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Maintaining Cognitively Challenging Discourse Through Student Silence

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Student engagement in high-level, cognitively demanding instruction is pivotal for student learning. However, many teachers are unable to maintain such instruction, especially in instances of non-responsive students. This case study of three middle school teachers explores prompts that aim to move classroom discussions past student silence. Prompt sequences were categorized into Progressing, Focusing, and Redirecting Actions, and then analyzed for maintenance of high levels of cognitive demand. Results indicate that specific prompt types are prone to either raise or diminish the cognitive demand of a discussion. While Focusing Actions afforded students opportunities to process information on a more meaningful level, Progressing Actions typically lowered cognitive demand in an effort to get through mathematics content or a specific method or procedure. Prompts that raise cognitive demand typically start out as procedural or concrete and progress to include students' thoughts or ideas about mathematical concepts. This study aims to discuss five specific implications on how teachers can use prompting techniques to effectively maintain cognitively challenging discourse through moments of student silence.

Keywords: questioning, silence, cognitive demand, discourse prompting.
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I. INTRODUCTION

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When students engage in cognitively demanding mathematical tasks and discussions, they are more likely to see larger gains in their mathematics understanding and achievement (Boaler & Staples, 2008; Stein & Lane, 1996). Providing these opportunities for student engagement in high-level tasks requires at least two skill sets from teachers: task selection and task maintenance (Wilhelm, 2014). Past research has focused on the task selection by mathematics teachers (Stein, Grover, & Henningsen, 1996), but only recently have researchers started to delve more deeply into the complex practice of maintenance of high-levels of cognitive demand by means of teacher prompting (Drageset, 2014; Wilhelm, 2014).

Promoting opportunities for cognitively demanding discussions in the classroom can help students develop a deeper understanding of mathematical concepts (Boston & Smith, 2009). Franke, Kazemi, and Battey (2007) note that "one of the most powerful pedagogical moves a teacher can make is one that supports making detail explicit in mathematical talk, in both explanations given and questions asked" (p. 232). However, studies have shown that many teachers have a difficult time maintaining these high levels of cognitive demand, and teacher moves commonly deprive students of these opportunities by guiding students towards a correct solution or method of solving a problem (Lithner, 2008). In doing so, the teacher may be doing most of the cognitive work and requiring students to do little more than single step basic arithmetic or recall of vocabulary (Lithner, 2008; Yackel, Cobb, & Wood, 1998). This same problem of lowering cognitive demand was also found to be a way for teachers to handle situations where students did not respond to teacher prompts. Knowing that students who are engaged in cognitively demanding work are more likely to achieve higher levels, it is important to find ways to provide cognitively demanding tasks as well as maintaining the level of demand. While many teachers do not notice students' lack of response, those who do notice are often at a loss for how to handle non-responsive students (Gal, Lin, & Ying, 2009).

Teacher prompting is at the heart of maintaining cognitive demand, and it directly affects the types of experiences students have in mathematics classes (Cullen, 2002). A few studies have focused on various ways to classify teacher prompts (Drageset, 2014; Wood, Williams, & McNeal, 2006), but research seems to be lacking in instructional tools that help move teachers past these moments of silence without giving up on cognitively demanding tasks or prompts (Gal et al., 2009). Further investigations that support the development of practices that maintain cognitive demand are pivotal in ensuring dialogical success in the classroom (Wilhelm, 2014). Drageset (2014) mentions that "knowledge of different progress actions can equip a teacher with tools that might be helpful to get a halted process to move forward" (p.300); hopefully without reducing the cognitive load.

These ideas and previous research have led us to the following research questions:

When a teacher aims to re-engage students in discourse after no student response to the initial prompt:

1) What types of prompts typically lead to maintained or raised cognitive demand when compared to the initial prompt?

2) What characteristics are common within and between types of prompts that are successful in maintaining or raising cognitive demand?

By analyzing the data through combined frameworks by Drageset (2014) and Stein et al. (1996) and answering these research questions, we aim to be able to list implications on how teachers can use prompting techniques to effectively maintain high levels of cognitive demand when students do not respond to an initial prompt. These techniques will provide teacher educators with a repertoire of strategies to aid pre-service and novice teachers in navigating through these difficult discourse instances. Additionally, future research can explore successful prompting types to investigate how teacher's intended level of cognitive demand aligns with students' enacted level of cognitive demand throughout various prompting patterns.

The purpose of this study is to gain a deeper insight into the ways in which teachers use prompting to engage students in high-level cognitive thought and discussion even after students have withdrawn from the discussion. Through an in depth look at teacher prompts in these moments of student silence, we will attempt to answer the two research questions.

II. LITERATURE REVIEW

1. CONSTRUCTIVISM

The theoretical lens we chose for this study is constructivism, as it highlights the importance of teachers' ability to lead student discourse by prompting students to explore ideas more deeply themselves. Constructivists believe that learning is active, and when a learner experiences a mathematical pattern or problem they must rely on their personal, current knowledge to construct new knowledge or understandings (Windschitl, 2002). Consequently, when students reach a point where their own knowledge is not enough to move them forward, the role of classmates and the teacher becomes pivotal. Teachers play the role of a "co-constructor" of knowledge "by seeding students' conversations with new ideas or alternatives that push their thinking" (p. 147), which requires the use of cognitively demanding prompts.

Through this theoretical belief, the instances of student silence discussed in this study could be explained by a mismatch of the teacher's mathematical conceptual language and the students' conceptual understandings (Gal et al., 2009). The ways in which we view a teacher's ability to handle classroom silence in response to a prompt is dependent on the researcher's theoretical beliefs. Therefore, prompts that encourage students to re-engage in cognitive processing after student silence are viewed as successful.

2. ROLE OF DISCUSSION IN LEARNING

Researchers discuss the interactive nature of learning in constructivist views (Von Glasersfeld, 1989; Windschitl, 2002). Through discussion, teachers can guide students' learning in real time by assessing where students are currently at in their understanding, and where the teacher wants them to go (Adhami, 2001). Prompts are one of the major tools that teachers can use to help fill in the gap between the students actual level of understanding and the teacher's desired level of student understanding (Brodie, 2010).

The types of prompts teachers give can influence the intended level of cognitive demand that they wish to engage students in. Several types of prompts have been noted as conducive for higher levels of intended cognitive demand. These prompts are typically open questions, which lead to more elaborate answers by students, or prompts that engage students in making connections or generalizations (Almeida, 2010; Course, 2014). Additionally, Adhami (2001) notes, prompts that are based on student ideas, and call for "negotiation of meaning, handling of misconceptions, and attention to minute and idiosyncratic steps of reasoning" (p. 28) are also of benefit to both teachers' and students' depth of mathematical understanding.

