticing: attending, interpreting, and responding. Teacher educators need to help PSTs understand that curriculum is composed of far more than just textbooks. Teachers need to consider tasks, games, exercises, content, pacing, and even grouping when working to plan and design instruction and choose curricular materials. Not only do all of these elements need to be taken into consideration, but PSTs also need to learn how to locate their curriculum and then spend time taking in the NTEs, thinking about how they are making meaning of those NTEs, and then spend time thinking about how their students might make sense of those visuals as well.

This practice — thinking about sense making — can potentially be another focus for teacher educators. Those who are tasked with preparing future educators should be spending time helping PSTs work to think about and understand the many layers of student thinking so that they can then use this understanding to inform their future planning and teaching. This is a crucial practice to attend to given that teachers must consider “the goals, experiences, and dispositions of students” (Dietiker et al., 2018, p. 526). This means that teacher educators should consider helping their students attend to visuals as a curricular element. Additionally, they can take that one step further and work to interpret them, using not only their own lenses, but attempting to use those of their students as well.

By taking the time to help their PSTs attend to NTEs, teacher educators can aid PSTs in developing their own understanding of the learning advantages NTEs afford learners. Through this process, PSTs are bound to see that NTEs are far more than a support for the text. Once we help PSTs work to attend to NTEs, we can then work with them to interpret the NTEs, thinking about their students and all of the prior knowledge, beliefs, and experiences they bring into the classroom. If PSTs work to dive into NTEs in this way, they will then develop stronger skills needed to enact the final step of curricular noticing: responding. They can gain the skills necessary to respond to curricular materials in a way that will be most beneficial to the students in front of them.

Teacher educators, just like teachers, have a great deal to focus on in any given course. This is especially true for teacher educators who focus on mathematics, as there are times when a love for mathematics is simply not present in PSTs. With this in mind, we need to meet PSTs where they are and work to help them build on their understanding of NTEs. This can give them the skills necessary to identify and make sense of strong NTEs and then implement them effectively in the classroom.

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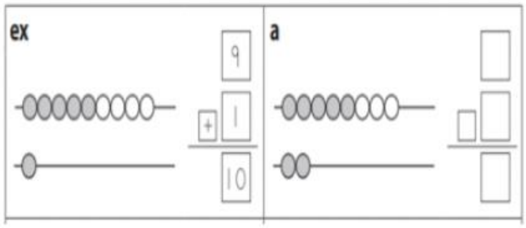
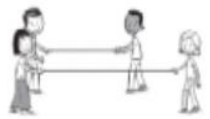
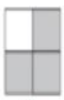
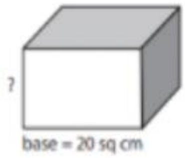
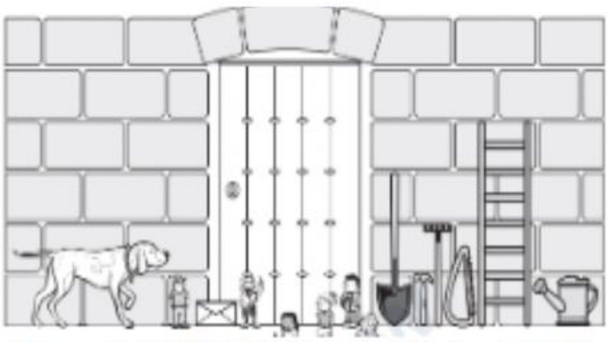
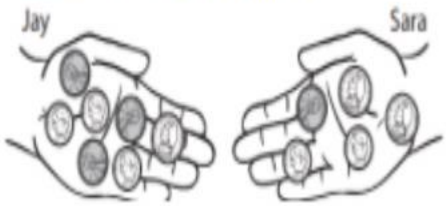
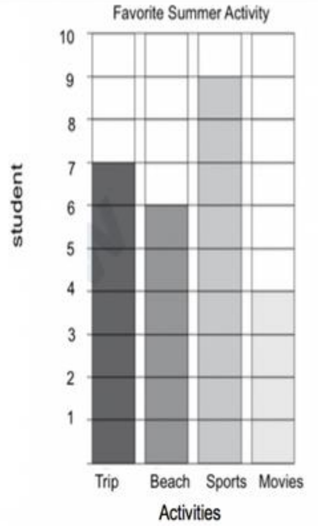
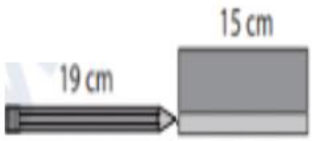
Appendix: Examples provided for the evaluation task

	Example	Grade/Content
1	Write and solve an addition fact to match each of the number racks. 	Grade 1 Addition and subtraction
2	There were some apples in the bowl. The kids ate 3 of them. Now there are only 7 apples left in the bowl. How many apples were in the bowl to start? Show your work.  There were ____ apples in the bowl to start.	Grade 1 Addition and subtraction
3	Ten children are riding on the train. Some of them have pulled down their shade so they can take a nap. Write an equation to show how many children are awake and how many are taking a nap in the picture. _______ + _______ = 10 awake sleeping 	Grade 2 Addition and subtraction
4	We have 2 jump ropes. The red one is 120 inches long. The blue one is only 84 inches long. How many inches longer is the red rope than the blue rope?  	Grade 2 Addition and subtraction
5	Choose the fraction that shows how much of each shape is filled in.  (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$  (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$  (a) $\frac{1}{2}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$  (a) $\frac{2}{1}$ (b) $\frac{1}{2}$ (c) $\frac{2}{2}$	Grade 3 Fractions

<table>
<tr><td>6</td><td>The volume of a rectangular prism is 140 cubic centimeters. The area of the base of the prism is 20 square centimeters. Do you have enough information to find the height of the prism? If not, what information do you need? Is so, what is the height?

</td><td>Grade 5
Multiplication and division
Geometry and measurement</td></tr>
<tr><td>7</td><td>Work with a partner to test how much you can rotate your knee.

</td><td>Grade 4
Geometry and measurement</td></tr>
<tr><td>8</td><td>The giant's hammer is 48" tall. Use this information to solve the problem below. Show your work and write an equation to represent the situation.

The shovel is ____ times as tall as the hammer. The shovel is ___ inches tall.
Equation: _______________
Answer: _______________</td><td>Grade 4
Multiplicative thinking</td></tr>
<tr><td>9</td><td>Jay and Sara are walking home from school and talking about how much money they have in their pockets. How much money do they have in all?

</td><td>Grade 2
Money
Place value</td></tr>
</table>

10		Samir and Sophia interviewed their classmates about their favorite summertime activities. They put the results into a bar graph. Use the bar graph to answer the following questions. How many people liked going to the beach best? How many people did Sophia and Samir survey in all?	Grade 3 Addition and subtraction Data analysis
11	How long are the pencil and the whiteboard eraser lined up end-to-end, 19 centimeters + 15 centimeters? 		Grade 3 Addition and subtraction