Prompts that end up diminishing high levels of intended cognitive demand typically include closed, short response questions (Almeida, 2010; Course, 2014) and leading questions where the "teacher assumed much of the mathematical work while supporting students when moving them through correct and complete explanations" (Franke, Webb, Chan, Ing, Freund, & Battey, 2009, p. 390). Furthermore, when the teacher exhibits control of over the conversation, student participation is largely based on the teacher's agenda and the teacher's line of questioning is reduced in complexity to narrow in on the desired content or methods to be learned (Emanuelsson & Sahlstrom, 2008).

One framework that has been used to study classroom discourse is the initiation-response-follow-up (IRF) framework (Sinclair & Coulthard, 1975). This framework is used to study patterns where a teacher initiates discourse with a student or the class, a student answers, and the teacher follows-up either through evaluation or some other form of feedback. As this prompting pattern tends to be associated with teacher-dominated discourse (Marzban, Yaqoubi, & Qalandari, 2013; Wiebe Berry & Kim, 2008) and does not appear to be aligned with practices encouraged by the United States Common Core State Standards (CCSS-I, 2010) where more student involvement is expected, it has been noted as the dominate discourse pattern used in classrooms across the United States (Wells, 1993).

Various studies have shown that the IRF framework is useful in analyzing various aspects of classroom dialogue (Cullen, 2002; Molinari, Mameli, & Gnisci, 2013; Wells, 1993). Wells (1993) recognized the potential of the model to exemplify more than just teacher-directed exchanges. For example, IRF can be used to achieve other goals "including the co-construction of knowledge on the basis of ideas and experiences contributed by the students as well as the teacher" (p. 35). In addition, Cullen (2002) discussed the importance of the follow-up portion of the IRF sequence in building on students' responses and supporting student learning.

Although IRF seems to be a fairly strict three-step process, "the combination and function of teacher moves can impact student learning in varying ways" (Brodie, 2010, p. 185). Molinari and colleagues (2013) examined chained IRFs and found that different types of questions led to different types of sequences, which affected student participation. A prompting sequence, which is the unit of analysis for the current study, is a set of prompts that are aimed at getting students to engage in discussion or thought about a mathematical idea.

Drageset (2014) states that IRF and similar concepts might be useful for generally describing teacher discourse practices, but more specificity, is needed to gain a deeper understanding of these practices. For this reason, he constructed a framework for analyzing teacher prompts in dialogue using patterns in IRF sequences. Through this framework, the teacher moves are described in detail through redirecting, progressing, and focusing actions, which can "give names to some of the actions the teacher uses in an appropriation process" (p. 301). *Redirecting actions* are prompts that the teacher uses to try to redirect students' attention to something else; *progressing actions* are used to try to help the process of learning move forward; and *focusing actions* direct the students to focus on certain details. Each category is then broken up into more specific prompting types within each action.

When the teacher initiates dialogue, students are expected to respond, but when there is no response teachers can use redirecting, progressing, and focusing actions to get the discourse moving again. In our study, we examine whole class discourse centered around teacher prompts, for which Drageset's framework can be used to provide the level of detail needed to identify the patterns of teacher prompts that attempt to move a halted dialogue forward. Knowing the different types of questioning practices that teachers embrace will help determine what moves may be beneficial for increasing or maintaining high levels of cognitive demand.

3. STUDENT SILENCE DURING CLASSROOM DISCOURSE

How vocal students are can affect how students are treated in classroom discourse and can result in varying teacher actions. A study by Emanuelsson and Sahlstrom (2008) recognized that teachers could be "constrained by the absence of answers and consequently [have] to do the major part of the interactional work by herself" (p. 216). Accordingly, teachers will lower the cognitive demand required of the students by assuming that their silence indicates a lack of knowledge. The way teachers behave in response to silence has not been extensively researched in terms of the level of cognitive demand assumed of these silent students. In classroom instances where students did not respond to a teacher prompt, "teachers coped by hinting, asking small-step questions, warning of mistakes in advance, or referring back to a previous lesson" (Gal et al., 2009, p. 407). Another way teachers commonly respond is by answering the question themselves and then moving on with the lesson, leaving gaps in students' understanding (Leinhardt & Steele, 2005).

In order for classroom discourse to work effectively, the majority of the class must be actively listening at any one point in time. Since this structure requires silence from the majority of students, it is difficult to define silence in terms of participation. A study by Gal et al. (2009), categorized silence into six levels according to varying numbers of students' verbal responses. The lowest level, total silence, is defined by no response from any student. The data in the current study is analyzed using this lowest level of silence as its definition since the lowest level of silence is a major area where many teachers struggle (Leinhardt & Steele, 2005).

4. COGNITIVE DEMAND

Stein et al. (1996) classified the cognitive demand of mathematical tasks into four different categories: 1) memorization, 2) formulas, algorithms, or procedures *without* connections to concepts, 3) formulas, algorithms, or procedures *with* connections to concepts, and the highest level, 4) "doing mathematics". Doing mathematics involves "complex mathematical thinking and reasoning… such as making and testing conjectures, framing problems, looking for patterns, and so on" (p. 466). Tasks that were considered to demand high cognitive levels were "doing mathematics" and "procedures with connections to concepts". On the other hand, the remaining two categories were considered to exhibit low-levels of cognitive demand. To incorporate high-levels of cognitive demand in the classroom, teachers have an important role in choosing tasks, which can scaffold students' understanding of concepts and ideas that require complex and non-algorithmic thinking. Teachers should be able to provide prompts that can help students make connections among conceptual ideas as well (Stein, Engle, Smith, & Hughes, 2008).

Henningsen and Stein (1997) state five main actions that teachers must enact to maintain high levels of cognitive demand and create opportunities for conceptual connections. Those five actions are: producing tasks that build on students' prior knowledge; scaffolding; giving appropriate amounts of time; modeling of high-level performance; and sustaining pressure for explanation and meaning. Teachers have a crucial role in encouraging students to explore justifications, explanations, and meaning through their questions, comments, and feedback (Stein et al., 2008). However, Stigler and Hiebert (2004) pointed out that U.S. teachers tend to teach mathematical tasks by turning them into procedural exercises instead of making connections to the task, which indicates teachers' lack of ability to maintain high cognitive demand during class.

Stein et al. (2008) and Henningsen and Stein (1997) state that teachers reduce the level of cognitive demand when they do not value the accuracy of students' explanation and focus on simply completing the tasks with correct answers. Along with such cases, another action that can reduce the level of cognitive demand is when teachers provide explicit procedures in order to complete tasks by "taking out" the difficult pieces and reduce the task complexity (Henningsen & Stein, 1997). Moreover, when students are not allowed to have an appropriate amount of time to work on tasks, it can reduce such levels as well (Stein et al., 2008).

Stein et al. (2008) stated that providing algorithmic tasks, including either reproducing previously learned information or memorizing procedures without understanding "why", are the elements that decline cognitive demand. Lither (2008) termed one such pattern of declining cognitive demand as "algorithmic reasoning," which is a step-by-step structured thinking procedure that uses memorization and a set of rules, to arrive at an expected standardized solution. Similarly, "funneling" instruction describes a teacher proceeding through a series of direct questions step-by-step, narrowing down the students' responses until they find the correct answer (Yackel et al., 1998). Practices such as algorithmic reasoning (Lithner, 2008) and funneling (Yackel et al., 1998) indicate instances where learning mathematics happens based on superficial, not intrinsic, mathematical properties (Haavold, 2010). This can limit the students' contribution by asking direct questions in order to reach the objective and directing students' thinking in a predetermined path based only on how the teacher would solve the problem (Herbel-Eisenmann & Breyfogle, 2005).

In order to maintain high cognitive demand, teacher prompting should move beyond these low level tactics and aim at developing students' explanations of mathematical concepts by building off of students' ideas (Franke et al., 2009). In order to gain higher levels of student achievement with essential mathematical skills, it is important to begin with high-level tasks that are cognitively demanding and complex (Stein & Lane, 1996). However, maintenance of such high levels of cognitive demand through questioning is also a crucial skill (Boaler & Staples, 2008). Students learn best in classrooms where high levels of cognitive demand are maintained (Kessler, Stein, & Schunn, 2015). However, Stigler and Hiebert (2004) point out that teachers do not commonly maintain high cognitive demand instruction during class and the way of learning mathematics is highly routinized, consisting of memorizing content or reproducing teacher-demonstrated procedures to solve problems.

Both the Drageset (2013) and Stein et al. (1996) frameworks are pivotal in analyzing teacher prompting practices, as Drageset (2013) focuses on the role of the teacher in fostering classroom discourse and Stein et al. (1996) focuses on the intended level of cognitive demand. Together these frameworks can be used to analyze the intended levels of cognitively demanding mathematical situations that teacher's use to prompt students' engagement. While previous studies have shown the importance of teacher prompting (Drageset, 2013; Henningsen & Stein, 1997) and maintenance of high levels of cognitive demand (Ellington, 2006; Stein et al., 1996), few study have shown how these two concepts can be used simultaneously by teachers to move past difficult discourse instances such as the non-responsive student.

III. RESEARCH METHODOLOGY

To answer two research questions: 1) What types of prompts typically lead to maintained or raised cognitive demand when compared to the initial prompt? And 2) What characteristics are common within and between types of prompts that are successful in maintaining or raising cognitive demand?, qualitative methods were used in this case study of three middle school mathematics teachers in order to take an interpretivist approach (Merriam, 2009) in looking at how specific teacher prompts can reengage students in mathematical discourse after moments of student silence. This framework was chosen because of the complex nature of the dialogical interaction between the teacher and students in a mathematics classroom environment. Through the interpretivist approach, the theory of constructivism was allowed to guide our interpretations of these interactions and any other factors that may have knowingly affected the data (Creswell, 2013).

Six class periods were observed and videotaped for each teacher to acquire a variety of lesson types, giving a total of eighteen observations. Since introductory lessons may have different questioning patterns than review lessons, this choice enabled us to code two introductory lessons, two developing lessons, and two review lessons for each teacher. The type of a lesson was determined by the number of exposures to the content students should have had based on the curriculum materials. Lessons where students had no previous exposure to the content were labeled *introductory*, lessons with one previous exposure were labeled *developing*, and lessons with two or more previous exposures were considered *review*. Although these lesson types are not distinguished during coding or analysis, the variety of lesson types ensured that the data were more representative of the various forms a lesson might take.

Multiple interviews were conducted throughout the study. Using semi-structured interviews, we were able to gain knowledge about teachers' background information and_explore each teacher's beliefs about student discourse and questioning strategies. At the end of each visit a short informal interview was conducted with each participant about their lesson and how they felt the lesson allowed students to communicate about their mathematical ideas. During the last interview, we conducted member checking to see if our interpretation of classroom events fit that of the participant. At this time, questions that came up during data analysis were fully discussed and examined. Interviews were audio recorded and transcribed. Field notes and memos were also used for triangulation of the data.

1. PARTICIPANTS

The participants comprised of three middle school mathematics teachers from a school district located in a Midwestern, suburban town in the United States of America. This was a purposive sample that was chosen from one of the top performing school districts in the Midwestern state. Teachers were also chosen based on their varying levels of student involvement in classroom discourse. One teacher, noted as Kev[1] below, recommended by a district instructional strategist, was identified as a teacher who stands out as one who frequently employs strategies that engage students in mathematical discourse. During the study, Kev was

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[1] All names mentioned in the study are pseudonyms.

in his seventh year of teaching mathematics, but this was his first year of teaching mathematics at the 8th grade level. All of his previous years of teaching were at the 5th through 7th grade levels. As seen through various observations, Kev teaches in a manner that regularly engages students in discussion, either through peer-to-peer conversations or whole class discussion.

The second teacher, noted as Jan below, was also in her seventh year of teaching, all of which took place in her current position in 7th grade mathematics. Jan's classes were very lecture driven with few opportunities for students to discuss or give input until the end of class when students worked on homework. Jan's lectures were often tied to conceptual or real world examples.

The third teacher, noted as Lacy below, was in her twelfth year of teaching, but her second year in her current position as a 7th grade mathematics teacher. Her previous experience was split between high school and middle school mathematics. Lacy's students talked freely during her class, but less frequently about mathematical ideas. Lacy was also lecture driven, and her lectures were typically more procedure-based.

2. METHODS OF ANALYZING DATA

All 18 video recordings were transcribed verbatim. These videos and transcripts were initially coded by identifying moments of total student silence, which is defined by no student response after teacher's prompt for student dialogue (Gal et al., 2009). These moments were then coded using the combined frameworks of Drageset (2013) and Stein et al. (1996) to identify prompt types and levels of cognitive demand, respectively. Our unit of analysis was a prompt sequence (Molinari et al., 2013): A sequence begins with an unanswered prompt and ends once students re-engaged in discourse after any teacher follow-up prompt within the sequence, or the teacher answered the prompt. Figure 1 shows the process of the unit of analysis, a sequence. Each sequence was coded as a whole for prompt type, but each prompt within a sequence was coded separately for cognitive demand so changes in cognitive demand could be noted.

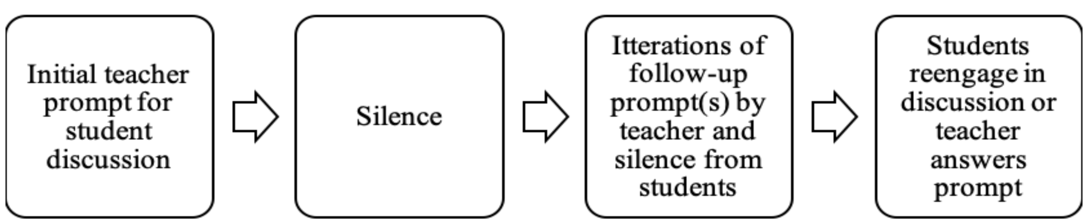

Figure 1. Unit of analysis. This figure illustrates the elements in a prompt sequence.

Drageset (2013) lists thirteen different prompt types. These classifications fell under three categories: Progressing Actions, Focusing Actions, and Redirecting Actions. Progressing Actions (PA) were actions that were intended to help students progress through a process or the lesson. There were four types of PAs identified in the coding procedure; demonstration, simplification, closed progress details, and open progress initiatives. Focusing Actions (FA) were actions that were intended to have students focus in on a specific detail. The six types of FAs were enlighten details, justification, apply to similar problems, notice, recap, and request assessment from other students. The last category, Redirecting Actions (RA), was intended to direct students back to a more successful path or way of thinking. The three types of RAs were correcting questions, advising a new strategy, and put aside. Table 1 shows these prompt types and the descriptions of each type.

Table 1. Prompt type descriptions

Progressing Action	
Demonstration	The teacher demonstrates the solution without involving or asking students (sometimes asking for confirmation, but not always requiring it)
Simplification	The teacher adds information that makes the task easier (could include adding information, giving hints, or telling students what to solve)
Closed progress details	The teacher splits the problem up into several smaller tasks and asks for answers to each of these
Open progress initiatives	The teacher initiates progress but still leaves it at least partly open to the students to choose or suggest which path to follow
Focusing Action	
Enlighten details	The teacher requests that students stop and explain what something means or how something happens
Justification	The teacher asks questions about why the answer found or the method used is correct
Apply to similar problems	The teacher invents a question on the spot to check whether students can apply knowledge to a new and different problem
Notice	The teacher emphasizes or points out important elements during a dialogue
Recap	The teacher repeats a comment in order to make it easier for others to follow the line of thought
Request assessment from other students	The teacher requests other students to assess a student answer or suggestion
Redirecting Action	
Correcting questions	The teacher asks a question to redirect the students towards another approach (usually a “yes, but…” comment)
Advising a new strategy	The teacher advises an explicit alternative approach or way of thinking
Put aside	The teacher puts aside or rejects a student’s comment without providing any help

Stein et al. (1996) categorized levels of cognitive demand of tasks used by mathematics teachers. These levels can also be used to explore cognitive demand of teacher prompts. Table 2 shows the various levels of cognitive demand used in the coding process. The initial framework by Stein et al. (1996) included four levels of cognitive demand; level 1, memorization or recall of a fact, level 2, use of procedures and algorithms without attention to concepts or understanding, level 3, use of procedures and algorithms with attention to concepts or understanding, and level 4, "doing mathematics," which includes employment of complex thinking and reasoning strategies such as conjecturing, justifying, interpreting, etc. Since this study focused on maintenance of high levels of cognitive demand, or prompts that raised cognitive demand from low to high levels, the two lower levels were combined and considered low cognitive demand, while the two upper levels were combined and considered high cognitive demand.

Table 2. Cognitive demand descriptions

Level of Cognitive Demand	Description
Low	Memorization or recall of a fact OR Use of procedures and algorithms without attention to concepts or understanding
High	Use of procedures and algorithms with attention to concepts or understanding OR "Doing Mathematics" – employment of complex thinking and reasoning strategies such as conjecturing, justifying, interpreting, etc.

We identified units of analysis during the first round of coding: teacher prompts that were followed by no student response to analyze further. Videos were transcribed and instances of no student response to prompts were marked in the transcript. When data were coded both transcripts and video recordings were used to ensure accuracy in coding procedures. Each coder found instances of silence in two videos from each teacher, which yielded a total of six videos for each coder.

When coding each moment of silence, we employed the "coding by committee" format (Miles, Huberman, & Saldaña, 2013), coders met to code nine class periods of instances of silence together using the Drageset (2013) and Stein et al. (1996) combined framework. Although the use of two coding schemes at a time made the coding process more complex, coding as a committee of three coders allowed for intensive discussion, which helped to improve and refine definitions of the codes. After coding nine class periods, less discussion was needed, and the remaining nine class periods were coded by pairs of coders. The head coder was present in all coding pairs to ensure a common vision of code meanings. Miles, Huberman, and Saldaña (2013) note, “team coding not only aids in definitional clarity but also is a good reliability check” (p. 84). Since all coding was completed as a group, any disagreements were discussed with the coding team until a consensus was reached. Consensus was reached on 100% of the codes.

For the second round of coding, we placed the initial and follow-up prompt(s) within a matrix of overlapping frameworks by Drageset (2013) and Stein et al. (1996). The columns signify the type of prompt sequence, and the rows signify the change in level of cognitive demand for that prompt sequence. Therefore, the cell that each sequence was placed in represents two pieces of information about that sequence: prompt type, and the change in level of cognitive demand. The data shows the cognitive demand of each sequence as being coded as one of four types: a) sequences that maintained high levels of cognitive demand, b) sequences that raised from low to high levels of cognitive demand, c) sequences that lowered cognitive demand from high to low levels, or d) sequences that maintained low levels of cognitive demand.

Since the goal was to learn from teachers maintaining and raising cognitive demand through prompting, sequence types a) and b) were considered the focus of the next level of analysis. Although the data in prompt types c) and d) could give information on what to avoid, we hoped to develop implications that build on teachers’ strengths, not deficits.

During the first two rounds of coding, intensive memos were written in another document. These memos were used to keep track of patterns noticed during the coding process. Along with these memos, all three coders grouped prompt sequences within the same group together to analyze common characteristics between prompt sequences. For example, we looked to see if there was a certain way that teachers used a specific prompt type to maintain or raise discussions to high levels of cognitive demand. The prompt types that fit this criterion were enlighten details and apply to similar problems. Common patterns within those specific cells were analyzed and discussed. We then analyzed the groups of prompts that fell within the same row of cognitive demand. These two rows were maintained cognitive demand and cognitive demand that was raised from low to high. Sequences that maintained high levels of cognitive demand or raised cognitive demand from the initial prompt were collected and analyzed for common characteristics that could possibly explain the success of the prompt. This allowed us to observe, “how interactive sequences foster a variety of forms in classroom discourse” (Molinari et al., 2013, p. 422).

IV. RESULTS

Throughout the data analysis, some clear patterns surfaced in the data. The percentage of prompt sequences that raised or maintained high levels of cognitive demand spanned from 24% to 52.6% of all prompt sequences that involved student silence. Different prompt categories resulted in different patterns of cognitive demand, but the patterns seemed to be consistent between all of the teachers. Progressing Actions and Focusing Actions were both used frequently by all teachers, however, Progressing Actions typically led to lower levels of cognitive demand, while Focusing Actions led to higher levels of cognitive demand, especially for Jan and Lacy. Redirecting Actions were rarely used by any of the three teachers during moments of student silence.

Table 3 shows the total number of prompts over the course of the six class periods for each teacher that resulted in student silence. Since a prompt that resulted in silence is needed to initiate a prompt sequence, the number of prompts that initially resulted in student silence is equal to, and can also be viewed as, the number of prompt sequences. The table lists the number of prompt sequences that each teacher gave that raised cognitive demand to high levels, or maintained high levels. Percentages listed represent the percentage of teacher prompts that raised or maintained high levels of cognitive demand through students' silence out of all prompts in the class periods considered. Even though Kev's instruction incorporated more frequent explanations from students, as viewed in the observational data, his percentage of maintaining/raising high cognitive demand prompts was the lowest at 24%. On the other hand, Jan and Lacy's instruction had fewer opportunities for student input, however the percentages on maintaining/raising high cognitive demand prompts were approximately two times higher than Kev's, with 52.6% and 42.9% of their prompts, respectively, maintaining or raising to high levels of cognitive demand.

Table 3. Numbers and percentages of teacher prompts that raised (R) or maintained (M) cognitive demand

Teacher	Raised or Maintained			Percentage of R or M	Total
	Raised	Maintained	Subtotal		
Kev	3	9	12	24%	50
Jan	5	25	30	52.6%	57
Lacy	3	15	18	42.9%	42

Research Question 1) what types of prompts typically lead to maintained or raised cognitive demand when compared to the initial prompt?

Table 4 shows the results of analysis in terms of teachers' prompts and cognitive demand simultaneously. Each row represents a specific prompt type while each column indicates whether the prompting sequence rose to high levels of cognitive demand (R) or maintained high levels of cognitive demand (M). Each cell shows the frequency of prompting sequences that fall within the corresponding prompt type and level of cognitive demand for each teacher.

When looking at the data collectively, all three teachers tend to use more focusing actions than progressing actions. Progressing actions also did not frequently lead to maintained or raised levels of high cognitive demand, while focusing actions tend to enable higher levels of cognitive thinking. This is not true for Kev, however, who tended to have low levels of cognitive demand when using focusing actions.

To be specific, prompts of Simplification, Closed Progress Details, and Notice led to maintained low cognitive demand levels. Moreover, all three teachers used very few redirecting actions without any particular patterns of cognitive levels.

Table 4. Prompt type and maintenance of cognitive demand

Low or Maintained/Raised CD		Kev		Jan		Lacy	
		L	M/R	L	M/R	L	M/R
Progressing Actions	Demonstration	1	-	7	3	-	-
	Simplification	6	2	4	-	4	3
	Closed Progress Details	2	1	3	2	11	-
	Open Progress Details	5	-	1	2	1	1
	Total Progressing Actions	14	3	15	7	16	4
Focusing Actions	Enlighten Details	6	7	1	5	3	3
	Justification	-	-	-	1	-	1
	Apply to Similar Problems	1	1	1	6	-	6
	Notice	10	1	6	5	3	2
	Recap	3	-	-	2	1	2
	Request Assessment	2	-	1	2	1	-
	Total Focusing Actions	22	9	9	21	8	14
Redirecting Actions	Correcting Questions	1	-	3	2	1	-
	Advising a New Strategy	1	-	-	-	-	-
	Put Aside	-	-	-	-	-	-
	Total Redirecting Actions	2	0	3	2	1	0

1) Focusing Actions

As mentioned previously, all three teachers tend to use more focusing actions than progressing actions when navigating discussions through student silence. This is true without regards to level of cognitive demand within the sequence, and in regards to greater amount of maintained or raised high cognitively demanding prompts. Two main types of prompts that were mostly used by all three teachers in maintaining high level cognitive demand were Enlighten Details and Apply to Similar Problems. Both of these prompt types focus students' attention to conceptual understanding and application. For example, Kev frequently used Enlighten Details to provoke students to interpret mathematical meaning. When looking at all of Kev's Enlighten Details prompts collectively, six out of the thirteen times that he used this prompt type, he used it to get students to focus on the underlying conceptual meaning of symbols or formulas. The following example shows an enlighten details prompt sequence where Kev tries to get students to focus in on the meaning of the y-intercept, but students meet his request with silence.

Kev: y-intercept really means what? [silence]

Kev: What does it mean? [silence]

Kev: This is why we don't… I don't like to be caught up on equation stuff. I just gave you that. We developed that yesterday. If I just gave you that, it doesn't mean anything. What does it mean for the y-intercept? [student response]

From this sequence, Kev kept asking students to recall their prior knowledge and explain what the y-intercept actually means, looking for a response that shows students have a conceptual understanding of the y-intercept being the point at which the line intersects with the y-axis. This request for mathematical meaning is the defining characteristic of the Enlighten Details prompt type. This is a common prompt type for Kev because he expects his students to co-construct knowledge as a class, based off of previous understandings, instead of simply having students memorize procedures.

Both Jan and Lacy used numerous prompts that fell under the Apply to Similar Problems prompt type. With seven and six Apply to Similar Problems prompting sequences, respectively, these teachers asked their students to apply their newly learned ideas to a different context. During the following discussion students were exploring slope in terms of a staircase. To make sure students understood how rise and run affect the steepness of the slope, Jan invented additional questions to see if students could transfer their knowledge about the slope of a staircase to more general ideas about what affects the slope. In this example of an Apply to Similar Problems prompting sequence, Jan asks students to think about the relationship between the rise, run, and steepness of a slope.

Jan: How would you make stairs less steep? [silence]

Jan: What would you do to the rise or to the run? [silence]

Jan: What would you do? Sam, what could you do to the rise to make them less steep? [student response]

Jan continued asking questions by giving different situations, which enabled her to see if they could apply their knowledge to different problems or situations.

On two occasions, Focusing Actions were found nested within Progressing Actions. Lacy stopped a Simplification (under Progress Action) to focus in on a detail she thought might be difficult for students and asked them a prompt from the Apply to Similar Problems category to check their depth of understanding. Students were working on explaining what a positive correlation between grip strength and arm strength means. After students explained that this would mean you would expect a stronger grip strength to be accompanied by stronger arm strength, the teacher asked students to interpret a contextual situation in which there is no correlation. The following prompt sequence shows her check for application of the mathematics to a different correlational relationship.

Lacy: And then, if there’s no correlation? [silence]

Lacy: What would I say? [silence]

Lacy: The two things, so say this was no correlation, I would say the arm strength and the grip strength what? [student response]

Here, Lacy asked the same question by adding information such as the labels of the x and y-axes, grip strength and arm strength, which reduces the cognitive load for students, while still allowing them to interpret the contextual situation.

The use of Justification prompts also led to maintained high level cognitive demand, however, only two prompts of this type were found. The nature of Justification prompts would lead us to believe that the majority of this prompt type would lend to maintained or raised high-level cognitive demand. Justification prompts ask students to explain or justify why an answer or method is correct. For example, Lacy asked her students to explain why they would keep the denominator the same when adding two fractions instead of adding them. Similarly, Jan asked her students to explain why the ratio of rise to run is the same at any step on the staircase. Both of these explanations require students to consider their conceptual understanding of the content, which would at least incorporate a level three cognitive demand. The low prevalence of this prompt type may show an untapped resource where teacher development could be beneficial.

2) Progressing Actions

One noticeable finding in the Progressing Action field was that Jan tended to use a lot more Demonstration prompts than the other two teachers. However, the levels of cognitive demand for these ten prompts were at both ends of the cognitive spectrum. Although she used this prompt type more frequently, these prompts followed no pattern in terms of cognitive demand; some prompts were at the lowest level, while some were at the highest level. One sequence that led to maintained high cognitive demand occurred during a discussion about a scale balancing activity that allowed students to understand the concept of equality.

Jan: When I'm solving the equation, what did you notice when we were doing it? [silence]

Jan: We wanted our bag [the variable] alone.

Here, Jan demonstrated the solution and asked students to make a connection between the process of solving an equation and the concept of using equality to find unknowns. However, sometimes she would use the Demonstration prompt type in a low level manner, such as the following prompt.

Jan: And how do we do that? [silence]

Jan: To isolate it, we undo… We're having *x* plus 3 here, and we "undo" that addition by using inverse operations.

In this case, Jan asked a question in the process of demonstrating a procedure. It is noticeable that she asked students to recall their prior knowledge and they are expected to remember how to isolate the variable. This indicates that the teacher's demonstration led students to follow her memorized procedure without connecting it to any conceptual understanding. It was not uncommon for Demonstration prompts to be high-level when the teacher was discussing a conceptually-based task, and low-level when a more procedural task was the focus. Keeping the prompts focused on conceptual ideas or connections seemed to help students create a meaningful understanding of the mathematics.

Another pattern in the data is Lacy's use of the Closed-Progress Details prompt. This prompt type was used eleven times within prompting sequences that led to student silence. All eleven of these prompts led to low cognitive demand. Since this prompt type splits a problem up into several smaller tasks, it easily leads to a shift in cognitive load from student thinking to teacher thinking, while still leaving students involved in smaller, more procedural tasks. Within these eleven prompting sequences, there were 25 individual prompts from Lacy. Remarkably, 24 of these 25 prompts were a level two cognitive demand, which asks students to use a procedure or algorithm without connections to concepts or understanding. The other prompt was a cognitive demand level 1. When using prompts of this type, Lacy would walk the students through a computational problem, step by step, such as the following sequence that took place during a lesson where students were adding fractions.

Lacy: And what else happens with these? [silence]

Lacy: So we added our tops, we kept the bottom, and then what? [student response]

Here, Lacy has walked students through the first few steps of the addition problem and then prompts them to think about simplifying the answer. There is not much student thought going into these answers other than the enactment of a procedure that the teacher is walking the students through.

3) Redirecting Actions

Redirecting actions were very rare in instances of student silence, most likely because redirecting actions are typically directed at a specific student. It was uncommon for a prompt to go unanswered when the teacher directed the prompt at a particular student. The most commonly used redirecting action was Correcting Questions, but out of the seven prompts of this type, only two of them were higher cognitive demand, both of which were given by Jan. The lower level use of this prompt type shows correction of students' incorrect use of a procedure. The higher level use of this prompt can be captured in the follow prompt of Jan's, where she tries to help a student think more generally about the concept of slope. Prior to the sequence, a student responds to a prompt about what changes result in a steeper slope by stating that the rise is what affects the steepness. Jan wanted to correct this student's generalized idea.

Jan: What about run? [silence]

Jan: Could you do something to the run to make it steeper? [student response]

Although this student was not wrong in their thinking about an increased rise creating a steeper slope, Jan used the Correcting Questions prompt type to push the student's thinking to a deeper level of connection between the concepts of rise and run and slope.

Research Question 2) What characteristics are common within and between types of prompts that are successful in maintaining or raising cognitive demand?

As seen in Table 5, Out of the 60 prompts that reached high cognitive demand without dropping to low levels, only eleven of them raised the cognitive demand from lower to higher. With the current sample, it was much less likely for teachers to reach high levels of cognitive demand through their prompts if they did not start out with high level prompts. This should make sense, as it is unlikely that teachers would raise the cognitive demand of their questioning if they are met with student silence.

Table 5. Prompt type and maintenance of cognitive demand

		Kev		Jan		Lacy	
Maintained/Raised CD		R	M	R	M	R	M
Progressing Actions	Demonstration	-	-	-	3	-	-
	Simplification	1	1	-	-	-	3
	Closed Progress Details	1	-	2	-	-	-
	Open Progress Details	-	-	-	2	-	1
	Total Progressing Actions	2	1	2	5	0	4
Focusing Actions	Enlighten Details	-	7	1	4	-	3
	Justification	-	-	-	1	-	1
	Apply to Similar Problems	-	1	-	6	2	4
	Notice	1	-	1	4	1	1
	Recap	-	-	-	2	-	2
	Request Assessment	-	-	-	2	-	-
	Total Focusing Actions	1	8	2	19	3	11
Redirecting Actions	Correcting Questions	-	-	1	1	-	-
	Advising a New Strategy	-	-	-	-	-	-
	Put Aside	-	-	-	-	-	-
	Total Redirecting Actions	0	0	1	1	0	0

1) Maintained High Cognitive Demand

There were two main types of prompts that the teachers utilized in order to maintain high cognitive demand through student silence: Enlighten Details and Apply to Similar Problems. Across all three teachers, the Enlighten Details prompts encouraged students to recall important conceptual aspects of mathematical concepts that were previously learned. For example, Lacy posed the following problem while discussing addition of the two fractions 1/6 and 2/6:

Lacy: Why don't we add the bottoms? [silence]

Lacy: Why don't we make this three twelfths? [silence]

Lacy: If I add them, the bottom? Why wouldn't I do that? [student response]

This sequence indicates a situation where the teacher is trying to get students to think about the meaning of fractions and why denominators are not added together while adding fractions. Notice how Lacy maintains the high level of conceptual prompting even when she is met by silence through these three prompts. This maintenance communicates her belief of the importance of conceptual understanding, and the ability of her students to explain the reasoning behind the mathematical algorithm.

Kev commonly used prompts that asked students to find connections between different mathematical ideas, which led to maintained high cognitive demand. The following excerpt is an example of an Enlighten Details sequence where Kev tries to help students understand why any two points on a line will result in the same slope.

Kev: What do those… If I were to just look at the fractions, so one half, two fourths, and three sixths, okay? What do those all have in common? [silence]

Kev: Who can raise their hand and tell me what all of those have in common? One half, two fourths, and three sixths... [silence]

Kev: If I go back to my original equation, okay? Do I see any similarity? [silence]

Kev: What's the connection? [silence]

Kev: Why did we take the time to plot all of these points, tell the directions and then simplify it down? [silence]

Kev: Why would we do that? [student response]

In this example, high cognitive demand is maintained because Kev is attempting to get students to make the connection between equivalent fractions, ordered pairs, and the slope of a line. Although students don't pick up on the prompt right away, he continues to give them time to process the question by asking it numerous times by rewording the question without it ever losing its meaning or high level of cognitive demand.

In addition to Kev's prompts that fell under Enlightened Details, Jan and Lacy also maintained cognitive demand through this prompt type by posing prompts that asked students to describe what something means or how to do something. For example, Jan asked students to explain what it *means* for two things to have a positive correlation, instead of just asking if the two characteristics were positively correlated. Prompts that ask students to explain a concept's mathematical meaning push students' understanding to a deeper level than prompts that focus on procedural skills or ideas.

Prompts that were categorized as Apply to Similar Problems also led to maintained high cognitive demand across all teachers, although Kev only used this prompt type in a cognitively demanding way one time. In general, teachers tended to create more than one question that applied to other problems within a given situation. Most of the mathematical situations involved students interpreting meaning from a real-life context. Prompts from Jan illustrated how stairs are used as a context for exploring slope. For example, while the class was exploring the relationship of rise, run, and steepness, within four different prompting sequences, Jan asked students for two different ways you could make stairs steeper, and two different ways you could make stairs less steep. These prompts show the teacher trying to identify how well students understand the concept of slope by approaching it from many different angles.

Lacy also used the Apply to Similar Problems prompt type multiple times within one situation when interpreting correlations between grip strength and arm strength. Although the original problem only asked students to interpret the correlation between the two variables, and to explain the relationship in words, Lacy asked numerous questions about interpreting the meaning of various kinds of correlations between these two variables to make sure students could interpret any kind of correlation. Noticeably, both Lacy and Jan used prompts that were embedded within a real world scenario that required students to make sense of the situation using their mathematical understanding.

Between Enlighten Details and Apply to Similar Problems, there were also noticeable trends in ways they maintained high cognitive demand. Many of the prompts in both categories asked students to apply prior knowledge to new mathematical notation, vocabulary, or procedures. Another commonality was that these prompts requested students to interpret meaning from problems, such as understanding what the y-intercept represented. Many of the prompts were centered on mathematical ideas that were very general, but asked students to use those ideas to solve or consider other problems. These prompts gave students opportunities to make sense of the mathematical concepts they were learning about instead of staying focused on more minute procedural details.

2) Raised Cognitive Demand

As shown in Table 5, prompts that raised cognitive demand from lower levels (1 and 2) to higher levels (3 and 4) did not have any specific patterns in the prompt type because the few prompts that achieved this were scattered across multiple prompt types. In general, prompts that raised cognitive demand started out as a procedural or concrete idea and then changed into a prompt that asked students to interpret a situation or understand a mathematical concept. For example, when Kev was introducing the concept of a line on a coordinate grid, the prompts increased from concrete to conceptual:

Kev: Do you see any (points on the line)? [silence]

Kev: We don't have any points, like actual dots on there, but does that mean that there aren't any points on it? [student response]

Here Kev begins by discussing the physical line in a mathematical problem, but connects this idea to the conceptual understanding of a line representing an infinite set of points along a straight path in both directions. Similarly to prompts that maintained high cognitive demand, these prompts took the time to discuss the concepts on a more general level.

Another way teachers were able to raise cognitive demand was through the inclusion of students' thoughts or ideas, such as asking what they thought or how to solve a problem. When more than one method or idea was presented, students could then draw connections between ideas or compare and contrast various strategies. These prompts did not show up very often in our coded data, since only moments that led to student silence were coded, however, there were multiple instances within the videos where the inclusion of various student methods raised the cognitive demand from a procedural level to a higher analytical level of student discussion.

Generally, prompts that were successful in raising or maintaining high levels of cognitive demand after student silence were embedded within a real-life context and included questions that were aimed at understanding mathematical concepts. If prompts started as procedural, they were able to raise the cognitive demand by asking questions about the meaning of mathematical ideas. Similarly, when students were asked to make connections between previous ideas and newer concepts, they were given the opportunity to engage in more in depth mathematical thought.

V. DISCUSSION

The purpose of this study was to gain a deeper insight into the ways in which teachers use prompting to reengage students in high-level cognitive thought and discussion. Through this analysis, we are able to list implications on how teachers can use prompting techniques to maintain high levels of cognitive demand through student silence. Our results correspond with previous research by Stein et al. (1996) that mathematics needs to be taught as something to be understood, rather than memorized, if students are to be engaged in high levels of cognitive thought. Prompts that communicated this value tended to be high level prompts that focused students' attention on the meaning of the concepts at hand. Two teachers in this study, Jan and Kev continued to rephrase or reform questions until students answered them, to prove to students that they are able to understand the mathematics if they think deeply enough about the concepts. However, during these moments, Jan tended to maintain high levels of cognitive demand, while Kev dropped the cognitive demand to enable student answers. Evidence from Kev's interviews lead us to believe that his drops in cognitive demand stem from his belief that students need to "feel successful" in mathematics in order to remain engaged. This habit of reducing cognitive demand to ensure student success reduces student opportunities to think deeply about mathematics, and could also be an explanation for Kev's low numbers of high level prompts, even though his students were engaged in discussion for a higher percentage of class time than the other two teachers. It is important for teachers to believe in the mathematical ability of their students (Franke et al., 2009). Without this belief, students might be deprived of cognitively demanding discussion prompts.

Through analysis of the data gathered from these three middle school teachers we come away with six implications for teacher education and development: 1) Ask for justifications; 2)

Take time to focus on meaning; 3) Refer to conceptual or real-life concepts when discussing procedures; 4) Wording is Important; 5) Discuss concepts in general; and 6) Be prepared with high level prompts. These results correspond with previous research, but add in detail to how teachers can develop more meaningful classroom discussions, with or without student silence.

1) Ask for Justifications

The justification prompt was used only two times in moments of student silence throughout the eighteen recorded lessons, both of which maintained high levels of cognitive demand. Stein et al. (2008) discuss the crucial role of teachers in letting students explore justifications and explanations, while they make sense of mathematical ideas. This prompt type is certainly an untapped resource for the teachers in this study. Teachers can create more opportunities for justification prompts by having students explain their own ideas and strategies and back them up with evidence. Asking a student "why" a mathematical idea is true or not is a quick and easy way to engage students in discussion that is focused on mathematical meaning.

2) Take Time to Focus on Meaning

Yackel et al. (1998) discussed the idea of "funneling" where teachers walk students through a task step-by-step, reducing the cognitive load. Prompts under the closed-progress-details category were very similar to funneling prompts and almost never lead to high cognitive demand prompts. Instead of reducing the cognitive demand by "taking out" the difficult pieces (Henningsen & Stein, 1997; Yackel et al., 1998) teachers can use focusing actions to "put in" more difficult details or complexities to test students' depth of understanding, while also helping them build a sound understanding of the concepts involved in a task. Focusing actions led to higher cognitive demand more often than progressing actions and prompted students to take time to focus on the meaning of various concepts involved in the problem. So often teachers feel the need to progress through a problem in a timely manner in order to stay on schedule, however, the importance of the educational opportunities that are afforded by focusing actions should not be overlooked.

3) Refer to Conceptual or Real Life Concepts When Discussing Procedures

Similar to the study by Stigler and Hiebert (2004), our data showed that teachers' tended to generate low levels of cognitive demand when they turned tasks into procedural exercises. However, there were numerous instances where teachers in this study were able to enrich a procedural task by adding a quick conceptual or real life context to reference during the procedure. This was a regular occurrence in Jan's classroom; once using a scale to represent the conceptual idea behind solving an equation for an unknown, another time using pizzas to discuss adding fractions with unlike denominators, and also using stairs to discuss slope and constant rate of change. Although Jan's lessons were often focused on lectured procedures, she was able to generate opportunities for higher levels of cognitive thought and discussion by using language that tied procedures to more conceptual meaning. Using conceptual language during procedural tasks helps students make connections and strengthen their level of understanding.

4) Wording is Important

Similar to the last implication, the use of language that focuses on conceptual meaning instead of procedures seems to be an important aspect of leading a class discussion. However, even the slightest changes in wording can increase the cognitive demand of a prompt. For example, within one lesson Lacy discussed adding fractions in both a procedural and conceptual way. The procedural prompt discussed how they "added our tops" and "kept the bottom". This prompt requests that students remember a procedure without making sense of the mathematical meaning behind what is happening when two fractions are added. The conceptual prompt, which was also more cognitively demanding, asked students what they would get if they had "seven out of eight and take three of them away". Although the problem could have been exactly the same for these two prompts, the conceptual prompt helps students build a more solid understanding of why we "add the tops and keep the bottom". If teachers use more conceptual language, even during procedural tasks, students may begin to see mathematics as what it is, an intricate set of patterns that make sense and build on one another.

5) Discuss Concepts in General

Many of the moments where teachers raised cognitive demand from low to high happened when teachers stepped back from the problem to discuss more conceptual or general patterns. One example of this was the previously mentioned moment when Kev discussed a line consisting of an infinite amount of points. A deep understanding of these simpler concepts is needed in order for students to build a strong understanding of more complex concepts such as the slope of a line. Asking students to notice or think about patterns after more procedural work will ensure that students are being given the opportunity to make these connections, and see mathematics as something to be understood. In order to do this, teachers must be given opportunities to develop an understanding of what big ideas the mathematical ideas are connected to, and in turn, they must know how to help students make these connections themselves.

6) Be Prepared with High Level Prompts

Maintenance of high level cognitive demand was not uncommon among the three teachers, but raising cognitive demand from low to high levels through student silence was uncommon. This makes sense, since a teacher would not intuitively raise the demand of a question where students were already having difficulty. Every prompt that raised cognitive demand from low to high during student silence started out as a procedural question, which was answered with student silence, and then shifted to a conceptual question. Bringing in conceptual aspects to a question allowed students to view the problem from a different perspective and ultimately respond to the prompt. Teachers should become more aware of the benefits that conceptual prompts have on enabling student understanding. However, since raising the cognitive demand of prompts is not common, teachers need to have high level prompts at the ready when preparing to teach a lesson. If teachers do not start with high level prompts, there is less of a chance that students will be engaged in meaningful conversation. Pre-service teacher education and professional development should focus on writing cognitively demanding prompts in preparation for classroom discussions about mathematical tasks.

IV. CONCLUSIONS

The complex nature of the coding scheme required a large amount of time for coding, which limited the sample size of the study. The use of three teachers allows us to gain insights on some practices that are happening within classrooms, but a larger sample size would allow us to see how common different prompting practices are across various classrooms. Concurrently, it would be beneficial to get access to students work to see the connection between teachers' intended level of cognitive demand and students enacted level of cognitive demand.

Another limitation to this study was the coding scheme used for cognitive demand (Stein et al., 1996). The coding scheme was designed for use on mathematical tasks, and while the majority of the prompts fit within one of the four given categories, there were numerous prompts that dealt with the development of vocabulary that were difficult to code. Many of Kev's low level prompts stem from making connections from one category to another, while helping students understand vocabulary in a different way. Vocabulary development is very important in mathematics, but it often involves recall of terms. Connecting vocabulary to conceptual ideas seems to be a higher level skill, but since it is not a procedure, or a higher analytical skill, these prompts often ended up being placed at the lowest level of cognitive demand. Future research on a more appropriate coding scheme for cognitive demand of teacher prompts would allow for more precise research that could help teachers lead more meaningful discussions with their students.

Another area for research would be the connection between conceptually based tasks and higher cognitively demanding questions. In this study, higher level prompts seemed to be connected to more conceptually based tasks. It would be useful to research whether these conceptual tasks are needed, or if teachers are able lead cognitively demanding discussions about procedural tasks as well.

Through analyzing the data in this study, we have gained insights into ways that teachers can develop cognitively demanding and meaningful mathematical discussions that can be sustained through student silence. Although reform efforts call for major changes in the methods that teachers use to teach mathematics (CCSSI, 2010), we found that there are small changes teachers can make in their prompting patterns to shift student thinking to higher levels. These small changes include asking students to justify their answers by asking why a method works, or why their solution makes sense; tying procedural work to conceptual tasks, examples, or discussions; using vocabulary that highlights mathematical meaning; and taking time to discuss concepts or ideas in general. Through development of these skills preservice and veteran teachers can learn to lead more cognitively engaging discussions, which should in turn lead to deeper levels of students' mathematical understanding (Franke et al., 2007).

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